# Modeling of changes in the Debye length for the collision processes at high energies

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# *Parameters for a plasma*

- Coulomb coupling parameter with $a$ as Landau length at $\Gamma = 1$:

$$\Gamma = \frac{a^2}{3\,r_D^2} = \frac{q^2}{a\,k_B\,T}; \quad a \propto \left(\frac{3}{4\pi n}\right)^{1/3}$$

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when of $\Gamma \geq 1$ we have a strongly coupled plasma.

- Knudsen's physical number for plasma

$$K_{ph} = \frac{l_{ph}}{L} \propto \frac{r_D}{L}\,.$$

Smallness of this parameter determines possibility for hydro description of matter.

# *Vlasov equation*

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- We can apply Vlasov equation when

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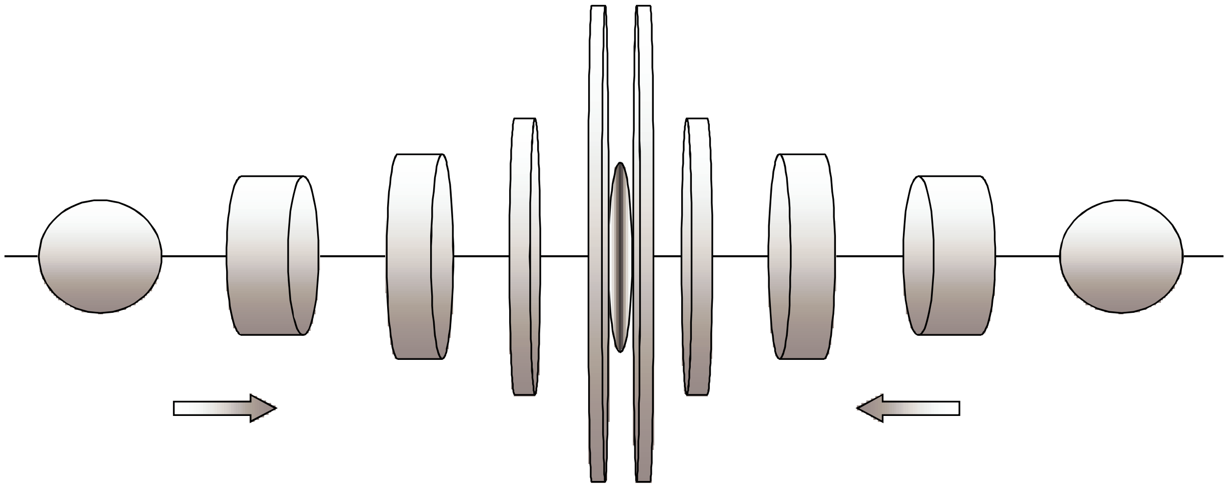

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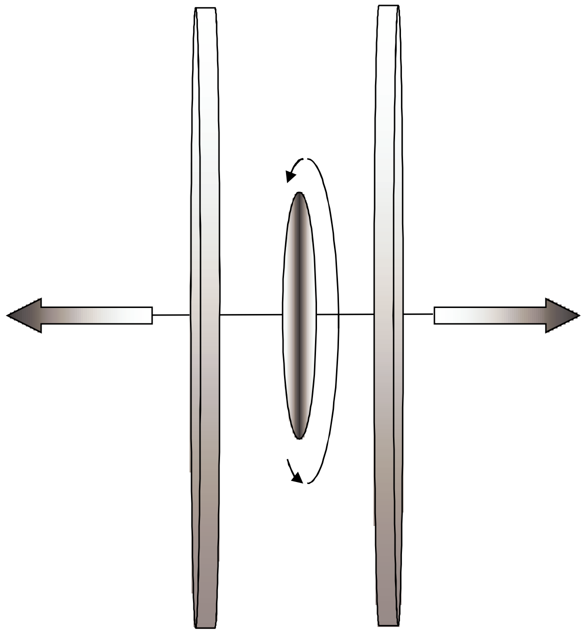

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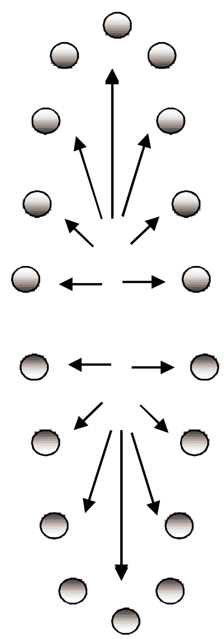

# *Hot spot of dense matter*

Matter which is not "globally in equilibrium" but instead "locally in equilibrium" is considered. Parameters of the matter can vary from point to point. The size of the hot spot is given by Debye length $r_D$. Particles inside this drop we will call by "slow" particles, outside there by "fast". Our non-equilibrium problem is reduced , therefore, to the problem of "local equilibrium" of hot spot in the external field of "fast particle".

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How to distinguish this "hot spot" in the matter?

Hot spot in the matter may be distinguished by the value of Knudsen number for the spot: $K_{ph} = \frac{r_D}{r_{D0}}$. If $r_{D0}$ is a characteristic length of the matter before interaction, for the spot we will obtain $K_{ph} < 1$

# *Spot of incident matter: frame of rest*

- Vlasov equation for the non-neutral plasma in the rest

$$\frac{\partial f_s}{\partial t} + \vec{v}\,\frac{\partial f_s}{\partial \vec{r}} + q\vec{E}\,\frac{\partial f_s}{\partial \vec{p}} = 0 \, .$$

$$\nabla \cdot \vec{E} = 4\,\pi\,q\,n \int f_s(x,t)\,d^3p \; .$$

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- Stationar solution of this equation is simple: screened potential

$$\varphi = C_0\,\frac{e^{-r/r_{DExt}}}{r} + 4\,\pi\,q\,n\,r^2_{DExt}$$

$$r^2_{DExt} = \frac{k_B\,T_{Ext}}{4\pi\,q^2\,n}$$

# *Spot of incident matter: c.m. f.*

- In the rest frame of the "slow" particles: Proca equation

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial^2 t} - \frac{\varphi}{r^2_{DExt}} = -4\pi\, q\, n\, dV_{Ext}\, \delta(\vec{r} - \vec{r}_s(t))$$

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- Solution for the field of "fast" particles:

$$\varphi(\vec{r},t) = \frac{q\, n\, dV_{Ext}}{2\pi^2} \int d^2k_\perp\, e^{k_\perp(r_\perp - b)} \int_{-\infty}^{\infty} \frac{e^{k_z(z - v\,t)}}{k_\perp^2 + k_z^2/\gamma + 1/r^2_{DExt}}$$

where $\vec{r} = (r_\perp, z)$ is position of the hot drop and the position of the incident matter is given by $\vec{r}_s = (b, vt)$ with $r_\perp = (r_x, r_y)\,,\ b = (b_x, b_y)$.

# *Hot spot point of view*

- At the relativistic limit when $v \approx c$:

$$\varphi(\vec{r},t) = 2\,q\,n\,dV_{Ext}\,\delta(z - ct)\,K_0(|r_\perp - b|/r_{DExt})$$

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- Denoting $r_\perp = r$ , $n = n_{0Ext} e^{\gamma_1}$ and taking $b = 0$:

$$A_r(\vec{r}, t) = q\, r^2_{DExt}\, n_{0Ext}\, s^{\gamma_1} \frac{m_N\, c^2}{s^{1/2}}\, \theta(z - ct)\, F(r/r_{DExt})\,.$$

# Hot spot point of view: Vlasov equation

- External field is two-dimensional and given by:

$$\vec{E}_{ext} = \frac{\partial A_r}{\partial \eta} \hat{e}_r = \delta(\eta) \tilde{F}(s, r/r_{DExt}) \hat{e}_r \,, \; \eta = z - ct \,.$$

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- Radial Vlasov equation for the self-consistent field of the hot drop

$$-c \frac{\partial f_s}{\partial \eta} + v \frac{\partial f_s}{\partial r} + q \left( E_s - E_{ext} \right) \frac{\partial f_s}{\partial p} = 0 \, ,$$

$$\nabla \cdot \vec{E}_s = 4 \pi q n \int f_s(x, p, t) \, d^3 p \, .$$

# Hot spot point of view: Vlasov equation

- Approximation of infinite incident media:

$$\vec{E}_{ext} = q\, r^2_{DExt}\, n_{0Ext}\, s^{\gamma_1}\, \frac{m_N\, c^2}{s^{1/2}}\, F(r/r_{DExt})$$

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- Equilibrium state equation:

$$v\,\frac{\partial\, f(r,p,t)}{\partial\, r} + \left(\frac{\partial\, V_{ext}(s,r)}{\partial\, r} - q\,\frac{\partial\, \phi_0(r)}{\partial\, r}\right)\frac{\partial\, f(r,p,t)}{\partial\, p} = 0$$

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- Solution of this equation

$$f_0(r,p) = \frac{1}{(2\,\pi\, k_B\, m\, T\,)}\, e^{-\frac{p^2}{2mk_BT} + \frac{V_{ext}(s,r)}{k_BT} - \frac{q\,\phi_0(r)}{k_BT}}\;.$$

# Debye´s length change

- Fix plasma´s parameter

$$\frac{1}{\mu} = 4\pi\, n \int^{r_D(n)} r^2\, dr \int d^3p\; f_0(r,p)$$

constant for the cases $V_{ext}(s,r) = 0$ and $V_{ext}(s,r) \neq 0$.

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- After some integration

$$\Delta\, r_D = r_{0D} - r_D \propto C\, r_{D0}^2\, r_{DExt}^2\, n_{0Ext}\, s^{\gamma_1}\, \frac{m_N\, c^2}{s^{1/2}}$$

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- Knudsen number:

$$K_{ph} = \frac{r_D}{r_{DExp}} \propto 1 - C\frac{r^2_{D0}}{r^2_{DExt}} N_{Ext}$$

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- Even when $r_{D0} = r_{DExt}$ we have during an interaction $K_{ph} \propto 1 - C\, N_{Ext}$ .
- It means $N_{Ext} \propto 1$– strong correlated plasma approximation.
- Local very quick thermalization of hot spots of very small size plus external field provides transition of the drop into a hydro state.
- After the external pressure is vanished the spot begins explode under the action of internal pressure.

# References


(1) L.D.Landau, Izv. Akad. Nauk: Ser.Fiz. **17**, 51 (1953).

(2) M.A.Leontovich, ZhETF **8**, 7, 844 (1938); Yu.L.Klimontovich, Sov. Phys. Usp. 26, 366 (1983).

(3) Yu.L.Klimontovich, Sov. Phys. Usp. 167, 23 (1997).

(4) M. Gyulassy, D. H. Rischke and B. Zhang, Nucl. Phys. A **613**, 397 (1997).