AdS/CFT and applications to strongly interacting systems

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Plan

- AdS/CFT correspondence, gauge/gravity duality
- Viscosity
- Hydrodynamics and black holes
- Gravity dual models of unitarity fermions?

Not covered:

- Interaction of fluid with hard probes (quarks, mesons)
- Holographic superfluids

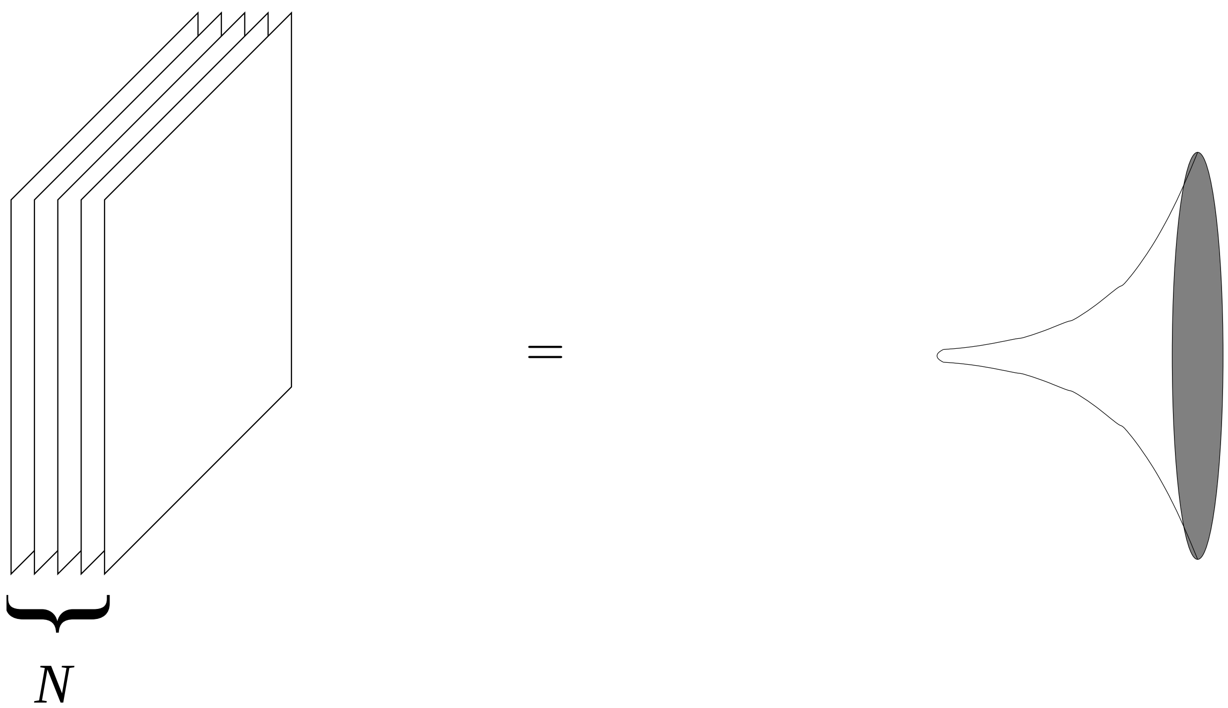
AdS/CFT correspondence (gauge/gravity duality)

Maldacena 1997: stack of N D3-branes in type IIB string theory can be described in two different pictures:

As a quantum field theory describing fluctuations of the branes: $\mathcal{N} = 4$ super-Yang-Mills theory

As string theory on a a curved spacetime $AdS_5 \times S_5$

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\mathbf{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$



Mapping of parameters

- Parameters of gauge theory g, N; 't Hooft coupling $g^2 N$.
- String theory side has three parameters
  - String length ℓ_s:
  - String coupling $g_{\rm st}$
  - Curvature of space R

Mapping between parameters:

$$
\begin{aligned}
g^2 &= 4\pi g_{\rm st} \\
g^2 N_c &= \frac{R^4}{\ell_s^4}
\end{aligned}
$$

$g^2 N_c \gg 1 \Leftrightarrow \ell_s \ll R$

Einstein gravity instead of string theory

Reliable calculation in a strongly coupled field theory through its gravity dual.

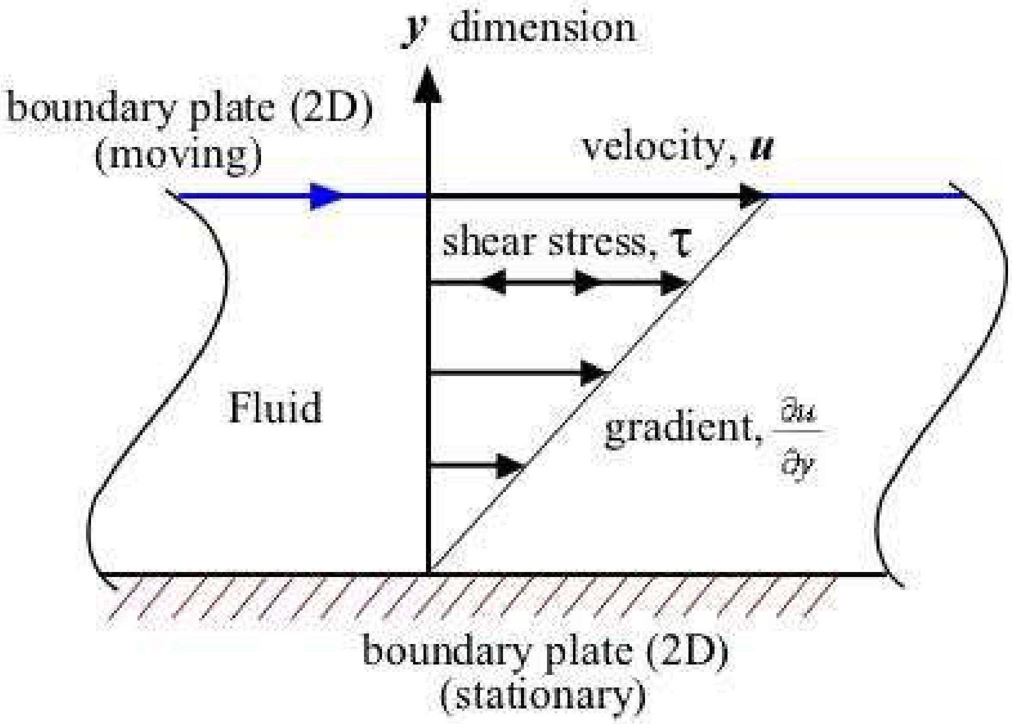
How about QCD?

- No gravity dual for QCD
- There exist theories with gravity duals which are similar to QCD: confinement, chiral symmetry breaking
- but no asymptotic freedom

What can we do?

- Use the $\mathcal{N} = 4$ SYM plasma as a simplest model of a strongly coupled plasma
- Many similarities to real quark-gluon plasma: deconfinement, Debye screening
- Recently has shed light on the behavior of viscosity at strong coupling

Viscosity

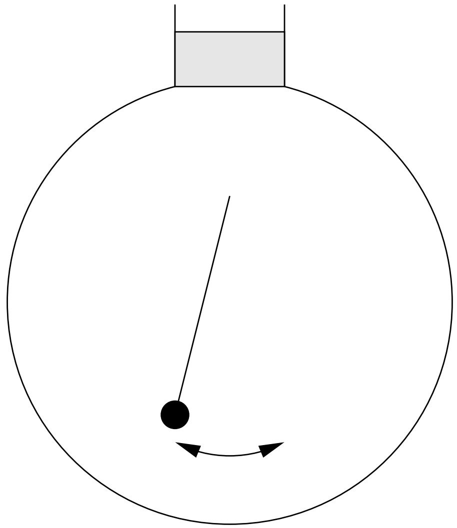


Friction force between two plates:

$$F = \eta A \frac{\partial u_x}{\partial y}$$

Viscosity: counter-intuitive behavor

Boyle and Hooke's experiment # 26 (1660)



pendulum in a glass container

pump out air: naively one expects oscillations to last longer: less dissipation

Actual result: no appreciable change in damping rate

Maxwell's formula for viscosity

Viscosity *diverges* when interaction is turned off.

Maxwell's estimate of the viscosity:

$$\eta \sim \rho v \ell = \text{mass density} \times \text{velocity} \times \text{mean free path}$$

$$\ell = \frac{1}{n \sigma v}$$

At fixed temperture $\sigma =$ const, $\ell \sim \rho^{-1}$

$\Rightarrow$ viscosity does not depend on pressure (at fixed T)

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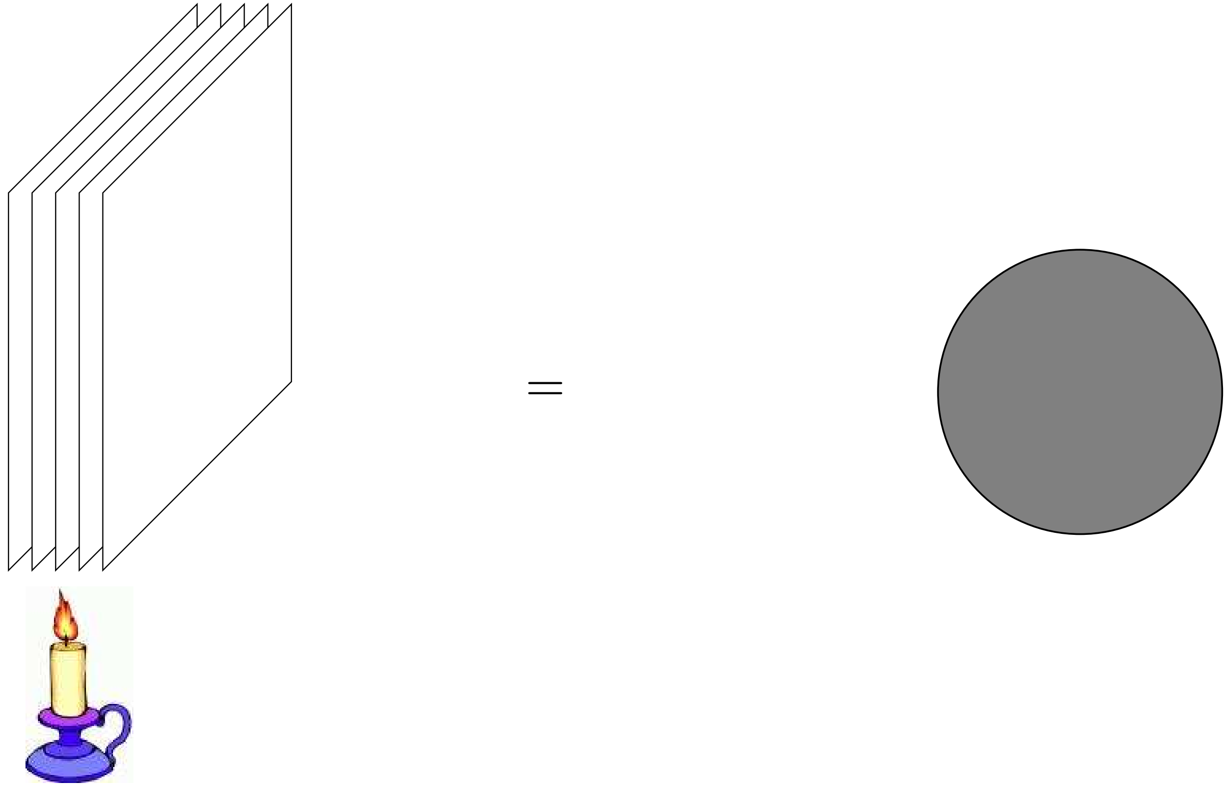
Experimentally verified by Maxwell, with his wife's help

Gauge/gravity duality at finite temperature



“Quark gluon plasma” = black hole (in anti de-Sitter space)

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\mathbf{x}^2] + \frac{R^2}{r^2 f}dr^2 + R^2 d\Omega_5^2$$

$f(r) = 1 - r_0^4/r^4$, $T = r_0/\pi R^2$

Viscosity/entropy density ratio

One of the earliest results is the universal value of η/s.

- Viscosity $\sim$ absorption cross section for low energy gravitons
  - From AdS/CFT rules and Kubo's formula for viscosity
- At low energies, absorption cross section $=$ area of horizon
- Entropy is also proportional to area of horizon (Bekenstein-Hawking formula)

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

where η is the shear viscosity, s is the entropy per unit volume.

- Universal for all theories with gravity duals

η/s and mean free path

In kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

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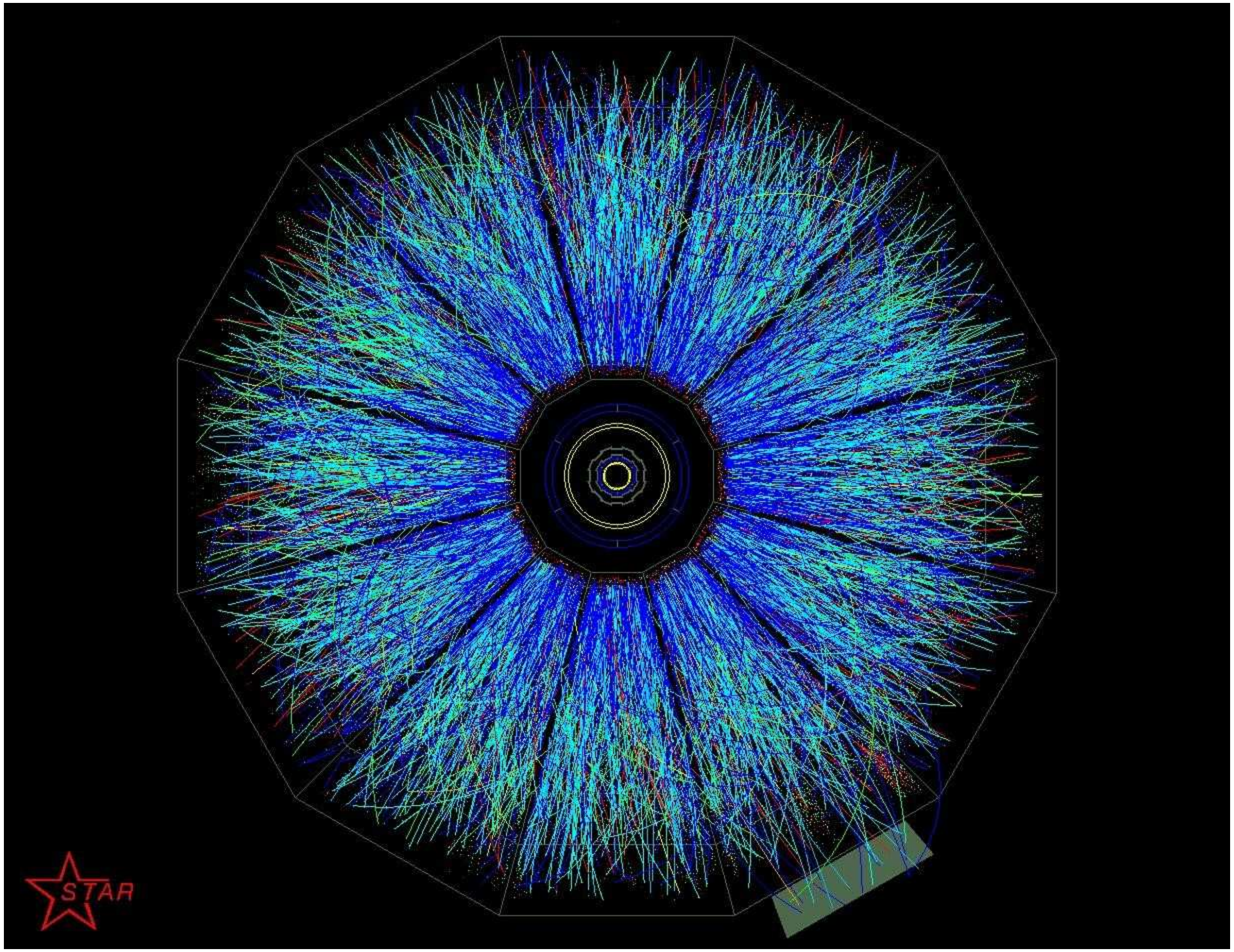
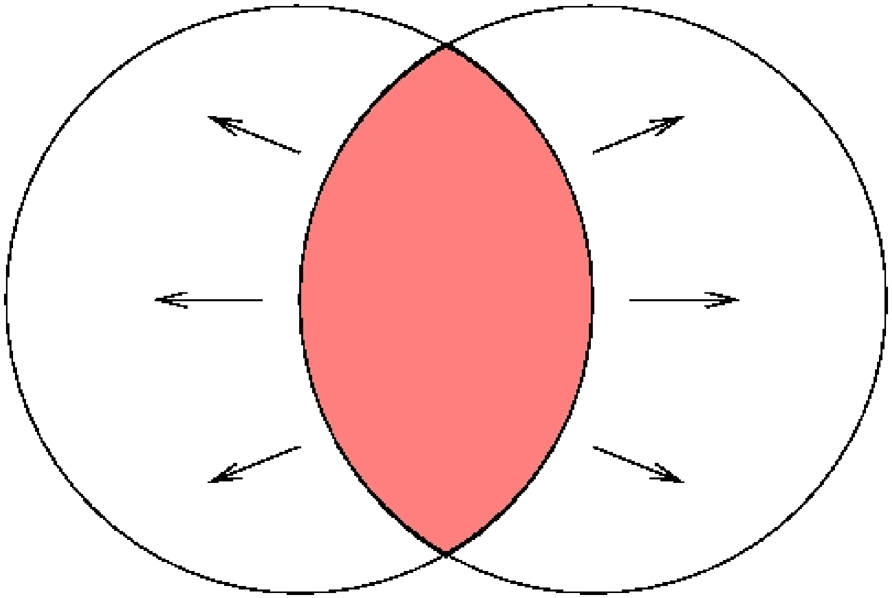
- Mean free path cannot be much shorter than de Broglie wavelength
  Danielewicz, Gyulassy 1980s
- The stronger the coupling, the smaller is η/s.
- Theories with gravity dual reach $\hbar/4\pi$: very short mean free path (of order the de Broglie wavelength)
- All laboratory liquids have η/s larger than $\hbar/4\pi$
- "The most ideal fluids" as those with η/s close to $\hbar/4\pi$
  - Quark-gluon plasma at RHIC?

Heavy ion collisions

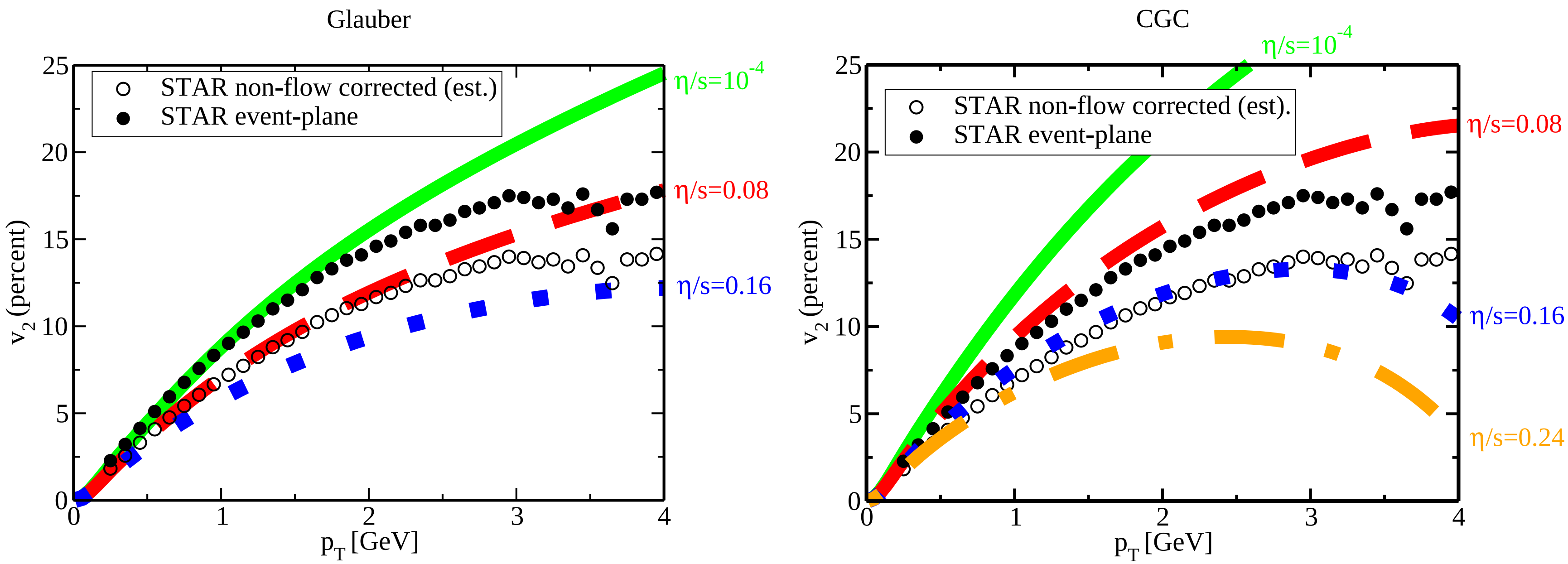


How one can try to determine η of QGP



- Look at scattering events with nonzero impact parameter (select by number of final particles)
- Distribution of particles over momentum is not axially symmetric: characterized by “elliptic flow” parameter v_2
- Naturally explained if the matter acts like a liquid: more pressure along the smaller axis of the hot region.
- Too large viscosity kills v_2
- Hydrodynamic simulations can give estimates for v_2

Recent numerical simulations

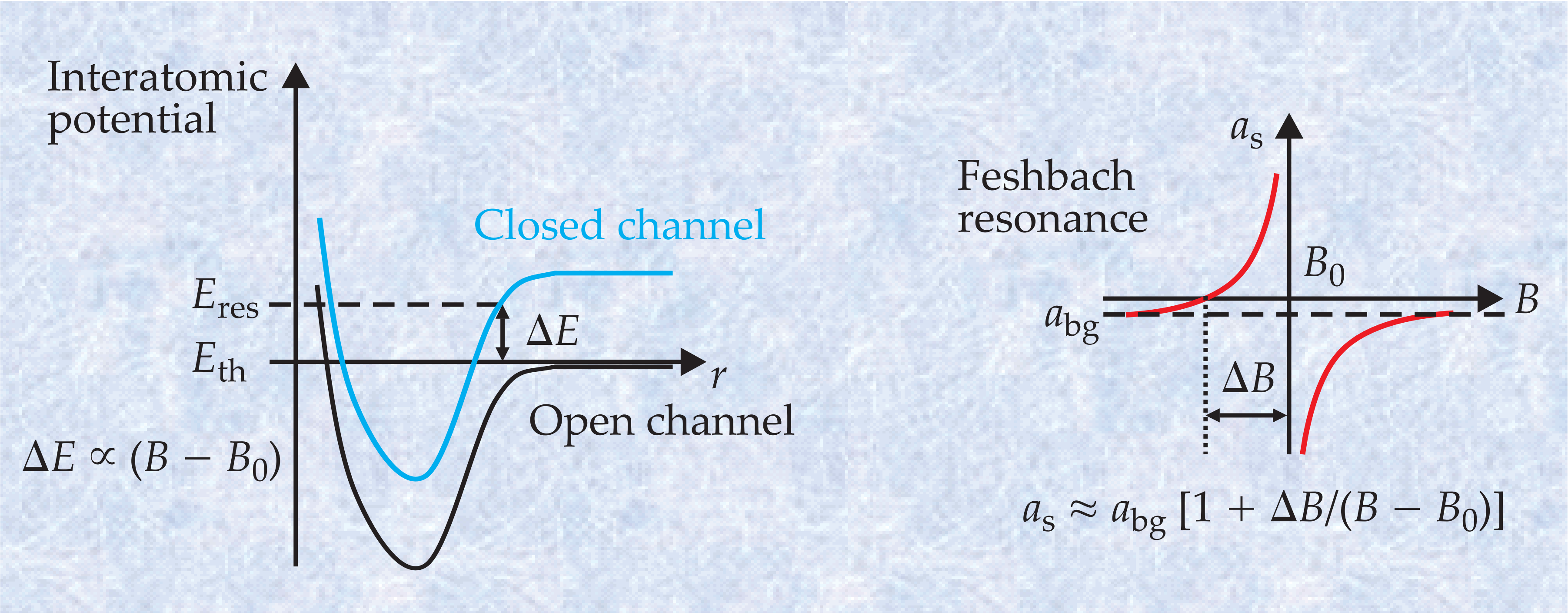


(from Luzum and Romatschke, arXiv:0804)
Remarks:

- Reproduce main features of the data
- Sensitive to the choice of initial condition
- η/s is at most a few times $\hbar/(4\pi)$

$$\frac{\eta}{s} = 0.1 \pm 0.1(\text{th}) \pm 0.08(\text{exp})$$

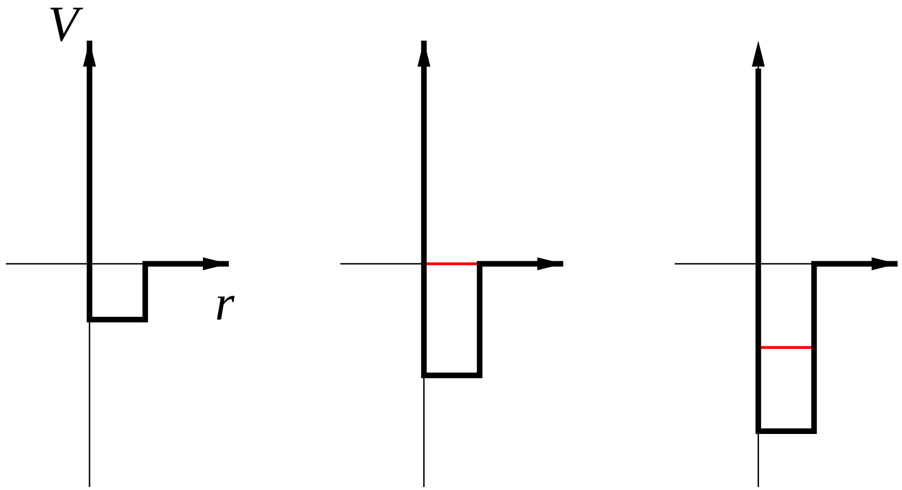
Non-relativistic AdS/CFT



Unitarity fermions

Consider two nonrelativistic fermions interacting through a potential
Assume a square-well potential, with fixed but small range r_0.

- $V_0 < 1/mr_0^2$: no bound state
- $V_0 = 1/mr_0^2$: one bound state appears, at first with zero energy
- $V_0 > 1/mr_0^2$: at least one bound state



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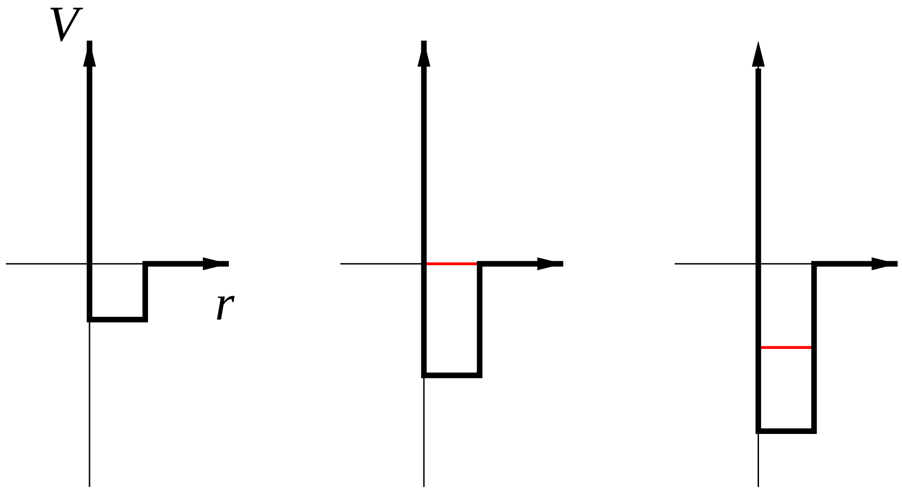


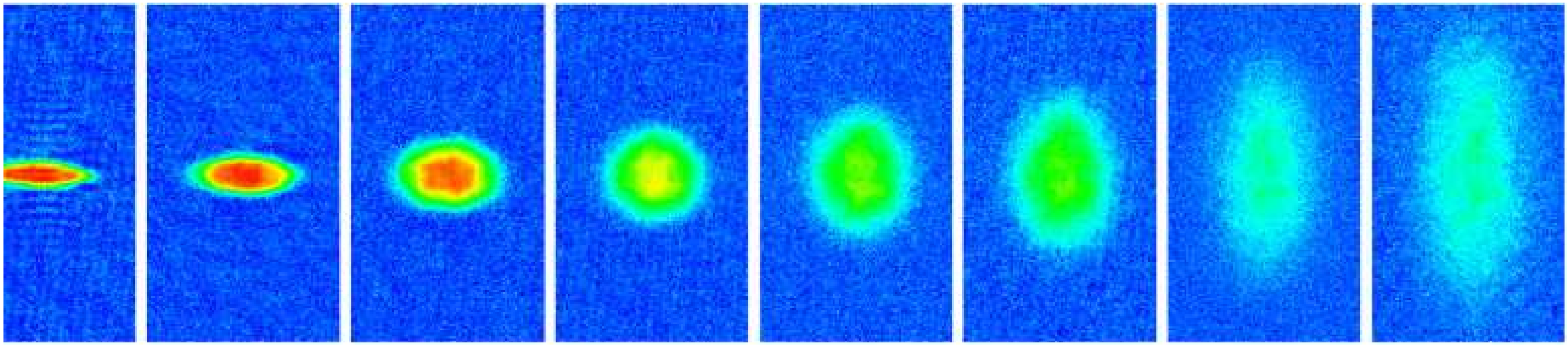
Unitarity regime: take $r_0 \to 0$, keeping one bound state at zero energy.

In this limit: no intrinsic scale associated with the potential

In the language of scattering theory: infinite scattering length $a \to \infty$

Examples of systems near unitarity

- Neutrons: $a = -20$ fm, $|a| \gg 1$ fm
- Trapped atom gases, with scattering length a controlled by magnetic field using Feschbach resonance
  - Analog of elliptic flow observed



Ground state energy

Finite density n, $T = 0$

Dimensional analysis: no intrinsic length/energy scale, only density n. So:

$$\epsilon \equiv \frac{E}{V} = \# \frac{n^{5/3}}{m}$$

The same parametric dependence as the energy of a free gas

$$\epsilon(n) = \xi \epsilon_{\text{free}}(n)$$

ξ is called the Bertsch parameter.

Numerically: $\xi \approx 0.4$ (Monte-Carlo, ϵ expansion)

Embedding the Schrödinger algebra

- Symmetry of unitarity fermions: also symmetry of Schrödinger's equation Sch(d)

$$i\frac{\partial\psi}{\partial t} + \frac{\nabla^2}{2m}\psi = 0$$

- CFT$_{d+2}$: is the symmetry of the Klein-Gordon equation

$$\partial_\mu^2\phi = 0, \qquad \mu = 0, 1, \cdots, d+1$$

In light-cone coordinates $x^\pm = x^0 \pm x^{d+1}$ the Klein-Gordon equation becomes

$$-2\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-}\phi + \partial_i\partial_i\phi = 0, \qquad i = 1, \cdots, d$$

Restricting ϕ to $\phi = e^{-imx^-}\psi(x^+, \mathbf{x})$: Klein-Gordon eq. $\Rightarrow$ Schrödinger eq.:

$$2im\frac{\partial}{\partial x^+}\psi + \nabla^2\psi = 0, \qquad \nabla^2 = \sum_{i=1}^{d}\partial_i^2$$

This means Sch$(d) \subset$ CFT$_{d+2}$

Holographic realization of Schrödinger symmetry

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

DTS; Balasubramanian and McGreevy, 2008

The additional term is invariant only under nonrelativistic dilatation:

$$x^+ \to \lambda^2 x^+, \quad x^- \to x^-, \quad x^i \to \lambda x^i$$

$x^+ = t$, $\omega = p^2/2m$

Break CFT$_{d+2}$ down to Sch(d).

Is a solution to Einstein gravity coupled to massive gauge field.

Can be realized ($d = 2$) in string theory
Herzog, Rangamai, Ross; Maldacena, Martelli, Tachikawa;
Adams, Balasubramanian, McGreevy (2008)

Holographic nonrelativistic fluids

- Introduce finite temperature T and finite chemical potential μ
- Black hole solutions found Herzog et al; Maldacena et al; Adam et al
- No superfluidity (lacking field that can condense), unrealistic equation of state:

$$P = \# \frac{T^4}{(-\mu)^2}, \qquad \mu < 0,$$

- Viscosity: $\eta/s = 1/(4\pi)$
- Thermal conductitvity: can also be computed
  Rangamani, Ross, DTS, Thompson, to be published
  -
    $$\kappa = 2\frac{\epsilon + P}{\rho T}\eta$$
  - Prandtl number $= 1$

Conclusion

- $\mathcal{N} = 4$ SYM plasma: a useful model of a strongly coupled gauge plasma
- Hydrodynamics $=$ long-distance dynamics of black hole horizons
- Small η/s as a defining characteristics of strongly coupled QGP
- We need to be careful about drawing conclusions: theories with gauge/gravity duality are not QCD, no asymptotic freedom
- Progress comes from both formal and the phenomenological sides
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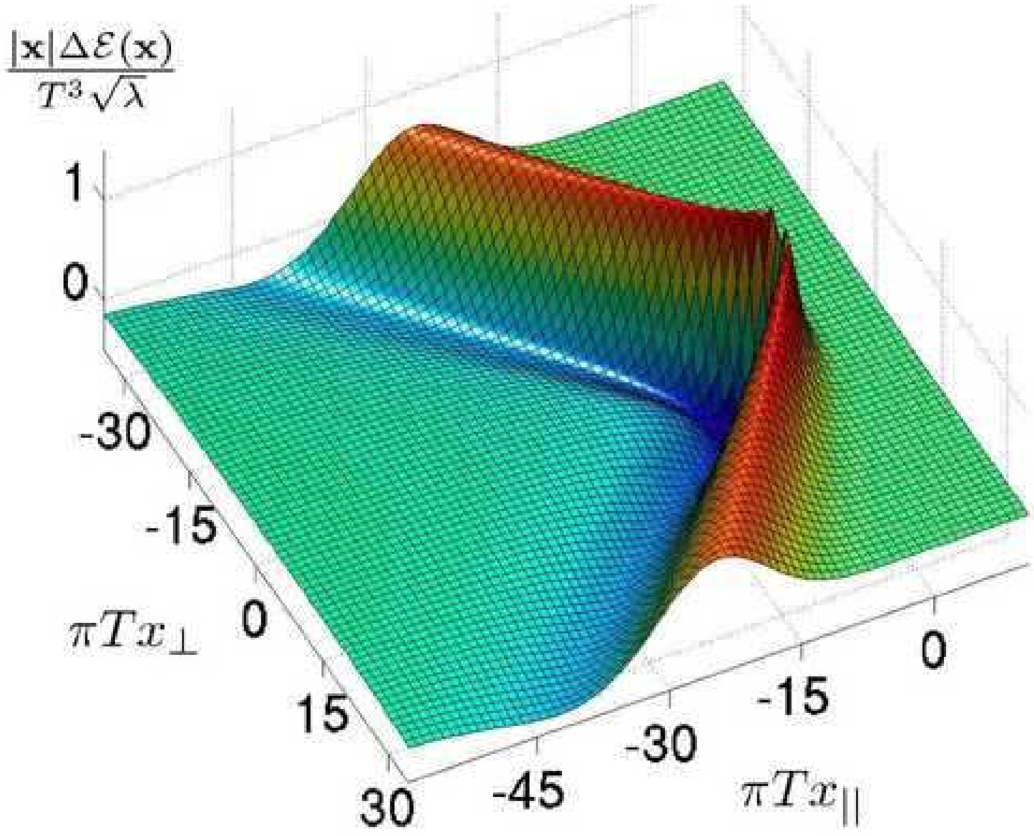
Looking foward to the LHC:

- Huge increase in the total c.m. energy: $0.2 \to 5.5$ TeV/nucleon
- Will be studied by ALICE, ATLAS, CMS
- Much more energetic jets, much more heavy quarks
- Medium is most likely still strongly interacting; interesting interplay between hard and soft physics
- Predictions are being made using AdS/CFT techniques, but clearly more theoretical advances are required.

Mach cone



(Chesler, Yaffe)