Difference between standard and quasi-conformal BFKL kernels

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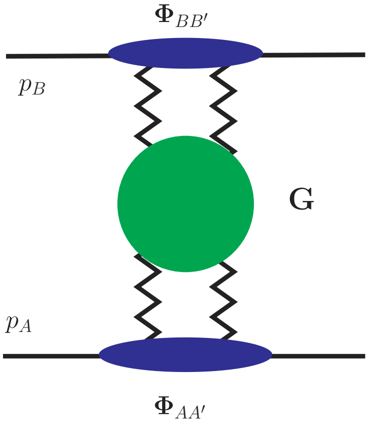
Contents

In the BFKL approach scattering amplitudes are represented by the picture



where $\Phi_{A'A}$ and $\Phi_{B'B}$ are impact factors of colliding particles, and G is the Green function of two interacting reggeized gluons.

Introduction

This picture is valid for any colour states of the two reggeized gluons and corresponds to the operator expression

$$\langle A'A|e^{Y\hat{\mathcal{K}}}|B'B\rangle,$$

$\hat{\mathcal{K}}$ is the BFKL kernel, Y is the total rapidity $(Y = \ln(s/s_0))$.
Now the kernel is known in the NLO both for forward scattering, i.e. for $t = 0$ and the colour singlet in the t–channel,
V.S. F., L.N. Lipatov, 1998
M. Ciafaloni, G. Camici, 1998
and for arbitrary t and any possible colour state in the t–channel
V. S. F., D. A. Gorbachev, 2000
V. S. F., R. Fiore, 2005

Introduction

The most important are the singlet (Pomeron) and adjoint representaions of the colour group. . Talking about the BFKL approach, one usually means BFKL Pomeron. But the adjoint representation is not less important. For two reggeized gluons the adjoint representation can be of two different types, symmetric and antisymmetric ones. In fact, antisymmetric adjoint representationis even more important, than the singlet one, first of all because of the gluon reggeization. The idea of the gluon reggeization appeared as the result of the fixed order calculations. Evidently it must be proved. It was done in using bootstrap relations, which follow from the requirement of compatibility of the multi-Regge form of amplitudes with the s-channel unitarity. Now fulfillment of these relations is proved in the NLO

V.S. F., M.G. Kozlov, A.V. Reznichenko, 2012

But there are at least two other reasons for significance of the kernel of the BFKL equation for the adjoint representations.
One is related to the BKP equation
J. Bartels,1980
J. Kwiecinski, M. Praszalowicz, 1980
-the generalization of the BFKL equation to bound states consisting of three and more reggeized gluons, in particular the C-odd three gluon system — Odderon. The BFKL kernel for symmetric adjoint representation appears in the BKP equation for the odderon because any pair of the three reggeized gluons are in the colour octet state.
Recently, another application of the BFKL approach, related to the BDS ansatz
Z. Bern, L. J. Dixon and V. A. Smirnov, 2005
was extensively developed

The approach was used for verification of the BDS ansatz for the inelastic amplitudes in $N=4$ SUSY and calculation of the remainder factor
J. Bartels, L. N. Lipatov, A. Sabio Vera, 2009
L. N. Lipatov and A. Prygarin, 2011
It was demonstrated that the BDS amplitude $M^{BDS}_{2\to4}$ should be multiplied by the factor containing the contribution of the Mandelstam cuts, and this contribution was found in the LLA and in the NLA
V.S. F. and L. N. Lipatov, 2011
At large N_c, when only planar diagrams are taken into account, there is degeneracy in signature, i.e. no difference between symmetric ant antisymmetric adjoint representations.

Colour singlet kernel

For scattering of colourless objects, due to gauge invariance of the impact factors, the BFKL equation can be written in the Möbius invariant form
L. N. Lipatov, 1989
In this form the BFKL kernel acts in the space of functions turning into zero at coinsiding transverce coordinates.
The Möbius invariance of the LO BFKL kernel can be made evident by transformation from the transverse momentum to the transverse coordinate representation
V.S. F, R. Fiore, A. Papa, 2007
In the NLO the Möbius invariance in QCD is violated by the running coupling, i.e. by the terms proportional to the β function. But it remains unbroken in $N = 4$ SUSY.

Colour singlet kernel

For $N = 4$ SUSY

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}_M^C | \vec{r}_1{}' \vec{r}_2{}' \rangle_{N=4} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{\rho} \frac{\vec{r}_{12}{}^2}{\vec{r}_{1\rho}{}^2 \vec{r}_{2\rho}{}^2}$$

$$\times \Big[\delta(\vec{r}_{11'}) \delta(\vec{r}_{2'\rho}) + \delta(\vec{r}_{1'\rho}) \delta(\vec{r}_{22'}) - \delta(\vec{r}_{11'}) \delta(r_{22'}) \Big] \left(1 - \frac{\alpha_s N_c \zeta(2)}{2\pi} \right)$$

$$+ \frac{\alpha_s^2 N_c^2}{4\pi^4} \left[\frac{\ln\left(\frac{\vec{r}_{12'}^{\,2} \vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2} \vec{r}_{22'}^{\,2}} \right)}{2 \vec{r}_{11'}^{\,2} \vec{r}_{22'}^{\,2}} \left(\frac{\vec{r}_{12}^{\,4}}{\vec{r}_{12'}^{\,2} \vec{r}_{21'}^{\,2} - \vec{r}_{11'}^{\,2} \vec{r}_{22'}^{\,2}} - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1'2'}^{\,2}} \right) + \frac{\vec{r}_{12}^{\,2} \ln\left(\frac{\vec{r}_{12}^{\,2} \vec{r}_{1'2'}^{\,2}}{\vec{r}_{12'}^{\,2} \vec{r}_{21'}^{\,2}} \right)}{\vec{r}_{11'}^{\,2} \vec{r}_{22'}^{\,2} \vec{r}_{1'2'}^{\,2}} \right.$$

$$\left. + 6\pi^2 \zeta(3) \, \delta(\vec{r}_{11'}) \delta(r_{22'}) \right] .$$

This form of the NLO kernel is immeasurably simple compared with the kernel in the momentum space.

Colour singlet kernel

In fact, there are three reasons for the simplicity:
— Möbius representation (i.e. limitation of space of functions),
— Simularity transformation $\hat{K} \to \hat{K} - [\hat{K}^B \hat{U}]$,
— use of impact parameter space.
The simplicity of the Möbius form of the quasi-conformal NLO BFKL kernel suggested to use just this form for finding the kernel in the momentum space.
The way to do that was not evident, and even the possibility to do it seemed doubtful, because the Möbius form is defined on a special class of functions in the coordinate space. However, it was shown
V.S. F., R. Fiore, A.V. Grabovsky, A. Papa, 2011
that such possibility exists due to the gauge invariance of the kernel and the way to obtain the kernel in the momentum space from its Möbius form was elaborated. But technically obtaining it turned out to be not easy.

Colour singlet kernel

An explicit form of the operator $\hat{U}$ in the momentum space
V.S. F., R. Fiore, A.V. Grabovsky, A. Papa, 2011

$$\langle\vec{q}_1,\vec{q}_2|\alpha_s\hat{U}|\vec{q}_1{}',\vec{q}_2{}'\rangle = \delta(\vec{q}_{11'}+\vec{q}_{22'})\frac{\alpha_s N_c}{4\pi^2}R_u(\vec{q}_1,\vec{q}_2;\vec{k})$$

$$-\frac{\alpha_s\beta_0}{8\pi}\ln\left(\vec{q}_1{}^2\vec{q}_2{}^2\right)\delta(\vec{q}_{11'})\delta(\vec{q}_{22'})\ ,$$

where β_0 is the first coefficient of the Gell-Mann–Low function,

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$$

and

$$R_u(\vec{q}_1,\vec{q}_2;\vec{k}) = \frac{1}{\vec{q}_1{}^2}\ln\left(\frac{\vec{q}_1{}'^2\vec{q}_2{}^2}{\vec{k}^2\vec{q}^{\,2}}\right)+\frac{1}{\vec{q}_2{}^2}\ln\left(\frac{\vec{q}_2{}'^2\vec{q}_1{}^2}{\vec{k}^2\vec{q}^{\,2}}\right)+\frac{1}{\vec{k}^2}\ln\left(\frac{\vec{q}_1{}'^2\vec{q}_2{}'^2}{\vec{q}_1{}^2\vec{q}_2{}^2}\right)$$

$$-2\frac{\vec{q}_1\vec{k}}{\vec{k}^2\vec{q}_1{}^2}\ln\left(\frac{\vec{q}_1{}'^2}{\vec{k}^2}\right)+2\frac{\vec{q}_2\vec{k}}{\vec{k}^2\vec{q}_2{}^2}\ln\left(\frac{\vec{q}_2{}'^2}{\vec{k}^2}\right)-2\frac{\vec{q}_1\vec{q}_2}{\vec{q}_1{}^2\vec{q}_2{}^2}\ln\left(\frac{\vec{q}^{\,2}}{\vec{k}^2}\right)\ .$$

Colour singlet kernel

Note that R_u has the gauge invariance properties:

$$R_u(\vec{q}_1, \vec{q}_2; \vec{q}_1) = R_u(\vec{q}_1, \vec{q}_2; -\vec{q}_2) = 0,$$

$$(\vec{q}_1^{\,2}\vec{q}_2^{\,2} R_u(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_1=0} = (\vec{q}_1^{\,2}\vec{q}_2^{\,2} R_u(\vec{q}_1, \vec{q}_2; \vec{k}))|_{\vec{q}_2=0} = 0\ .$$

Indeed, these properties are required to conserve the gauge invariance.
The difference between the standard BFKL kernel, defined according to the prescriptions
V.S. F., R. Fiore,1998
and the quasi-conformal BFKL kernel turned out to be rather simple
V.S. F., R. Fiore, A. Papa, 2012

Colour singlet kernel

$$D(\vec{q}_1, \vec{q}_1{}'; \vec{q}) = \frac{\alpha_s^2 N_c^2}{8\pi^3} \left[-\frac{\beta_0}{2N_c} \left(\frac{2}{\vec{k}^{\,2}} - 2\frac{\vec{q}_1 \vec{k}}{\vec{k}^{\,2}\vec{q}_1^{\,2}} + 2\frac{\vec{q}_2 \vec{k}}{\vec{k}^{\,2}\vec{q}_2^{\,2}} - 2\frac{\vec{q}_1 \vec{q}_2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \right) \right.$$

$$\times \ln\left(\frac{\vec{q}_1{}'^2 \vec{q}_2{}'^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right) + \frac{\vec{q}_1{}'^2}{\vec{q}_1^{\,2}\vec{k}^{\,2}} \ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2{}'^2}{\vec{q}_2^{\,2}\vec{q}_1{}'^2}\right) \ln\left(\frac{\vec{q}_2^{\,2}\vec{q}_1{}'^2}{\vec{q}^{\,2}\vec{k}^{\,2}}\right) + \frac{\vec{q}_2{}'^2}{\vec{q}_2^{\,2}\vec{k}^{\,2}} \ln\left(\frac{\vec{q}_2^{\,2}\vec{q}_1{}'^2}{\vec{q}_1^{\,2}\vec{q}_2{}'^2}\right)$$

$$\times \ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2{}'^2}{\vec{q}^{\,2}\vec{k}^{\,2}}\right) - 4\left(\frac{[\vec{q}_1 \times \vec{q}_2]}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} + \frac{[\vec{q}_1 \times \vec{k}]}{\vec{q}_1^{\,2}\vec{k}^{\,2}} + \frac{[\vec{q}_2 \times \vec{k}]}{\vec{q}_2^{\,2}\vec{k}^{\,2}}\right)$$

$$\left.\left([\vec{q}_1 \times \vec{q}_2] I_{\vec{q}_1, \vec{q}_2} - [\vec{q}_1{}' \times \vec{q}_2{}'] I_{\vec{q}_1{}', \vec{q}_2{}'}\right)\right].$$

The most natural conclusion is that the simplicity of the Möbius form of the quasi-conformal kernel is caused mainly by using the impact parameter space. The other possibility is that the quasi-conformal kernel can be written in simple form also in the transverse momentum space.

Adjoint representaion

The modified (with subtracted gluon trajectory depending on total t-channel momenta) kernel in the antisymmetric adjoint representtion can be written as follows:

$$K(\vec{q}_1, \vec{q}_1{}'; \vec{q}) = K^B(\vec{q}_1, \vec{q}_1{}'; \vec{q}) \left(1 - \frac{\alpha_s N_c}{2\pi}\zeta(2)\right)$$

$$+\delta^{(2)}(\vec{q}_1 - \vec{q}_1{}')\frac{\vec{q}_1{}^2\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\frac{\alpha_s^2 N_c^2}{4\pi^2}3\zeta(3) + \frac{\alpha_s^2 N_c^2}{32\pi^3}R(\vec{q}_1, \vec{q}_1{}'; \vec{q}) \,,$$

K^B is the leading order kernel, which can be written in the explicitly Möbius invariant form:

$$K^B(\vec{q}_1, \vec{q}_1{}'; \vec{q}) = -\delta^{(2)}(\vec{q}_1 - \vec{q}_1{}')\frac{\vec{q}_1{}^2\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\frac{\alpha_s N_c}{4\pi^2}\int\frac{\vec{q}^{\,2}\, d^2 l}{(\vec{q}_1 - \vec{l})^2(\vec{q}_2 + \vec{l})^2}$$

$$\left(\frac{\vec{q}_1^{\,2}(\vec{q}_2 + \vec{l})^2 + \vec{q}_2^{\,2}(\vec{q}_1 - \vec{l})^2}{\vec{q}^{\,2}\vec{l}^{\,2}} - 1\right) + \frac{\alpha_s N_c}{4\pi^2}\left(\frac{\vec{q}_1{}^2\vec{q}_2{}'^{\,2} + \vec{q}_1{}'^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}} - 1\right) .$$

Adjoint representaion

$$R(\vec{q}_1,\vec{q}_1\,',\vec{q}) = \frac{1}{2}\left(\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\right) + \ln\left(\frac{\vec{q}_1\,'^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_2\,'^{2}}{\vec{q}^{\,2}}\right)\right.$$

$$\left. + \ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1\,'^{\,2}}\right)\right) - \frac{\vec{q}_1^{\,2}\vec{q}_2\,'^{\,2} + \vec{q}_2^{\,2}\vec{q}_1\,'^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1\,'^{\,2}}\right) - \frac{\vec{q}_1^{\,2}\vec{q}_2\,'^{\,2} - \vec{q}_2^{\,2}\vec{q}_1\,'^{\,2}}{2\vec{q}^{\,2}\vec{k}^{\,2}}$$

$$\times \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1\,'^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_1\,'^{\,2}}{\vec{k}^{\,4}}\right) + 4\frac{(\vec{k}\times\vec{q}_1)}{\vec{q}^{\,2}\vec{k}^{\,2}}\left(\vec{k}^{\,2}(\vec{q}_1\times\vec{q}_2)\right.$$

$$\left. -\vec{q}_1^{\;2}(\vec{k}\times\vec{q}_2) - \vec{q}_2^{\;2}(\vec{k}\times\vec{q}_1)\right) I_{\vec{q}_1,-\vec{k}} + (\vec{q}_1 \leftrightarrow -\vec{q}_2,\;\; \vec{q}_1\,' \leftrightarrow -\vec{q}_2\,')\;.$$

$$\vec{k} = \vec{q}_1 - \vec{q}_1\,' = \vec{q}_2\,' - \vec{q}_2,\;\; (\vec{a}\times\vec{b}) = a_x b_y - a_y b_x$$

$$I_{\vec{p},\vec{q}} = \int_0^1 \frac{dx}{(\vec{p}+x\vec{q})^2}\ln\left(\frac{\vec{p}^{\,2}}{x^2\vec{q}^{\,2}}\right)\;.$$

Adjoint representaion

The contribution $R(\vec{q}_1, \vec{q}_1{}', \vec{q})$ violates the Möbius invariance. But it is generally believed that the remaider function is conformal invariant. Therefore in the paper
V.S. F., L.N. Lipatov, 2012
it was assumed that there is a conformal invariant representation of the kernel. Since its eigenvalues do not depend on the representation and on the total momentum transfer, they were found using the limit

$$|\vec{q}_1| \sim |\vec{q}_1{}'| \ll |\vec{q}| \approx |\vec{q}_2| \approx |\vec{q}_2{}'|\,.$$

In this limit

$$K(z) = K^B(z)\left(1 - \frac{\alpha_s N_c}{2\pi}\zeta(2)\right) + \delta^{(2)}(1-z)\frac{\alpha_s^2\,N_c^2}{4\pi^2}3\zeta(3) + \frac{\alpha_s^2\,N_c^2}{32\pi^3}R(z)\,,$$

where $z = q_1/q_1'$,

$$K^B(z) = \frac{\alpha_s N_c}{8\pi^2}\left(\frac{z+z^*}{|1-z|^2} - \delta^{(2)}(1-z)\int \frac{d\vec{l}}{|l|^2}\frac{l+l^*}{|1-l|^2}\right),$$

Adjoint representaion

$$R(z) = \left(\frac{1}{2} - \frac{1+|z|^2}{|1-z|^2}\right) \ln^2|z|^2 - \frac{1-|z|^2}{2|1-z|^2} \ln|z|^2 \ln\frac{|1-z|^4}{|z|^2|}$$

$$+ \left(\frac{1}{1-z} - \frac{1}{1-z^*}\right)(z-z^*) \int_0^1 \frac{dx}{|x-z|^2} \ln\frac{|z|^2}{x^2}\,.$$

$p = p_x + ip_y$ and $p^* = p_x - ip_y$ for the two-dimensional vectors $\vec{p} = (p_x, ip_y)$.Vice versa, two complex numbers z and z^* are equivalent to the vector $\vec{z}$ with the components $(z+z^*)/2$ and $(z-z^*)/(2i)$.

If the reminder function is indeed conformal invariant, there must be simularity transformation connecting two forms of the kernel.

Adjoint representaion

Due to the Möbius invariance, the kernel $K_c(\vec{q}_1, \vec{q}_1{}'; \vec{q})$ can be written as $K(z)$ with the argument $z = q_1 q_2' / (q_2 q_1')$. If we denote

$$K(\vec{q}_1, \vec{q}_1{}'; \vec{q}) - K_c(\vec{q}_1, \vec{q}_1{}'; \vec{q}) = \frac{\alpha_s^2 N_c^2}{32\pi^3} \Delta(\vec{q}_1, \vec{q}_1{}'; \vec{q}) \,,$$

then

$$\Delta(\vec{q}_1, \vec{q}_1{}'; \vec{q}) = R(\vec{q}_1, \vec{q}_1{}'; \vec{q}) - R(z) \,,$$

Since $R(\vec{q}_1, \vec{q}_1{}'; \vec{q})$ is not conformal invariant, $\Delta(\vec{q}_1, \vec{q}_1{}'; \vec{q})$ cannot be written using the single variable z. Using relations between dilogarithms it can be written in the form

Adjoint representaion

$$\Delta(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}) = \ln\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\ln\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}} + \ln\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\ln\frac{\vec{q}_2^{\,\prime\,2}}{\vec{q}^{\,2}} + \ln\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}}\ln\frac{\vec{q}_2^{\,2}}{\vec{q}_2^{\,\prime\,2}}$$

$$-2\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}\vec{q}^{\,2}}\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}}\ln\frac{\vec{q}_2^{\,2}}{\vec{q}_2^{\,\prime\,2}}+\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}\vec{q}^{\,2}}\left(\ln\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\ln\frac{\vec{q}_2^{\,\prime\,2}}{\vec{q}^{\,2}}\right.$$

$$\left.-\ln\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\ln\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\right)+\frac{4}{\vec{q}^{\,2}\vec{k}^{\,2}}\left(\vec{k}^{\,2}[\vec{q}_1\times\vec{q}_2]-\vec{q}_1^{\,2}[\vec{k}\times\vec{q}_2]-\vec{q}_2^{\,2}[\vec{k}\times\vec{q}_1]\right)$$

$$\times\left([\vec{q}_1\times\vec{q}_2]I_{\vec{q}_1,\vec{q}_2}-[\vec{q}_1^{\,\prime}\times\vec{q}_2^{\,\prime}]I_{\vec{q}_1^{\,\prime},\vec{q}_2^{\,\prime}}\right)\ .$$

Important properties of Δ are its symmetries with respect to the exchanges $\vec{q}_1 \leftrightarrow -\vec{q}_2$, $\vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}$ and $\vec{q}_i \leftrightarrow -\vec{q}_i$, as well as the gauge invariance (vanishing at zero momentum of each reggeon), which are easily seen from this representation.

Adjoint representaion

If the kernels $\hat{\mathcal{K}}$ and $\hat{\mathcal{K}}_c$ coincide in the leading order and are related by a similarity transformation, there must exist an operator $\hat{\mathcal{O}}$ satisfying the commutation relation

$$[\hat{\mathcal{K}}^B, \hat{O}] = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{8\pi}\hat{\Delta}\,.$$

One can find a formal expression for this operator allowing to construct the similarity transformation in perturbation theory. Indeed, it is enough to calculate the matrix element of the above commutation relation between the eigenfunctions of the Born kernel with the corresponding eigenvalues $\omega^B_{\nu n}$ in the form

$$\left(\omega^B_{\nu' n'} - \omega^B_{\nu n}\right) \langle \nu' n' | \hat{\mathcal{O}} | \nu n \rangle = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{8\pi} \langle \nu' n' | \hat{\Delta} | \nu n \rangle.$$

It can be seen from this equation that the solution $\hat{\mathcal{O}}$ exists only if the operator $\hat{\Delta}$ has vanishing matrix elements between states with the same eigenvalues. In this case

Adjoint representaion

$$\hat{O} = \frac{\alpha_s^2}{32\pi^3} \sum_{n,n'} \int d\nu d\nu' \frac{|\nu' n'\rangle\langle\nu' n'|\hat{\Delta}|\nu n\rangle\langle\nu n|}{\omega^B_{\nu' n'} - \omega^B_{\nu n}}$$

$$\langle\vec{q}_1, q_2|\hat{\mathcal{O}}|\vec{q}_1{}', \vec{q}_2{}'\rangle = \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{8\pi} \sum_{n,n'} \int d\nu \int d\nu'$$

$$\frac{\langle\vec{q}_1, q_2|\nu' n'\rangle\langle\nu' n'|\hat{\Delta}|\nu n\rangle\langle\nu n|\vec{q}_1{}', \vec{q}_2{}'\rangle}{\omega^B_{\nu' n'} - \omega^B_{\nu n}} \; .$$

Since the kernel $\hat{\Delta}$ is known in the momentum space, we can transform it into the (n, ν) representation,

$$\langle\nu n|\hat{\Delta}|\nu' n'\rangle = \int \frac{\vec{q}^{\,2} d\vec{q}_1}{\vec{q}_1^{\,2}(\vec{q} - \vec{q}_1)^2} \int \frac{\vec{q}^{\,2} d\vec{q}_1{}'}{\vec{q}_1{}'^{\,2}(\vec{q} - \vec{q}_1{}')^2} \langle\nu' n'|\vec{q}_1{}', \vec{q}_2{}'\rangle$$

$$\times \Delta(\vec{q}_1, \vec{q}_1{}'; \vec{q})\langle\vec{q}_1, \vec{q}_2|\nu n\rangle$$

using the known eigenfunctions in the momentum space, which allows to find the matrix element $\langle\vec{q}_1, \vec{q}_2|\hat{\mathcal{O}}|\vec{q}_1{}', \vec{q}_2{}'\rangle$.

Adjoint representaion

Calculations are rather complicated, but the final result is very simple.

V.S. F., R. Fiore, L.N. Lipatov, A. Papa, 2013

It turned out that

$$\hat{\mathcal{O}}_t = \frac{1}{4}\left[\ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right), \hat{\mathcal{K}}_r^B\right] \, ,$$

It meads that the conformal invariant kernel can be obtained using the substraction procedure different from the standard one.

In fact, this form of the operator $\hat{O}$ can be guessed.

In the momentum representation

$$O(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}) \; = \; \frac{\alpha_s N_c}{16\pi^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} + \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,\prime\,2}}\right)$$

Summary

- It is proved that complete colour singlet BFKL kernel can be restored from its Möbius form
- The difference between quasi-conformal and standard colour singlet BFKL kernels in the momentum space is found
- This difference turned out to be rather simple
- It is proved that in the adjoint representation of the colour group quasi-conformal and standard BFKL kernels are connected by simularity transformation.
- The simularity transformation is found explicitly