Absolute measurements of the fine structure constant (α)

François Nez

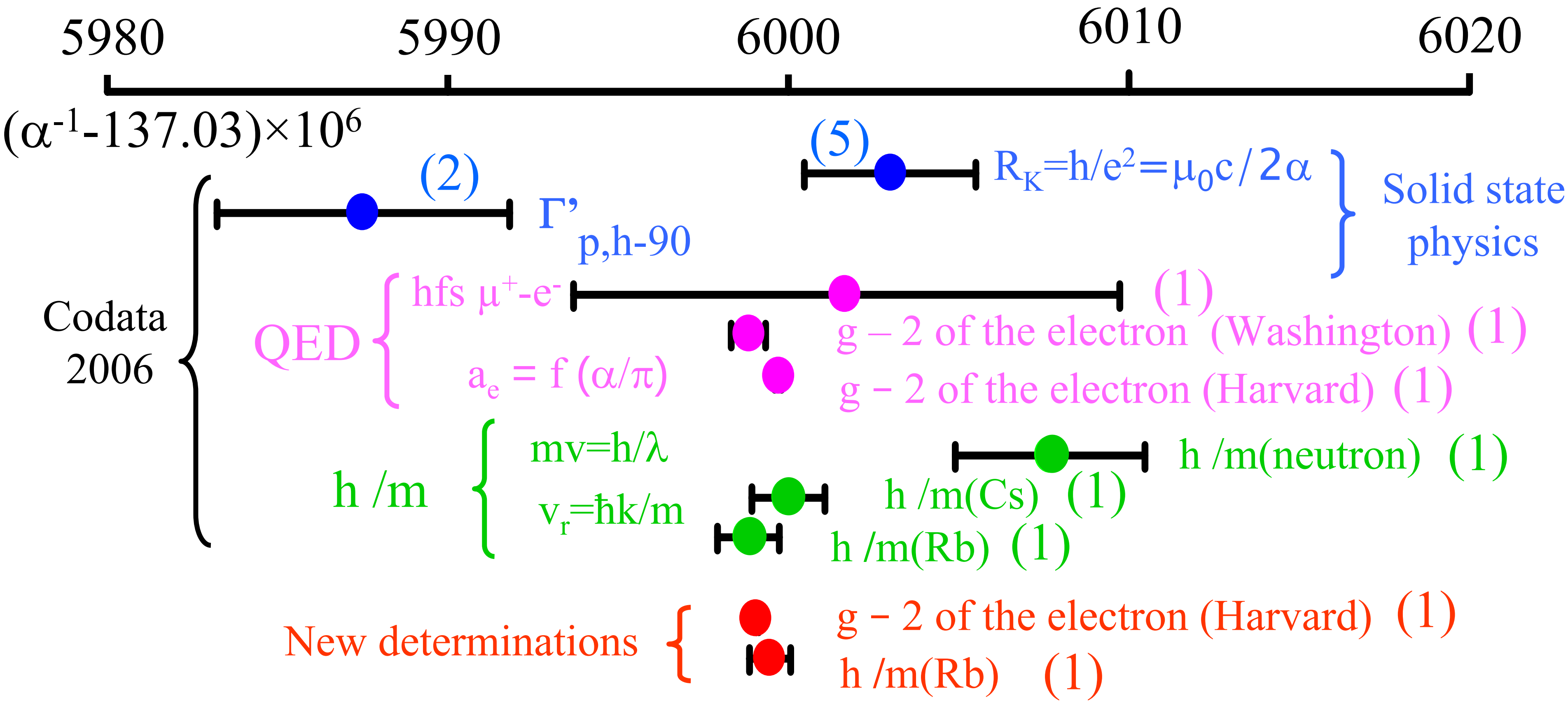
Laboratoire Kastler Brossel
Physique quantique et applications

Determinations of the fine structure constant (α)

(physics.nist.gov/constants)
(www.bipm.org/extra/codata/)

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

- dimension less
- scale electromagnetic interaction (α @ q =0)

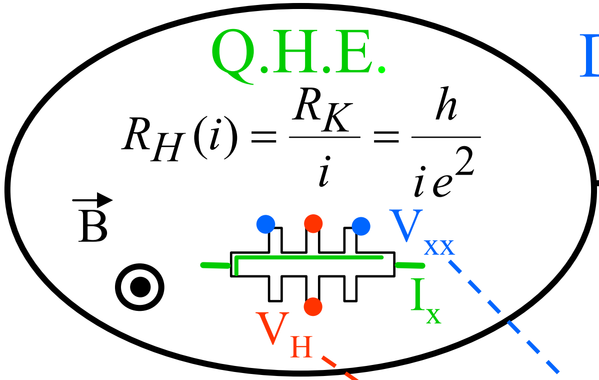


Codata 2006, P. Mohr, B. Taylor and D. Newell, Rev. Mod. Phys. 80, n°2 April 2008

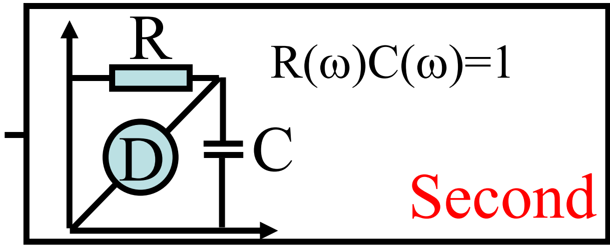
Quantum Hall effect → α (1.8×10^{-8})

Q.H.E.

$$R_H(i) = \frac{R_K}{i} = \frac{h}{i e^2}$$



DC

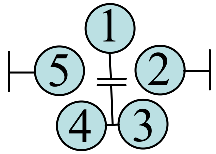


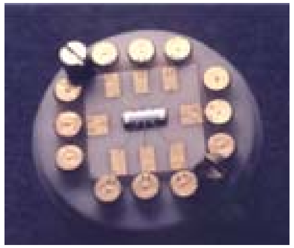
Calculable capacitor

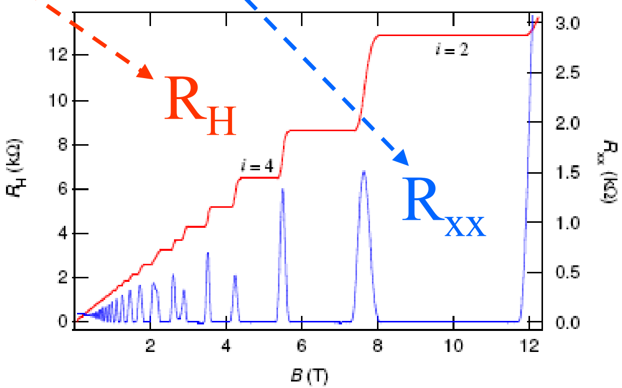
$$\gamma = \frac{\varepsilon_0}{\pi}\ln\frac{2}{\sqrt{5}-1}\,\mathrm{pF/m}$$

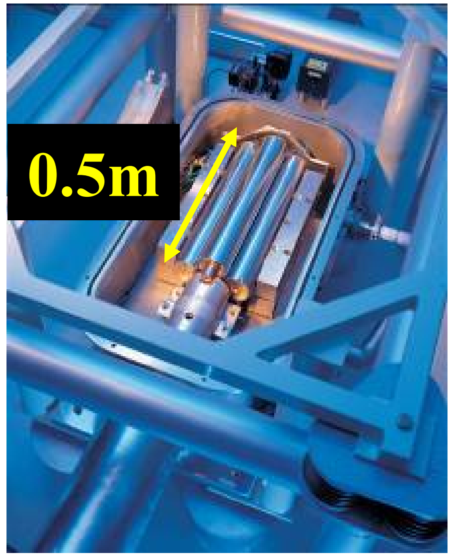
AC

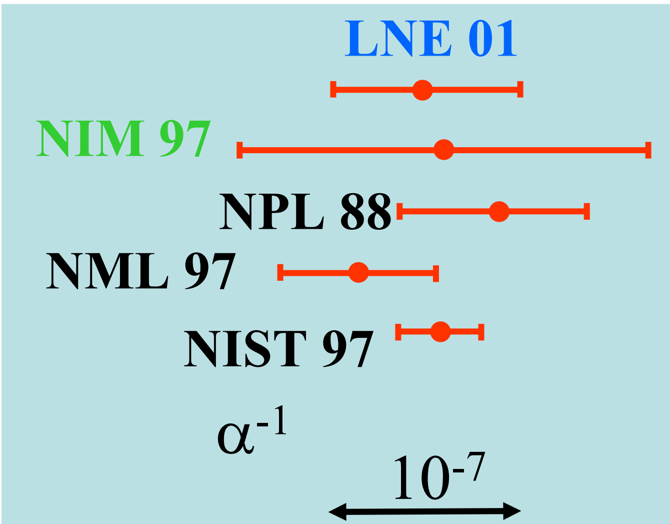
Meter









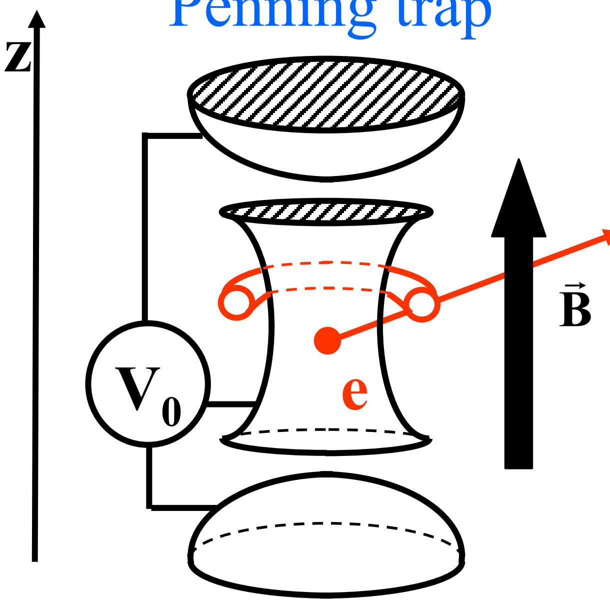


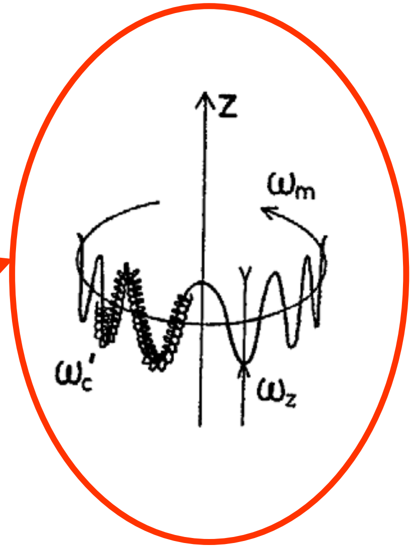
$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

Universality of QHE ? : graphene structure

Electron magnetic moment anomaly $a_e \to \alpha$ (3.7×10^{-10})

Penning trap



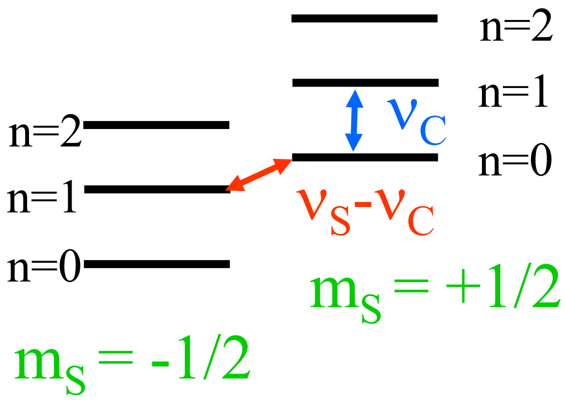


$$\nu_c = eB/2\pi m_e \quad \text{(cyclotron)}$$

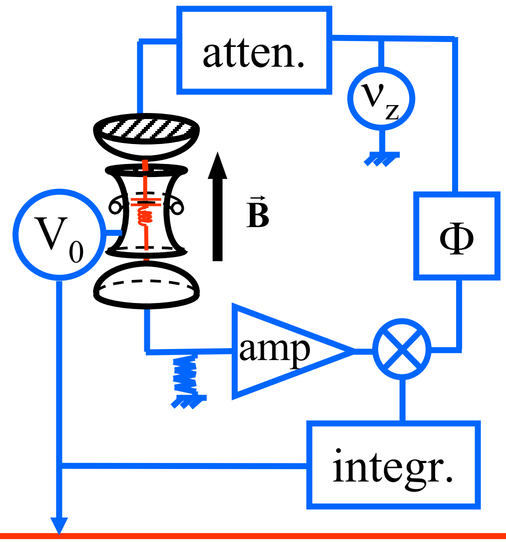
$$\nu_s = g_e \mu_B B/h \quad \text{(spin)}$$

$$\nu_z \approx \nu_{z0} + (n + m + 1/2)\,\delta$$

cyclotron — spin — depend on the trap



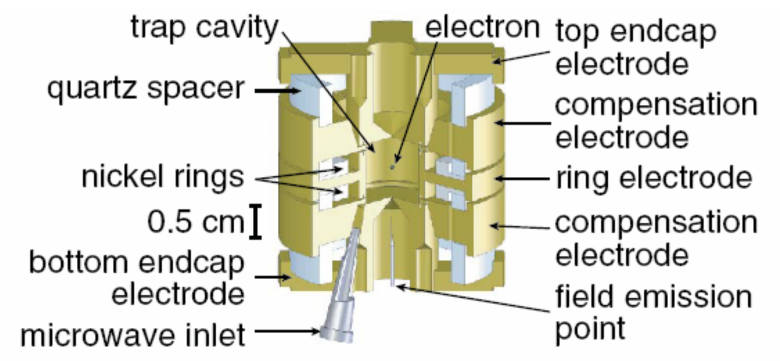
Self electron oscillator



$$a_e = \frac{g_e - 2}{2} = \frac{\nu_s - \nu_c}{\nu_c}$$

error ⇔ frequency shift ⇔ spin state

Electron magnetic moment anomaly a_e



Phys. Rev. Lett. 97, 030801 (2006)

- Cylindrical Penning trap
  → high Q cavity (no spontaneous emission)
  → calculable cavity : experimental tests of frequency shift
- Trap @ 100mK → no blackbody radiation
- quantum jump spectroscopy
- …. etc = 20 years improvements

$$a_e = \frac{g_e - 2}{2} = \frac{\mu_e}{\mu_B} - 1 = \frac{\nu_s - \nu_c}{\nu_c} = \sum_n C_e^{(2n)} \left(\frac{\alpha}{\pi} \right)^n + a_e(weak) + a_e(had)$$

Q.E.D.

Most accurate determination of α (3.7×10^{-10})

D. Hanneke, S. Fogwell and G.Gabrielse, Phys. Rev. Lett. 100, 120801 (2008)
T.Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. 99, 110406 (2007)

α deduced from the ratio h/M(atom) → α (4.6×10^{-9})

- Rydberg constant in terms of energy : $h\,c\,R_\infty = \frac{1}{2} m_e c^2\,\alpha^2$
- measurement of h/M allows to determine α

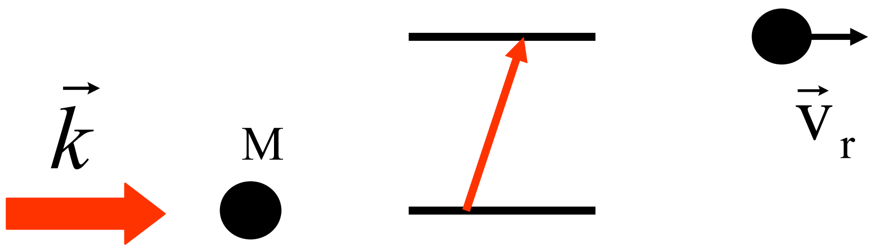
$$\alpha^2 = \frac{2\,R_\infty}{c} \times \frac{A_r(^{87}Rb)}{A_r(e)} \times \frac{h}{M(^{87}Rb)}$$

Relative uncertainty :
- Rydberg constant : 7×10^{-12}
- relative atomic mass : 2×10^{-10}
- relative atomic mass of the electron : 4.4×10^{-10}

h/m is given by the recoil effect

The recoil velocity is directly related to the h/M ratio

J.L. Hall, Ch.J. Bordé, K. Uehara: Phys.Rev.Lett ***37****,1339 (1976)*



$$\vec{v}_r = \frac{\hbar \vec{k}}{M}$$

and can be measured very precisely in terms of frequency

Experiments : Cs atoms (Stanford) Rb atoms (Paris)

Principle of our experiment

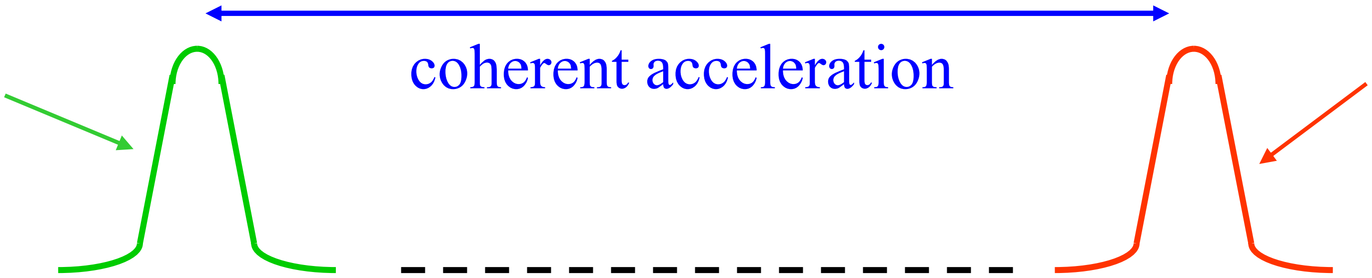
selection
(Raman transition
or Ramsey fringes)

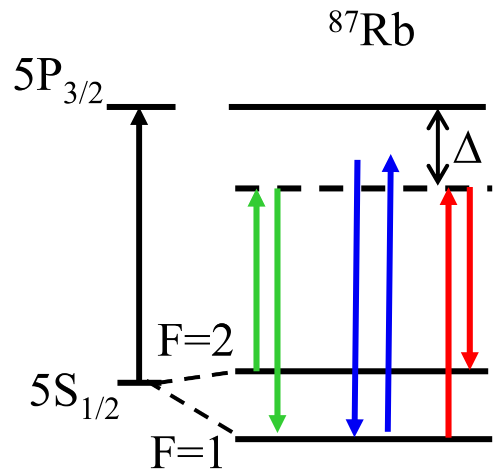
$N \times 2\hbar k$

measurement
(Raman transition
or Ramsey fringes)

coherent acceleration

Magneto-
Optic
Trap
+
molasses

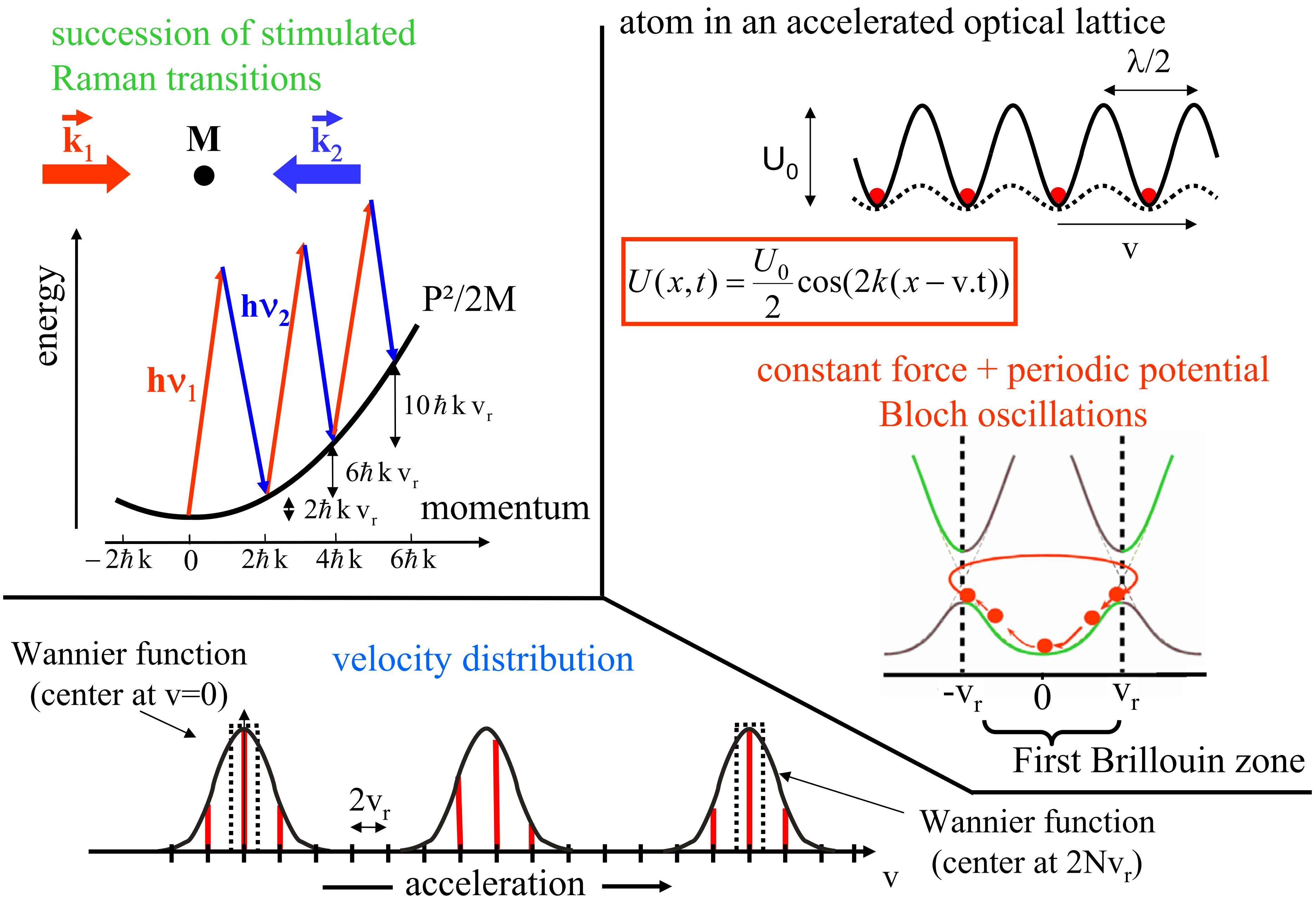




- selection of an initial sub-recoil velocity class
- coherent acceleration : N Bloch oscillations,
  $2N\hbar k$ momentum transfer
- measurement of the final velocity class (σ_v)

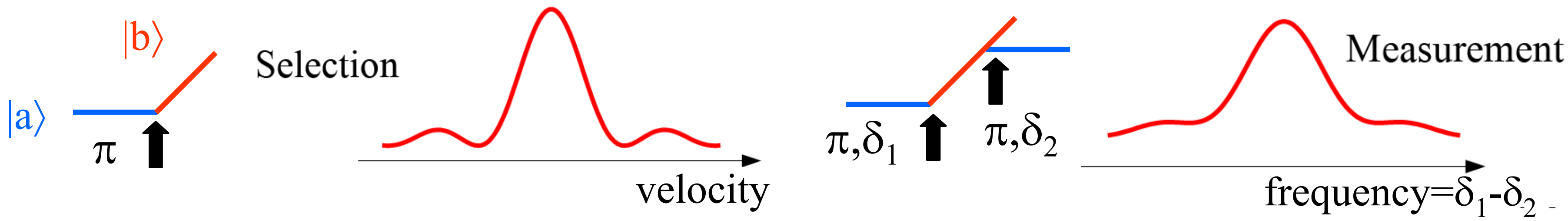
$$\sigma_{v_R} = \frac{\sigma_v}{2N}$$

Bloch oscillations : η ~ 99.97 % per recoil

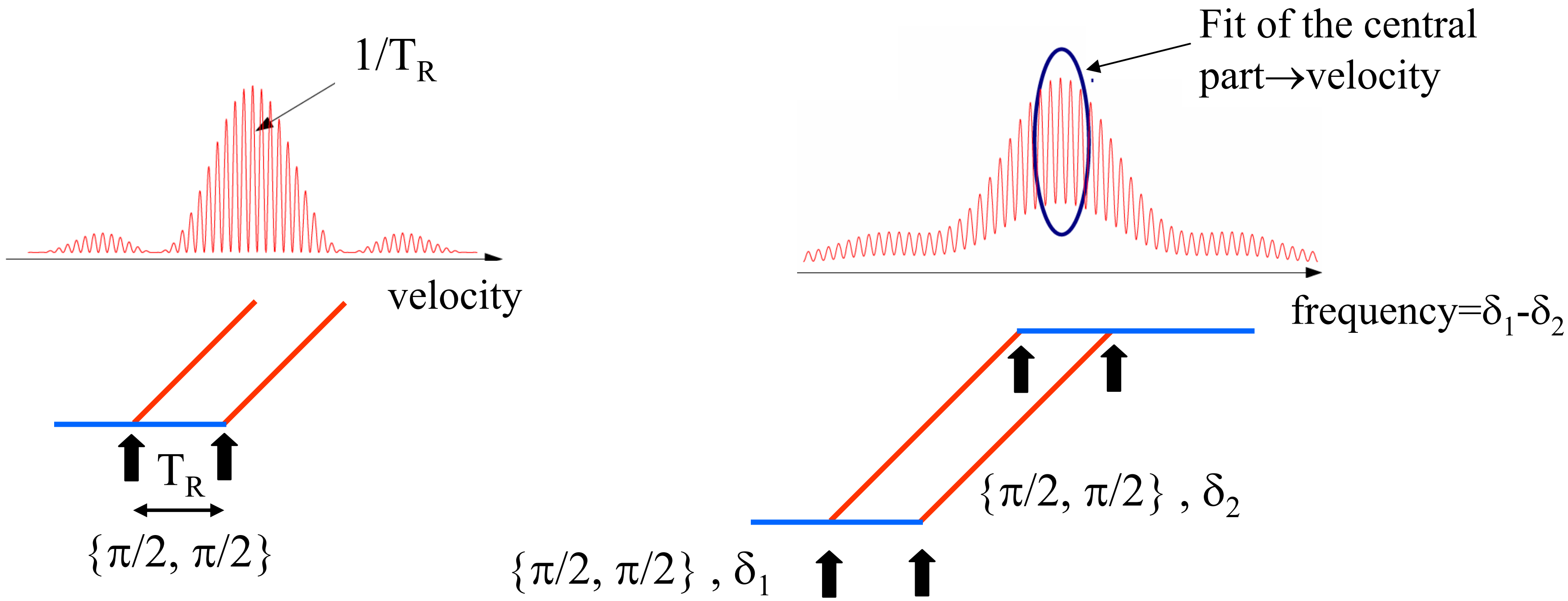


Velocity sensor based on Raman transition

π–π configuration : selection of subrecoil velocity distribution (Doppler effect) and measurement of the final velocity distribution

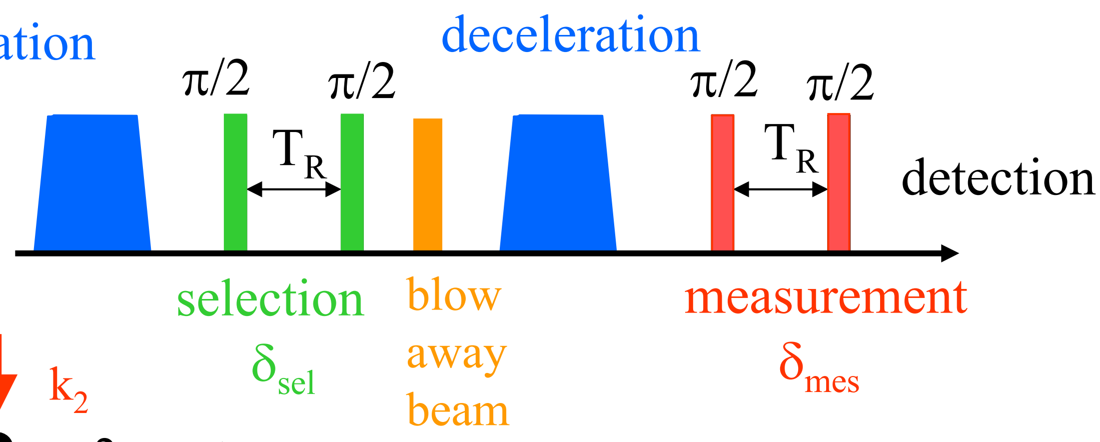


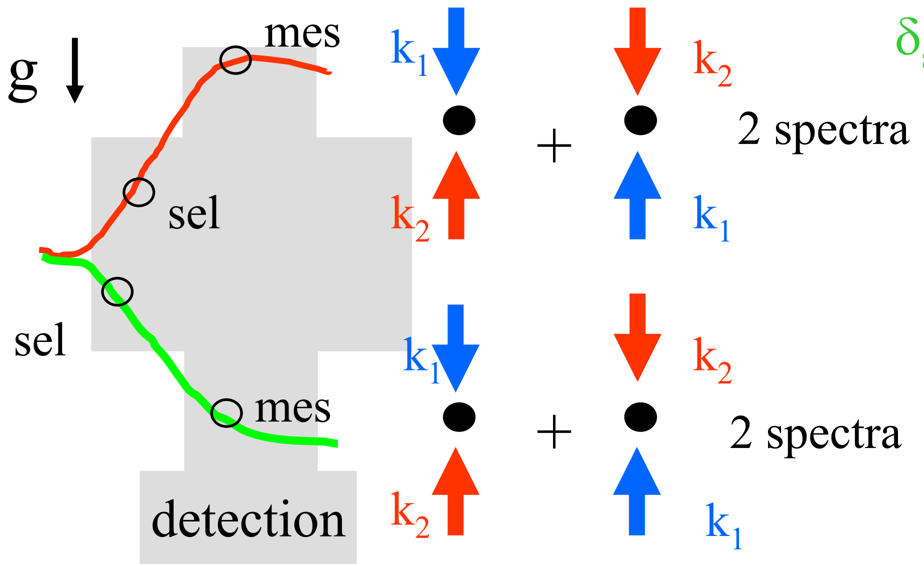
{π/2, π/2} – {π/2, π/2} configuration (interferometer) : selection and measurement of a velocity comb → higher resolution with the same number of atoms



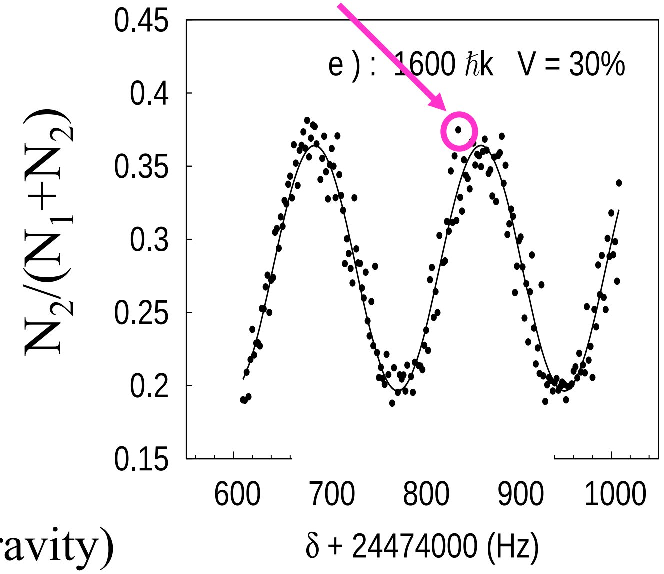
Combining Bloch oscillations with interferometry

temporal sequence :





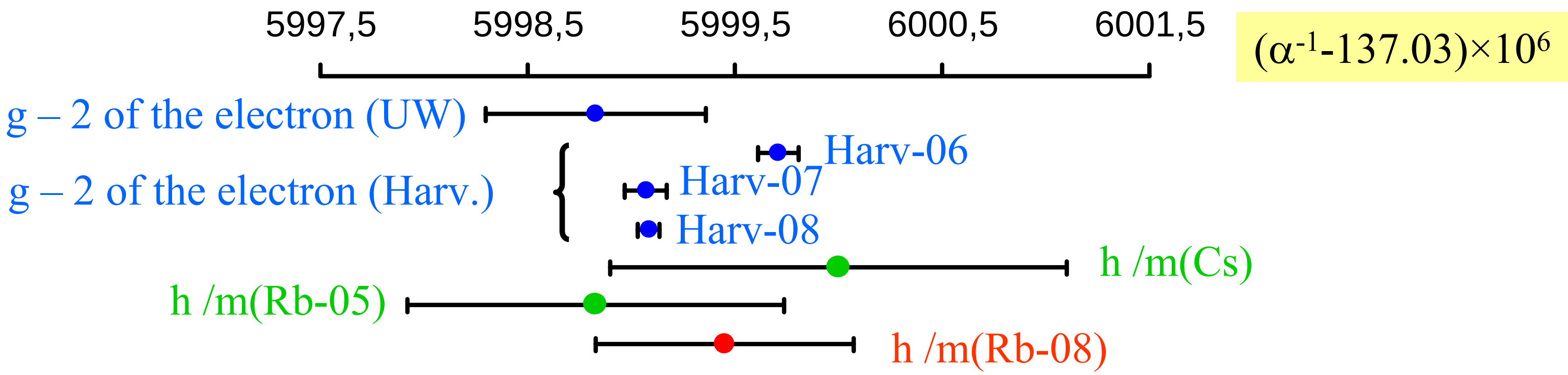
1 dot= 1 temporal sequence (δ_{sel}, δ_{mes})



4 spectra :

- upwards and downwards accelerations ("cancellation" of gravity)
- 2 directions of Raman beams (compensation of levels shift)

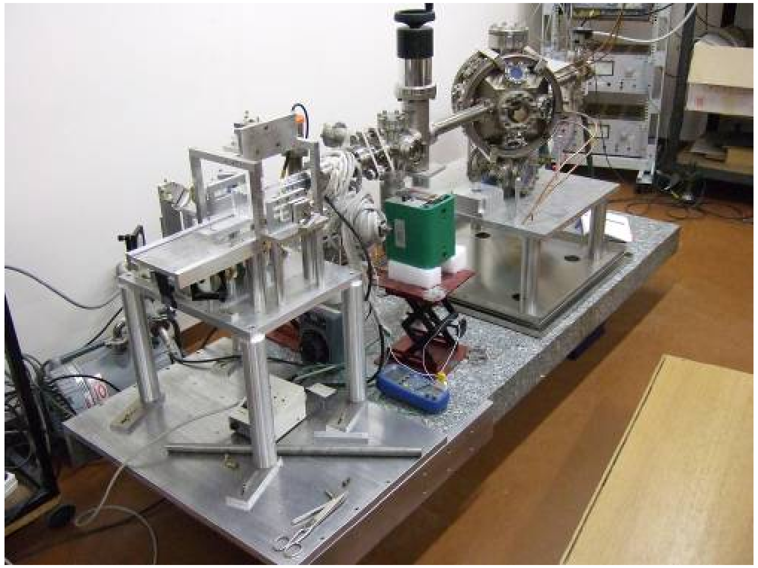
Conclusion and prospects



✓ The interferometric measurement is in good agreement with our previous measurement : it is a good test of the atomic interferometry

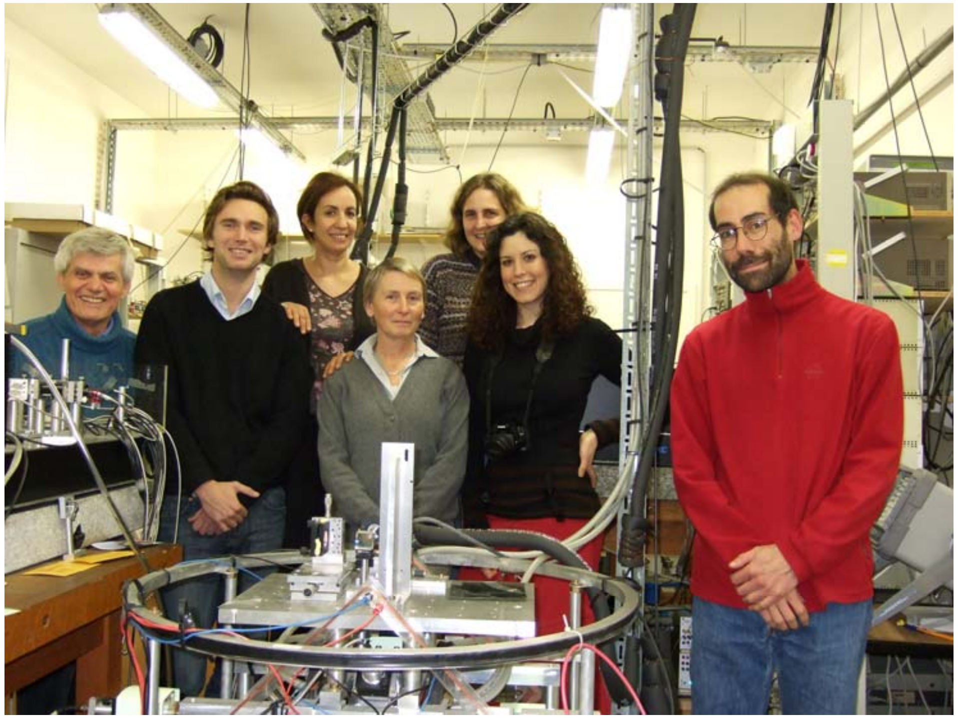
✓ Our goal is now to obtain a relative uncertainty of 1×10^{-9}. We are building a new experimental set-up.

✓ The comparison with the electron anomaly will be a test of the QED or, if we suppose the QED calculation exact, a test of a possible new physics (structure of electron, super-symmetry…).



h/m(Rb-O8) : M. Cadoret et al : ArXiv : 0810.3152 (accepted in PRL)

Laboratoire Kastler Brossel (UPMC, ENS, CNRS)



2007

2008

Ph.D Students :

R. Battesti (2003), P. Cladé (2005), M. Cadoret (2008)

Postdoc

E. De Mirandes (2007)

Permanents :

P. Cladé, S. Guellati-Khélifa,

C. Schwob (2007), F. Nez, L. Julien,

F. Biraben

IFRAF

UPMC PARISUNIVERSITAS

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Cnam

Electron substructure ?

Ref : Gabrielse et al Phys. Rev. Lett. 97, 030802 (2006)

$$a_e(theo) = \frac{g_e - 2}{2} = \sum_n C_e^{(2n)} \left(\frac{\alpha}{\pi}\right)^n + a_e(weak) + a_e(had)$$

independent value of α   OK

If $a_e(theo) \neq a_e(exp)$ and expression of a_e exact (QED) then correction δa

Electron with constituents (m*>>m)

- if $\delta a \approx \frac{m}{m^*}$ then m* > 34000TeV/c² and R < 6×10^{-24} m !!!
- if $\delta a \approx \left(\frac{m}{m^*}\right)^2$ then m* > 130GeV/c² and R < 1×10^{-18} α limited

Independent value of α @ 7×10^{-9} then m* > 600GeV/c² (not competitive with LEP)

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA

Metrologia 42 (2005) 71–80 doi:10.1088/0026-1394/42/2/001

Redefinition of the kilogram: a decision whose time has come

Ian M Mills[1], Peter J Mohr[2], Terry J Quinn[3], Barry N Taylor[2] and Edwin R Williams[2]

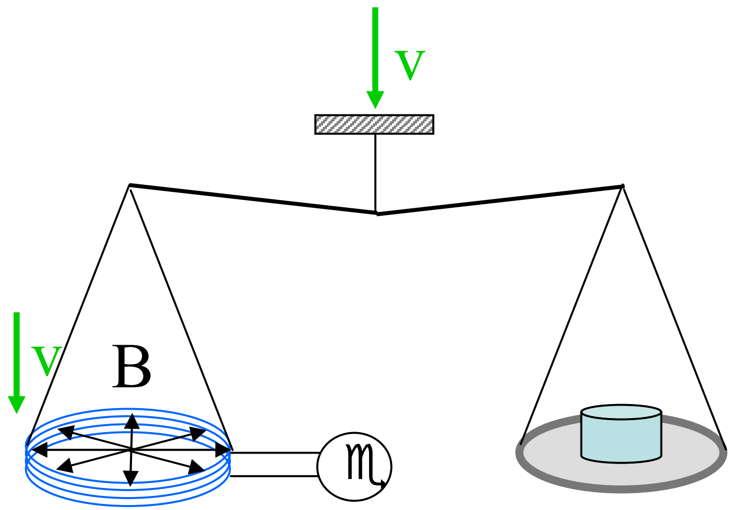
« Mechanical power »

=

« Electrical power »

$$m\,g\,v = U\,I = \frac{U^2}{R}$$

Watt balance



$$m\,g\,v = \frac{A}{K_J^2 R_K} = \frac{h}{4} \cdot A$$

U : Josephson effect : $K_J = 2\,e/h$

R : Quantum Hall effect : $R_K = h/e^2$

$$h_{WB}(3.6\times10^{-8}) \neq h_{Si\text{-}sphere}(2.9\times10^{-7}) \quad @10^{-6}$$

Comparison with others determinations (Codata 98-02, Codata 06, news)

a_e (Harvard)	137,035 999 084 (51)	QED, exp. ok	$0{,}37\times10^{-9}$
a_e (Harvard)	137,035 999 711 (96)		$0{,}7\times10^{-9}$
a_e (UW)	137,035 998 83 (52)	QED	$3{,}8\times10^{-9}$
a_e (UW)	137,035 998 58 (52)		$3{,}8\times10^{-9}$
h/m(Rb)	137,035 999 45 (91)		$4{,}6\times10^{-9}$
h/m(Rb)	137,035 998 83 (91)		$6{,}7\times10^{-9}$
h/m(Cs)	137,036 000 0 (11)		$7{,}7\times10^{-9}$
R_K	137,036 003 0 (25)		$1{,}8\times10^{-8}$
Γ'_{90}	137,035 987 5 (43)		$3{,}1\times10^{-8}$
h/m(neutron)	137,036 007 7 (28)		$3{,}4\times10^{-8}$
h/m(neutron)	137,036 001 5 (47)	d_{220}	$3{,}4\times10^{-8}$
h/m(neutron)	137,036 008 4 (33)	d_{220}	$3{,}4\times10^{-8}$
$\Delta\nu_{Mu}$	137,036 001 7 (80)	QED	$5{,}8\times10^{-8}$
$\Delta\nu_{Mu}$	137,035 995 2 (80)		$5{,}7\times10^{-8}$

h/M$_{neutron}$ → fine structure constant α

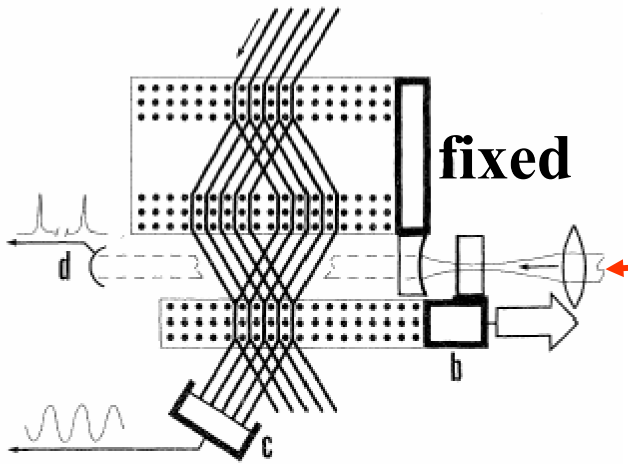
neutron

$$p = m\,v = \frac{h}{\lambda}$$

Time of flight (→ $m\,v$)

Bragg diffraction → d_{220} (→ λ)

$$\frac{h}{m_n d_{220}(W04)} \doteq \frac{c\,A_r(e)\alpha^2}{2\,R_\infty\,A_r(n)\,d_{220}(W04)}$$



fixed

HeNe

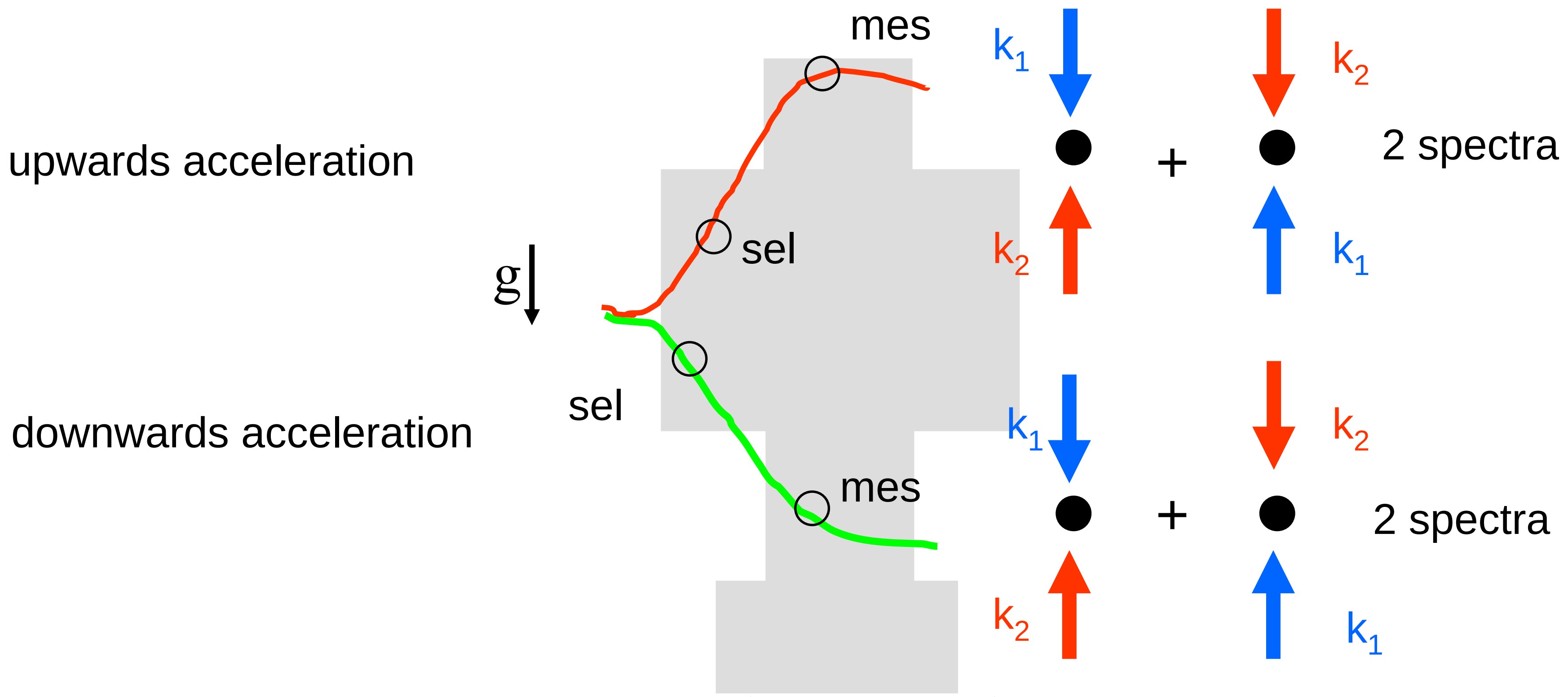
Deslattes et al,
PRL 31, p.972 (1973)

$$d_{220} = \left(\frac{m_{optical}}{n_{Xrays}}\right)\lambda_{HeNe}$$

Moving : 85 µm
$d_{220} \approx 0{,}192$ nm
$\lambda_{HeNe} \approx 633$ nm
$\lambda_X \approx 0{,}07$ nm

$$V_m(Si) \doteq \frac{\sqrt{2}\,c\,M_u\,A_r(e)\alpha^2 d_{220}^3}{R_\infty\,h}$$

Measurement of the recoil velocity

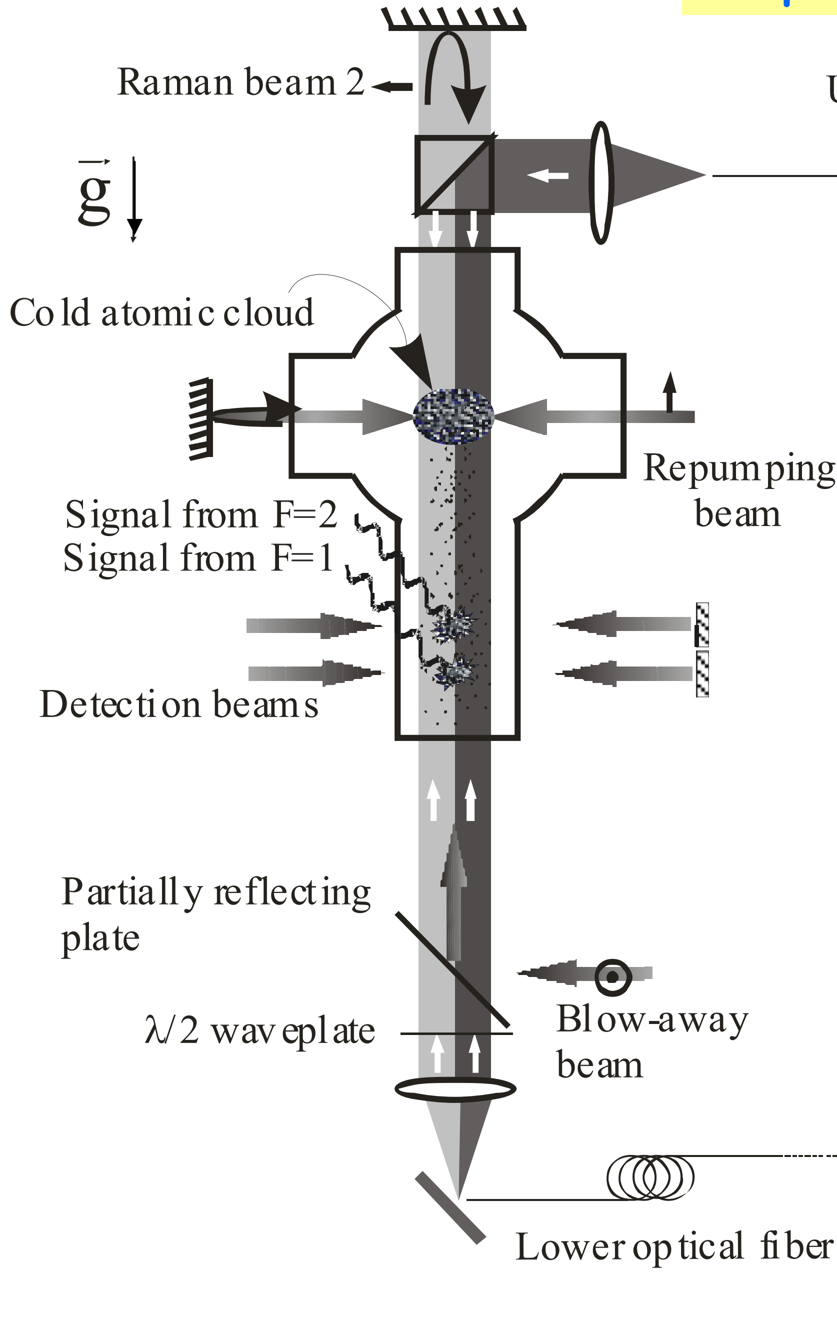


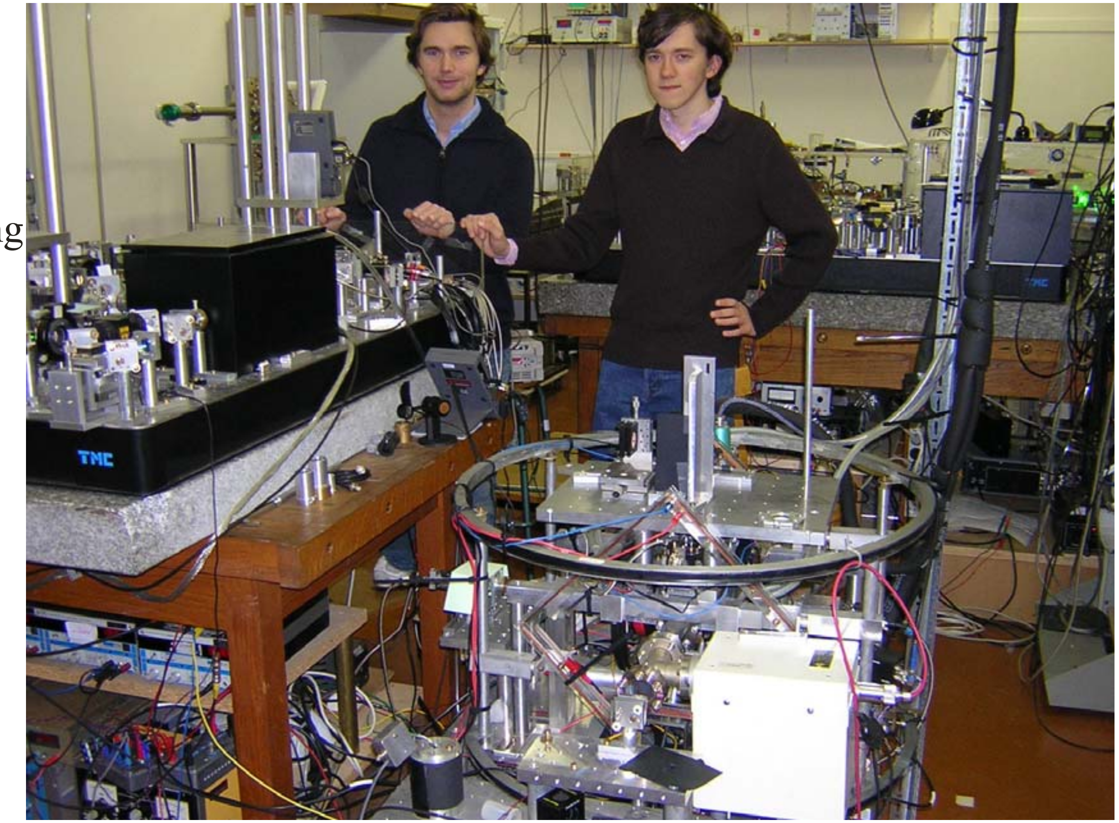
We measure (Doppler effect) : $\Delta V = \dfrac{\hbar(\delta_{sel} - \delta_{meas})}{(k_1 + k_2)}$ with $\Delta V = \mathrm{Avg}(\Delta V_{1,2}, \Delta V_{2,1})$

Acceleration in both opposite directions : $v_r = \dfrac{\Delta V^{up} - \Delta V^{down}}{2(N^{up} + N^{down})}$ (no contribution of g)

$$v_r = \frac{\hbar k_B}{m} \Rightarrow \boxed{\frac{\hbar}{m} = \frac{(\delta_{sel} - \delta_{meas})^{up} - (\delta_{sel} - \delta_{meas})^{down}}{2(N^{up} + N^{down})(k_1 + k_2)k_B}}$$

Experimental set up





Error budget (I)

Source	2005 results Correction $(\alpha^{-1})(\times 10^{-9})$	2005 results Uncertainty $(\times 10^{-9})$	2007-2008 results Correction $(\alpha^{-1})(\times 10^{-9})$	2007-2008 results Uncertainty $(\times 10^{-9})$
✓Laser frequencies	0	0.8	0	0.4
✓Beams alignment	- 2	2	-2	2
✓Wave front curvature and Gouy phase	- 8.2	4	-11.9	2.5
✓2nd order Zeeman effect	6.6	2	7.0	1
✓Light shift (one photon)	0	0.2	0	0.1
✓Light shift (two photon)	- 0.5	0.2	0	0.01
✓Gravity gradient	- 0.18	0.02	-0.18	0.02
✓Quadratic magnetic force	- 1.3	0.4	-1.45	0.2

Error budget (II)

Source	2005 results Correction (α^{-1})($\times 10^{-9}$)	2005 results Uncertainty ($\times 10^{-9}$)	2007-2008 results Correction (α^{-1})($\times 10^{-9}$)	2007-2008 results Uncertainty ($\times 10^{-9}$)
✓**Light shift (Bloch oscillations)**	**0.46**	**0.2**	**0,58**	**0,1**
✓**Index of refraction (cold atomic cloud)**	**<0.1**	**0.3**	**<0.1**	**0.3**
✓**Index of refraction (background vapor)**	**-0.37**	**0.3**	**-0.41**	**0.3**
Global systematic effects	**- 5.49**	**5.0**	**-8.36**	**3.4**
Statistical uncertainty		**4.4**		**3.0**
TOTAL		**6.7**		**4.6**

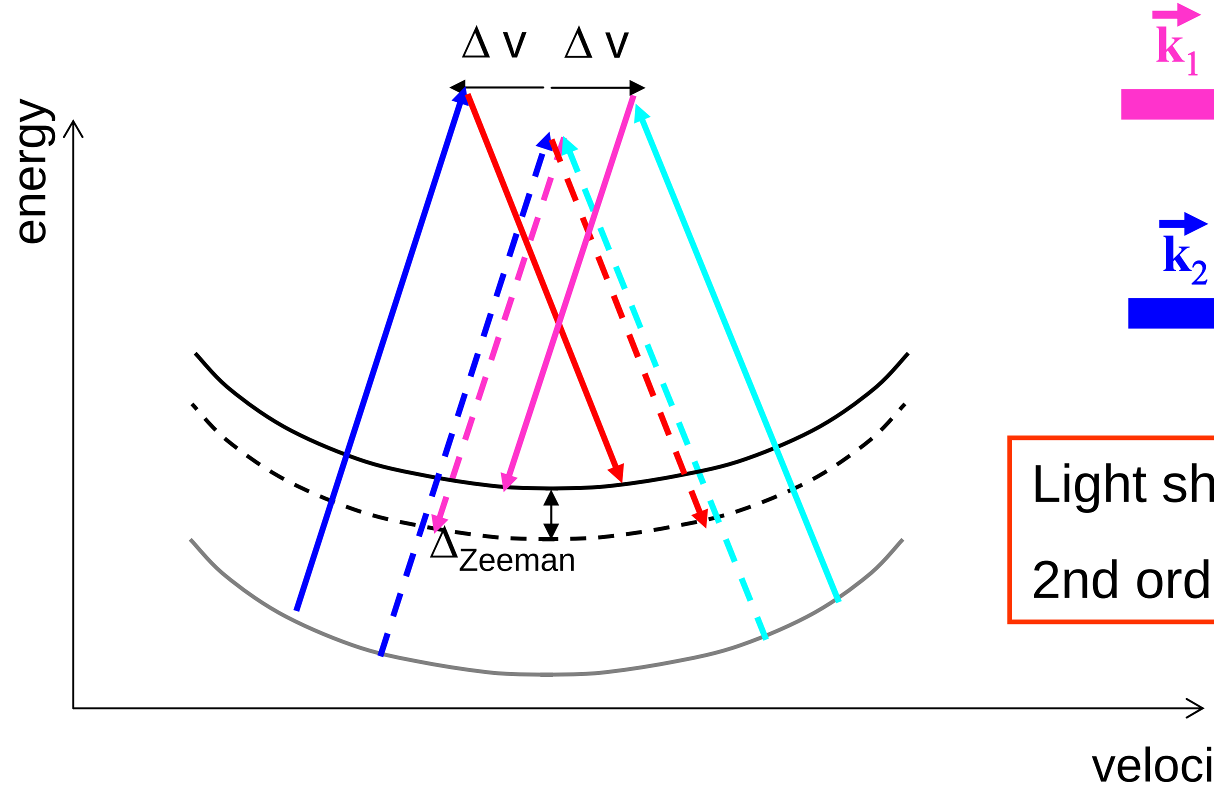
$\alpha^{-1} = \mathbf{137.035\,998\,84\,(91)}$

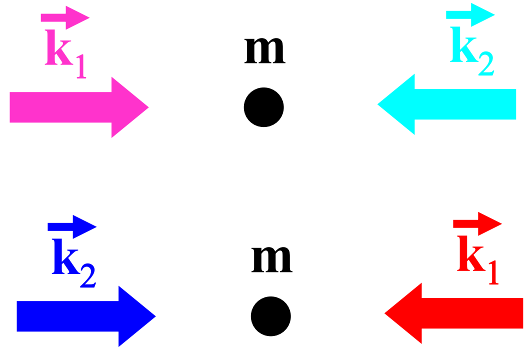
*P. Cladé et al, Phys. Rev. A **74**, 052109 (2006)*

New result

$\alpha^{-1} = \mathbf{137.035\,999\,45\,(62)}$

Reduction of levels shifts



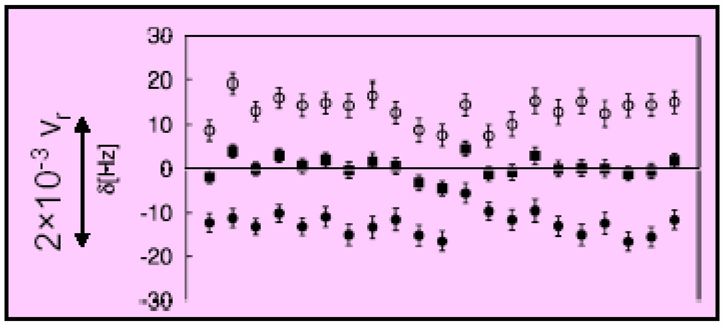


Light shifts

2nd order Zeeman effect

Compensation of levels shifts by exchanging

the Raman beams directions.



Refractive index

Recoil transmitted by one Bloch oscillation : $2\hbar k$ or $2n\hbar k$?
Doppler effect for the Raman transitions : $2kv$ or $2nkv$?

$$(n-1) = \pi \rho \frac{\Gamma}{\Delta}\left(\frac{\lambda}{2\pi}\right)^3$$

ρ: density
Γ: natural width
Δ: detuning

$$\Delta k = \frac{n\sigma}{2}\frac{\Gamma/2}{\Delta}$$

For the cold atoms : density: 10^{10} atoms/cm^3

Raman beams : Δ= 250 GHz :
$(n-1)=1,4.10^{-9}$ (selection)
$(n-1)\sim 1,4.10^{-10}$ (measure)

Bloch beams : Δ = 40 GHz:
$(n-1)=9.10^{-10}$ (selected atoms)

For the background vapor : density: 8.10^{8} atoms/cm^3

Raman beams : Δ= 250 GHz : $(n-1)=1,1.10^{-10}$

Bloch beams : Δ = 40 GHz: $(n-1)=7.10^{-10}$

Index of refraction

*PRL **94** 170403 (2005) (MIT): Photon Recoil Momentum in Dispersive Media*

Observation : modification of recoil energy in a dispersive medium (BEC).

	Dispersive medium	Atoms
N_{tot}	N_0	N_1

n : index of refraction

$N_1 << N_{tot}$

Dispersive medium	Atoms
$2(1-n)N_1/N_0\hbar k$	$2n\hbar k$

Bloch oscillations :

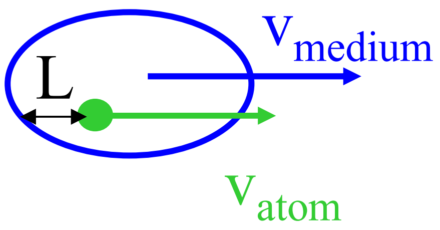
$$p_{final} = 2n\hbar k + 2(1-n)\,\hbar k\, N_{tot}/N_{tot} = 2\hbar k \quad \text{if } \eta = 100\%$$

Accelerated atoms ⇔ dispersive medium

otherwise $\sim(1-\eta)(n-1)$

Raman transition :

Atomic cloud



$$\omega' = \omega - 2nkv_{atom} + 2(n-1)kv_{medium}$$

$$\omega' = \omega - 2\,kv_{atom} + 2\,(n-1)k(v_{medium} - v_{atom})$$

$dL/dt = 0 \iff v_{medium} = v_{atom}$ no effect