QCD Reggeon Field Theory from the JIMWLK/KLWMIJ evolution.

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Open questions/Goals

- **What is the high energy limit of QCD?**
  many candidates: BFKL Pomeron Calculus, Lipatov's effective action, elements of Field Theory of Bartels, JIMWLK+KLWMIJ Hamiltonians for Wilson line operators....

- **Is it possible to rigorously derive an effective theory of QCD in terms of color singlet exchange amplitudes, the Reggeon Field Theory (RFT) of QCD?**

- **Will this RFT reduce to the BFKL Pomeron Calculus with a universal triple Pomeron vertex and that is it or there will be more Reggeons and more vertices?**

- **Grand Unification: Can we relate between various approaches to HE QCD?**

High Energy Scattering

Target (ρ^t) **Projectile (ρ^p)**

$$\langle \mathbf{T} | \quad \rightarrow \qquad\qquad \leftarrow \quad | \mathbf{P} \rangle$$

S-matrix:

$$\mathbf{S}(\mathbf{Y}) \;=\; \langle \mathbf{T} \, \langle \, \mathbf{P} | \;\; \hat{\mathbf{S}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; | \mathbf{P} \rangle \, \mathbf{T} \rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T} \, \langle \, \mathbf{P} | \;\; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; | \mathbf{P} \rangle \, \mathbf{T} \rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged operators:

$$\langle\, \mathbf{P}|\ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ |\mathbf{P}\rangle\ \ =\ \int \mathbf{D}\rho^{\mathrm{p}}\ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \ \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

evolve with rapidity as $\mathbf{H}\ \rightarrow$ **the HE effective Hamiltonian**

$$\frac{\mathbf{d}\langle\mathbf{P}|\hat{\mathcal{O}}|\mathbf{P}\rangle}{\mathbf{dY}}\ =\ -\int \mathbf{D}\rho^{\mathrm{p}}\ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \mathbf{H}[\rho^{\mathrm{p}},\ \delta/\delta\rho^{\mathrm{p}}]\ \ \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

or in other words

$$\frac{\mathbf{dW}^{\mathrm{p}}}{\mathbf{dY}}\ =\ -\,\mathbf{H}\ \ \mathbf{W}^{\mathrm{p}}$$

Spectrum of $\mathbf{H}$ defines the energy dependence of the average.

KLWMIJ Hamiltonian

A. Kovner & ML 2005

The KLWMIJ Hamiltonian is a limit of H for dilute partonic system ($\rho_{\mathrm{p}} \rightarrow 0$) which scatters on a dense target. It account for linear gluon emission + multiple rescatterings off the target. Most suitable for low x DIS.

$$H_{KLWMIJ} = \frac{\alpha_s}{2\pi^2}\int_{x,y,z}\frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2}\left\{J^a_L(x)J^a_L(y) + J^a_R(x)J^a_R(y) - 2J^a_L(x)R^{ab}_z J^b_R(y)\right\}$$

$$\mathbf{R(x)} \;=\; \mathbf{e}^{\mathbf{T^a}\frac{\delta}{\delta\rho^{\mathbf{a}}_{\mathbf{p}}(\mathbf{x})}}$$

Scattering matrix of a projectile quark

The left and right SU(N) generators:

$$\mathbf{J^a_L(x) = tr\left[\frac{\delta}{\delta R^T_x}T^a R_x\right] - tr\left[\frac{\delta}{\delta R^*_x}R^\dagger_x T^a\right]}$$

$$\mathbf{J^a_R(x) = tr\left[\frac{\delta}{\delta R^T_x}R_x T^a\right] - tr\left[\frac{\delta}{\delta R^*_x}T^a R^\dagger_x\right]}$$

PROJECTING KLWMIJ ONTO RFT

$\mathbf{H_{KLWMIJ}}$ defines a 2+1 dimensional non-local QFT of unitary matrix R, but not a QFT of Reggeons. Reggeons are physical scattering amplitudes - color singlets.

Is it possible to project $\mathbf{H_{KLWMIJ}}$ onto color singlets and derive the RFT ?

First step is to choose effective degrees of freedom and make sure to preserve symmetries

$\mathbf{SU_L(N)} \;\times\; \mathbf{SU_R(N)}$ – effective degrees of freedom must be scalars.

Charge conjugation Z_2: $\quad \mathbf{R(x)} \;\rightarrow\; \mathbf{R^*(x)}$

Time reversal (Signature) Z_2: $\quad \mathbf{R(x)} \;\rightarrow\; \mathbf{R^\dagger(x)}$

Natural condition: in the linearized regime $(\mathbf{R = 1 - T}\frac{\delta}{\delta\rho}\ldots)$ we shell reduce to BKP.

In a sense, we study the low energy limit of high energy QCD.

There is infinite number of independent color singlets, but there is a natural hierarchy

Dipole: $d(x, y) = \frac{1}{N_c}\mathrm{Tr}[R(x)R^\dagger(y)]$

Qudrupole: $Q(x, y, u, v) = \frac{1}{N_c}\mathrm{Tr}[R(x)\,R^\dagger(y)\,R(u)\,R^\dagger(v)]$

Naturally decomposes into

Pomeron: - C, T even $P(1, 2) = \frac{1}{2}[2 - d(1, 2) - d(2, 1)]$

Odderon: - C, T odd $O(1, 2) = \frac{1}{2}[d(1, 2) - d(2, 1)]$

B-Reggeon: C,T even, perturbatively orthogonal to P

$$B_{1,2,3,4} = \frac{1}{4}\left[4 - Q_{1,2,3,4} - Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right] - \left[P_{12} + P_{14} + P_{23} + P_{34} - P_{13} - P_{24}\right]$$

Other 'ONs

C-Reggeon odd, T even: $C_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} + Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right]$

T odds: $D^{\pm}_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} - Q_{4,1,2,3}\right] \pm \frac{1}{4}\left[Q_{3,2,1,4} - Q_{2,1,4,3}\right]$

And higher multipoles

$$H_{KLWMIJ} = H_P + H_O + H_B + H_C + H_D + \cdots$$

$$H_P = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} \frac{(x - y)^2}{(x - z)^2\,(y - z)^2} \Big\{ [P_{x,z} + P_{z,y} - P_{x,y} - P_{x,z} P_{z,y} + O_{x,z} O_{z,y}]\, P^{\dagger}_{x,y} \Big\}$$

$$H_O = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} \frac{(x - y)^2}{(x - z)^2\,(y - z)^2} \Big\{ [O_{x,z} + O_{z,y} - O_{x,y} - O_{x,z} P_{z,y} - P_{x,z} O_{z,y}]\, O^{\dagger}_{x,y} \Big\}$$

$$H_B = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{xyuvz} \Bigg\{ \Big[- \, [M_{x,y;z} + M_{u,v;z} - L_{x,u,v,y;z}] \, B_{xyuv} + 4 L_{x,v,u,v;z} B_{xyuz} \Big] B^{\dagger}_{xyuv}$$

$$-2 L_{x,y,u,v;z} \Big[P_{xv} P_{uy} + O_{xv} O_{uy} \Big] B^{\dagger}_{xyuv} - 2 P_{xz} P_{yz} \Big[2 L_{x,y,u,v;z} B^{\dagger}_{xyuv} - \Big(L_{x,u,y,v;z} + L_{x,v,y,u;z} \Big) B^{\dagger}_{xuyv} \Big]$$

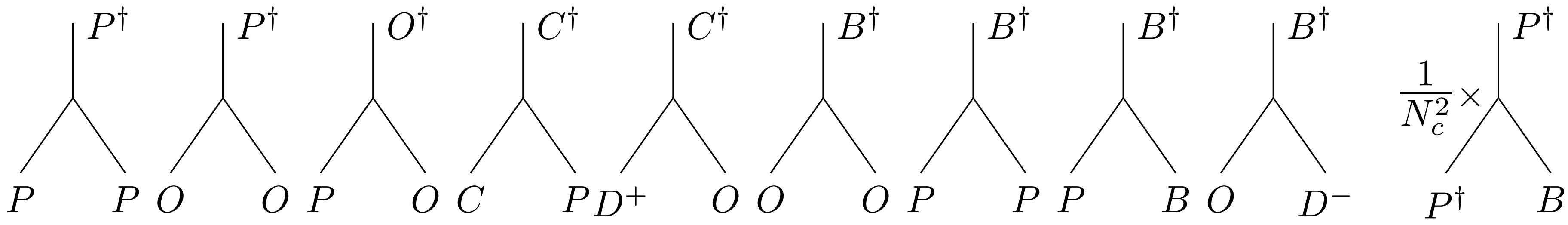
$$-4 P_{xz} P_{yu} \Big[2 L_{x,y,x,v;z} B^{\dagger}_{xyuv} - L_{x,y,x,u;z} B^{\dagger}_{xyvu} \Big] - 4 B_{xyuz} P_{zv} L_{x,v,u,v;z} B^{\dagger}_{xyuv}$$

$$-4 D^{+}_{xyuz} O_{zv} L_{x,v,u,v;z} B^{\dagger}_{xyuv} \Bigg\}$$

All vertices allowed by the symmetries

At leading N_c all of them have the nature of splitting: one Reggeon going into two

$$PPP^\dagger, \quad OOP^\dagger; \quad POO^\dagger; \quad PPB^\dagger; \quad BPB^\dagger; \quad CPC^\dagger \quad \dots$$



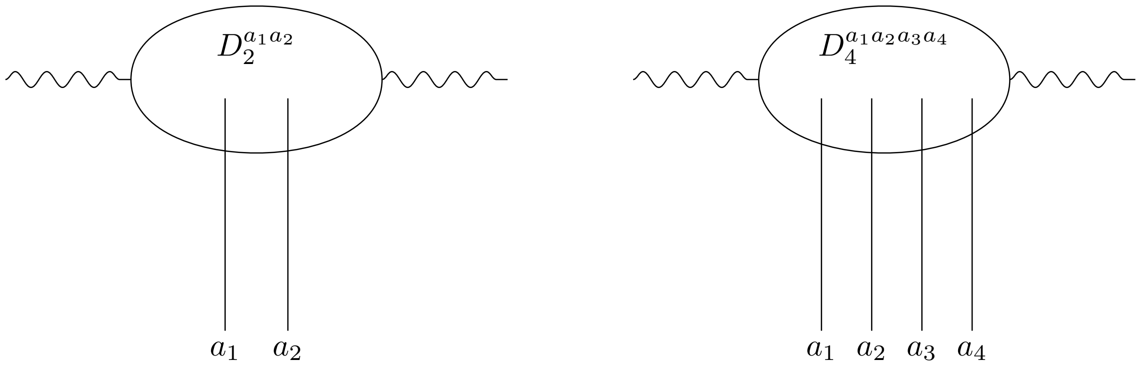
At subleading N_c one gets also merging vertices

REGGEONS vs BARTELS' D's

J. Bartels & Wusthoff (95); Bartels & Ewerz (99)

$$\mathbf{D}^{\mathbf{n}}_{\mathbf{a_1,\ldots a_n}}(\mathbf{x_1\ldots x_n}) \;=\; (-\mathbf{ig})^{\mathbf{n}}\;\langle\hat\rho^{\mathbf{a_1}}(\mathbf{x_1})\ldots\hat\rho^{\mathbf{a_2}}(\mathbf{x_n})\rangle_{\mathbf{photon}}$$

$\hat\rho^{\mathbf{a}}(\mathbf{x})$ - color charge density operator



$$\frac{\mathbf{dD^2(xy)}}{\mathbf{dY}} \;=\; \bar{\mathbf{K}}(\mathbf{xy},\mathbf{uv})\otimes\mathbf{D^2}(\mathbf{uv});$$

$\bar{\mathrm{K}}$ $-$ BFKL up to kinematical factor

Reggeization in D^4: $D^4_{irreducible} \; = \; D^4 \; - \; \sum D^2$

$$\frac{dD^4_{irreducible}}{dY} = \bar{K} \otimes D^4_{IRREDUCIBLE} + V \otimes D^2$$

V has been conjectured to be universal

$$\frac{dD^6_{irreducible}}{dY} = \bar{K} \otimes D^6_{irreducible} + V \otimes D^4 + (\text{NO } D^2 \text{ term})$$

The D-functions can be related to conjugate Reggeons.

$$\mathbf{D}^2_{\mathrm{aa}}(\mathbf{1},\mathbf{2}) \approx \Big[\mathbf{1} - \mathbf{P}_{12}\Big]\mathbf{P}^{\dagger}_{12}\,; \qquad \mathbf{D}^4_{\mathrm{aabb}}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) \approx \mathbf{P}^{\dagger}_{12}\,\mathbf{P}^{\dagger}_{34} - \delta_{12,34}\mathbf{P}^{\dagger}(\mathbf{1},\mathbf{2})$$

The last term is exactly the reggeization in D^4

"Reggeized" terms are exactly the extra terms in the relation between $\mathrm{D^{2n}}$ and $(\mathrm{P^\dagger})^\mathrm{n}$

The vertex $\mathbf{V}$ is exactly the same as appears in the KLWMIJ Hamiltonian $PPP^\dagger$ term.

Equations for $\mathrm{D^n_{irreducible}}$ are exactly equations for $\mathrm{P^\dagger\ P^\dagger}$, $\mathrm{B^\dagger}$, etc...

There will be new vertices in the evolution of $\mathrm{D^6}$, $\mathrm{D^8}$ etc – they can all be read off the Hamiltonian

Advantage of our approach: once we have chosen the basic variables,

(a) there is no ambiguity anywhere, that is all reggeization corrections and vertices are determined uniquely and to any order.

(b) Our Reggeon is developed at an operator level. This means we can consider at the same level any (dilute) projectile and not be limited to just a photon.

CONCLUSIONS

- KLWMIJ is indeed the Reggeon Field Theory, or rather half of it, it has only Reggeon splittings. We need to include JIMWLK with perhaps other corrections to account for Reggeon mergings, and Reggeon loops.

- Even at large N_c we see more than just a Pomeron. There are many Reggeons and it is not clear if we can throw them away. Studying their intercepts in more details is needed.

- Are all Reggeons relevant for phenomenology? Some yes. B-Reggeons contribute to particle production, while we are not aware of any process in which a six-point Reggeon would appear. So, maybe after all the hierarchy can be truncated.

- On the path to Grand Unification we have been able to prove that the KLWMIJ Hamiltonian contains without any approximations all the elements of the Field Theory of Bartels.