Calculation of glueball spectra in supersymmetric theories via holography

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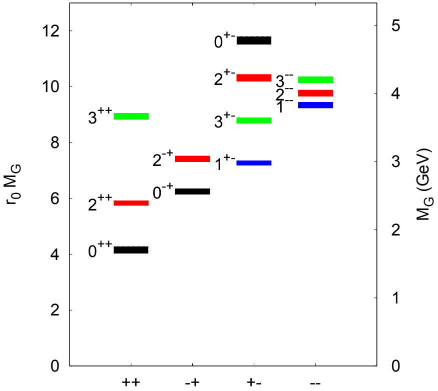
May'13

Pure glue world

Glueball spectrum of $SU(3)$ pure gauge [Morningstar,Peardon'99]

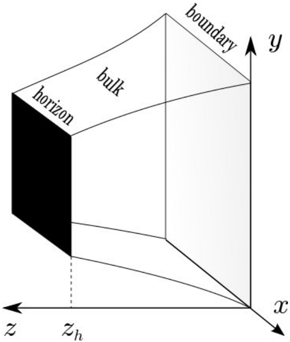


[Chen *et al.*'05]

No widely accepted results for supersymmetric theories

Holographic Correspondence

The main idea of holography is a possibility of a dual description of a (strongly coupled) (gauge) theory as gravity (string theory) living in a space with (at least one) extra dimensions [Thorne,'t Hooft,Polyakov,Susskind. . .] [Maldacena'97]



It is convenient to start with conformal theory

$$ds^2 = \frac{r^2}{R^2}\,\eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2}\,dr^2 + \dots$$

- isometries of $AdS \equiv$ conformal symmetries
- extra dimensions $\Leftrightarrow$ target space

One additional dimension plays the role of the energy scale and dynamical equations of gravity are equivalent to the RG-equations

Correlation functions

Mathematically the statement of the duality is very explicit

[Gubser,Klebanov,Polyakov'98][Witten'98]

$$Z_{\text{gravity}}\left[\phi^I(x^i, z=0)=\phi_0^I\right] = \langle \exp \int \mathrm{d}^D x\, \phi_0^I \mathcal{O}_I \rangle_{\text{QFT}}$$

In the most tractable case the curvature of the space on the lhs is small and loop corrections are suppressed (classical gravity limit) $\Longleftrightarrow$ strongly coupled ($\lambda \to \infty$) large N theory on the rhs

Clear prescription to evaluate the correlation functions:

$$\langle \mathcal{O}_{I_1} \cdots \mathcal{O}_{I_n} \rangle = \left. \frac{\delta^n S_{\text{gravity}}}{\delta \phi_0^{I_1} \cdots \delta \phi_0^{I_n}} \right|_{\phi_0^I = 0}$$

Particles – poles in the 2-point functions. Sufficient to look at linearized equations

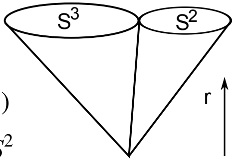
Known examples

As far as the gauge sector is concerned there is a handfull of (top-down) models

- $\mathcal{N} = 4$ supersymmetric Yang-Mills $\Leftarrow AdS_5 \times S^5$
- Witten-like models $\Leftarrow AdS_6 \times S^4$
- ABJM-like models $\Leftarrow AdS_4 \times S^7$
- extensions

How to construct a gravity dual to $\mathcal{N} = 1$ SYM?

- start with $\mathcal{N} = 4$ theory ($AdS_5 \times S^5$)
- break SUSY $\mathcal{N} = 4 \to \mathcal{N} = 1$ ($AdS_5 \times S^3 \times S^2$)
- break conformal invariance $M^{1,3} \times \underbrace{R^+ \times S^3 \times S^2}_{\text{conifold}}$



[Klebanov,Witten'98][Klebanov,Nekrasov'99][Klebanov,Tseytlin'00]

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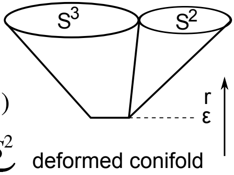


[Klebanov,Strassler'00]

Type IIB supergravity

Fields (bosonic sector)

- scalars: Φ, C_0
- 2-form potentials: B_2 and C_2 ($H_3 = \mathrm{d}B_2$, $F_3 = \mathrm{d}F_3$, $\tilde{F}_3 = F_3 - C_0 H_3$)
- metric $g_{\mu\nu}$
- 4-form potential C_4 ($F_5 = \mathrm{d}C_4$, $\tilde{F}_5 = F_5 + B_2 \wedge F_3$)

Equations of motion

$$\begin{aligned}
R_{MN} &= \frac{1}{2}\,\partial_M\Phi\partial_N\Phi + \frac{1}{2}\,\mathrm{e}^{2\Phi}\partial_M C_0 \partial_N C_0 + \frac{1}{4}\left(\mathrm{e}^{-\Phi} H_{MPQ} H_N{}^{PQ} + \mathrm{e}^{\Phi}\tilde{F}_{MPQ}\tilde{F}_N^{PQ}\right) \\
&\quad -\frac{1}{48}\,g_{MN}\left(\mathrm{e}^{-\Phi} H_{PQR} H^{PQR} + \mathrm{e}^{\Phi}\tilde{F}_{PQR}\tilde{F}^{PQR}\right) + \frac{1}{96}\tilde{F}_{MPQRS}\tilde{F}_N^{PQRS} \\
\mathrm{d}\star\mathrm{d}\Phi &= \mathrm{e}^{2\Phi}\mathrm{d}C_0 \wedge \star\mathrm{d}C_0 - \frac{1}{2}\,\mathrm{e}^{-\Phi} H_3 \wedge \star H_3 + \frac{1}{2}\,\mathrm{e}^{\Phi}\tilde{F}_3 \wedge \star\tilde{F}_3 \\
\mathrm{d}(\mathrm{e}^{2\Phi}\mathrm{d}C_0) &= -\mathrm{e}^{\Phi} H_3 \wedge \star\tilde{F}_3, \qquad \mathrm{d}(\mathrm{e}^{\Phi}\star\tilde{F}_3) = F_5 \wedge H_3 \\
\star\tilde{F}_5 &= \tilde{F}_5, \qquad \mathrm{d}\star(\mathrm{e}^{-\Phi} H_3 - \mathrm{e}^{\Phi} C_0 \tilde{F}_3) = -F_5 \wedge F_3
\end{aligned}$$

Klebanov-Strassler solution

An interesting analytic solution (smooth solution on a deformed conifold)

[Klebanov,Strassler'00]

$$\mathrm{d}s^2 = h(r)^{-1/2}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + h^{1/2}(dr^2 + r^2\mathrm{d}\Omega_5^2)$$

$h(r)$ - warp factor. $h(r) \propto \frac{\log r}{r^4}, \; r \to \infty, \qquad h(r) \to \text{const}, \; r \to \epsilon$

$$\int_{S^3\times S^2} \tilde{F}_5 \propto N\log(r), \qquad \int_{S_3} F_3 = M$$

- logarithmic behavior of the warp factor is related to the running of the coupling constant(s)
- deformation of the conifold is equivalent to the dynamical generation of scale
- breaking of the $U(1)_R$ symmetry: $U(1)_R \to \mathbb{Z}_{2M} \to \mathbb{Z}_2$

Dual theory

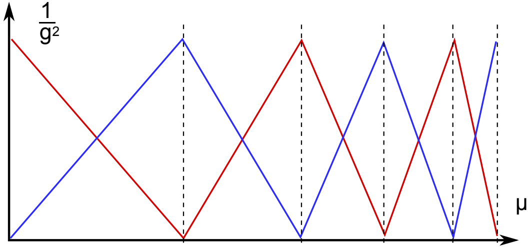
$\mathcal{N} = 1$ gauge theory with $SU(N+M) \times SU(N)$ gauge group and

- 2 doublets of bifundamental fields

$$A_i \in (N+M, \overline{N}), \qquad B_j \in (\overline{N+M}, N), \qquad i,j = 1,2$$

- superpotential $W = \lambda \,\mathrm{Tr}\, \epsilon^{ik}\epsilon^{jl} A_i B_j A_k B_l$

At low energy the theory runs (through a cascade of Seiberg dualities) to a $SU(M)$ theory with Goldstone modes of broken baryonic symmetry



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- new type of theory with no UV fixed point

$$SU(M) \leftarrow SU(2M) \times SU(M) \leftarrow \dots$$
$$\dots \leftarrow SU(N-M) \times SU(N) \leftarrow SU(N+M) \times SU(N) \leftarrow \dots$$

- in the IR the theory is not quite the $\mathcal{N} = 1$ SYM [Gubser,Herzog,Klebanov'04]

Glueballs

Operator-state correspondence

$$J^{PC} \quad \underset{\text{QFT}}{\longleftrightarrow} \quad \mathcal{O}_{\mu\nu\dots} \quad \underset{\text{AdS/CFT}}{\longleftrightarrow} \quad \delta\phi_{MN\dots}$$

$$\Delta = 2 + \sqrt{m_5^2 R^2 + 4}$$

Poles of the 2-point functions are given be the spectrum of the linearized equations of supergravity

$$\begin{array}{ccccc} 2^{++} & \longleftrightarrow & T_{\mu\nu} & \longleftrightarrow & \delta g_{\mu\nu} \\ 1^{++} & \longleftrightarrow & J^5_\mu & \longleftrightarrow & \{\delta g_{\mu\psi}, \delta C_4\} \\ & & \dots & & \end{array}$$

The spectra of these 2 glueballs coincide: Ferrara-Zumino multiplet

States with the same quantum numbers can mix. Symmetries help sometimes

Quantum numbers

Symmetries of the model

	$SU(N+M)$	$SU(N)$	$SU(2)$	$SU(2)$	$U(1)_B$	$U(1)_A$	$U(1)_R$
A_i	$\mathbf{N+M}$	$\overline{\mathbf{N}}$	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{2N+N}$	$\frac{1}{2N+N}$	$\frac{1}{2}$
B_j	$\overline{\mathbf{N+M}}$	$\mathbf{N}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{-1}{2N+N}$	$\frac{1}{2N+N}$	$\frac{1}{2}$

- Parity $(x^0, x^i) \to (x^0, -x^i)$, $A_0(x^0, -x^i) \to A_0(x^0, x^i)$, $A_j(x^0, -x^i) \to -A_j(x^0, x^i)$

$$\int d^4x\sqrt{-\det(g_{\mu\nu} + e^{-\Phi/2}(B_{\mu\nu} + F_{\mu\nu}))} + \int (C_0 F\wedge F + C_2 \wedge F + C_4)$$

- $\mathcal{I}$-symmetry ($C \times A_i \leftrightarrow B_i$): interchanges $S^2 \times S^2$ in $T^{1,1} \simeq S^3 \times S^2$

in terms of Euler angles $\quad \theta_1 \leftrightarrow \theta_2\,, \quad \phi_1 \leftrightarrow \phi_2$

flips the sign of 3-forms $\quad H_3 \to -H_3\,, \quad F_3 \to -F_3$

SUSY

Glueballs must arrange themselves in the supermultiplets:

$$(j - 1/2, j, j, j + 1/2), \qquad j = 0, \frac{1}{2}, 1, \frac{3}{2}$$

Supermultiplets have been classified in the conformal theory [Ceresole *et al.*'99]

SUSY manifests itself in the form of Supersymmetric Quantum Mechanics

$$Q = \begin{pmatrix} 0 & \partial_u - W \\ \partial_u + W & 0 \end{pmatrix}, \qquad Q^2 \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = -m_4^2 \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$

SQM may help in the calculation of the spectrum [Dymarsky,DM'07]

Singlet sector

Gravity approximation only gives states with spin ≤ 2.

- $\mathcal{I}$-odd sector

[Benna,Dymarsky,Solovyov'07]
[Dymarsky,DM,Solovyov'08]

$0^{+-}, 1^{+-}$	$0^{+-}, 1^{+-}$	$0^{--}, 1^{--}$	$1^{+-}, 1^{--}$	$1^{+-}, 1^{--}$
$\mathrm{Tr}\,(A e^V \bar{A} e^{-V} - B e^V \bar{B} e^{-V})$	$\mathrm{Tr}\, e^V \bar{W}_{\dot\alpha} e^{-V} W^2,\ \mathrm{Tr}\, e^V W_\alpha e^{-V} \bar{W}^2$			

- $\mathcal{I}$-even sector

[Berg,Haack,Mueck'05'06]
[Dymarsky,DM'06'07][Gordeli,DM'09]

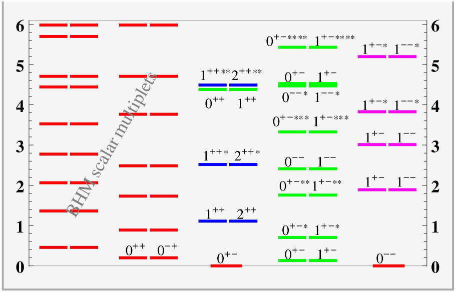
$1^{++}, 2^{++}$	$0^{++}, 1^{++}$
$\mathrm{Tr}\, W_\alpha e^V \bar{W}_{\dot\alpha} e^{-V}$	$\mathrm{Tr}\, W^2 \bar{W}^2$

$0^{++}, 0^{-+}$	$0^{++}, 0^{-+}$	$0^{++}, 0^{-+}$	$0^{++}, 0^{-+}$	$0^{++}, 0^{-+}$	$0^{++}, 0^{-+}$
$\mathrm{Tr}\, W^2,\ \mathrm{Tr}\, W^2 e^{-V} \bar{W} e^V$					

[Dymarsky,Gordeli,DM]

Summary

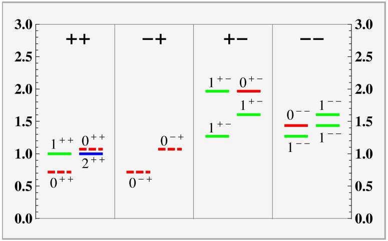
- Spectrum of $SU(2) \times SU(2)$ singlet glueballs in the large N, large λ Klebanov-Strassler theory [Gordeli,DM'11]

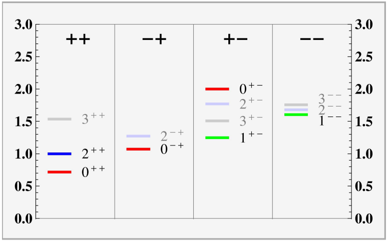


- no prediction for the higher spin states (α' corrections)
- typical scaling of the highly excited states $m_n^2 \sim n^2$

Summary

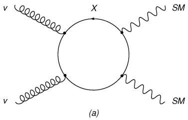
Pure gauge sector vs lattice calculations [Dymarsky,DM,Solovyov'08]





Outlook I

Imagine a hidden sector with a strongly coupled gauge group and heavy flavors, so that the lightest states are glueballs. Such a sector can be observed if flavor fields are charged under the SM group: [Juknevich,DM,Strassler'09]



Factorization of the amplitude

$$\frac{\alpha_i \alpha_v}{M^4}\, \chi_i\, C_S \langle G^a, G^b | \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu} | 0 \rangle \, \langle 0 | S | 0^{++} \rangle$$

$$\Gamma_{0^+ \to gg} = \frac{\alpha_s^2 \alpha_v^2}{16\pi M^8} (N_c^2 - 1) \chi_s^2 C_S^2 m_{0^+}^3 (\mathbf{F}^{\mathbf{S}}_{\mathbf{0}^{++}})^2, \qquad \frac{\Gamma_{0^+ \to \gamma\gamma}}{\Gamma_{0^+ \to gg}} = \frac{1}{2} \frac{\alpha^2}{\alpha_s^2} \frac{\chi_\gamma^2}{\chi_s^2}$$

It would be interesting to get estimates for the matrix elements in the strongly coupled sector:

$$\langle 0 | \mathcal{O} | J^{PC} \rangle$$

Outlook II

One can add matter to the Klebanov-Strassler theory (analog of Sakai-Sugimoto construction) and study mesons and baryons

[Dymarsky,Kuperstein,Sonnenschein'09][Dymarsky,DM,Sonnenschein'10]

- a reasonable model of nuclear force
- estimate glueball-meson and glueball-baryon couplings (alternative to the Sakai-Sugimoto)