Evolution equation for the 3-quark Wilson loop operator

A.V. Grabovsky

Budker Inst. of Nuclear Physics and Novosibirsk State University

Low x workshop, 04.06.2013
Israel, Eilat

Outline

- Motivation
- Previous work
- Shock wave formalism
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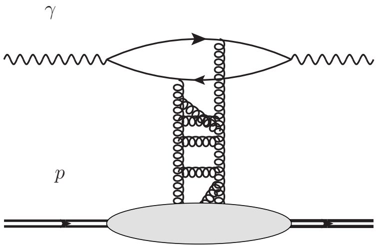
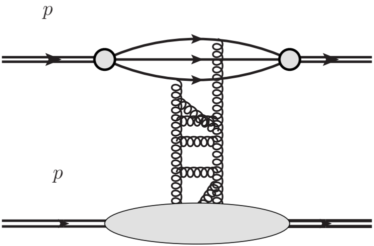
Motivation





Dipole picture ?

$$\sigma_{\gamma^*}(s, Q^2) = \int d^2\mathbf{r} \int_0^1 dx |\Psi_{\gamma^*}(\mathbf{r}, x, Q^2)|^2 \sigma_{dip}(\mathbf{r}, s).$$

$$\sigma_{dip}(\mathbf{r}, s) = 2 \int d\mathbf{b}(1 - \frac{1}{N_c} U_{12}(\mathbf{b}, \mathbf{r}, s))$$

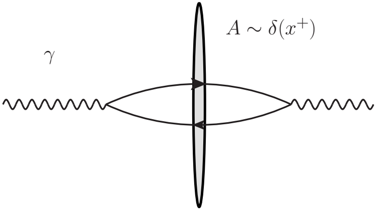
where U_{12} obeys the Balitsky-Kovchegov evolution equation

Previous work

- M. Praszalowicz and A. Rostworowski, 1998 - Proton wave function with one and two gluon emissions was studied. Indication that new color structures, not only dipoles and three-quark singlets (like proton) appear.
- Y. Hatta, E. Iancu, K. Itakura and L. McLerran, 2005 - Odderon in the color glass condensate was studied. Linear evolution equation for 3-quark Wilson line (its C-odd part) was obtained in the coordiante representation. It was shown that this equation is equivalent to the BKP equation in the momentum representation.
- J. Bartels and L. Motyka, 2008 - Wave function, impact factor were studied. Gluon radiation was diagonalized into the evolution of 2-quark, 3-quark, and 4-quark states in C-even and C-odd states obeying the BKP equations with nonlinear terms.

Shock wave formalism

LO I. Balitsky 1996, NLO I. Balitsky and G. Chirilli 2006-2010



Color field of a fast moving particle $A^- \sim \delta(z^+)A^\eta(z_\perp)$

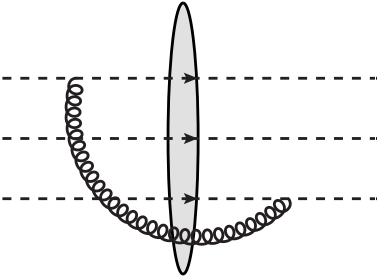
$A^\eta(z_\perp)$ contains slow components with rapidities $< \eta$

Quark propagator in such an external field $G(x,y) \sim U^\eta(z_\perp)$

DIS matrix element contains a Wilson loop = color dipole operator $U^\eta_{12} = tr(U^\eta(z_{1\perp})U^{\eta\dagger}(z_{2\perp}))$.

LO Evolution equation for a 3-quark Wilson line

$$B^{\eta}_{123} = \varepsilon^{i'j'h'}\varepsilon_{ijh} U^{\eta}(\vec{z}_1)^{i}_{i'} U^{\eta}(\vec{z}_2)^{j}_{j'} U^{\eta}(\vec{z}_3)^{h}_{h'} = U_1 \cdot U_2 \cdot U_3$$



$$\frac{\partial B^{\eta}_{123}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2}\int d\vec{z}_4 \left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}}\left(-B^{\eta}_{123} + \frac{1}{6}\left(B^{\eta}_{144}B^{\eta}_{324} + B^{\eta}_{244}B^{\eta}_{314} - B^{\eta}_{344}B^{\eta}_{214}\right)\right)\right.$$

$$\left. + (1 \leftrightarrow 3) + (2 \leftrightarrow 3)\right].$$

C-even case

We build C-even and C-odd operators with
$B^{\eta}_{\bar{1}\bar{2}\bar{3}} = \varepsilon^{i'j'h'}\varepsilon_{ijh} U^{\eta\dagger}(\vec{z}_1)^{i}_{i'} U^{\eta\dagger}(\vec{z}_2)^{j}_{j'} U^{\eta\dagger}(\vec{z}_3)^{h}_{h'} = U^{\dagger}_1 \cdot U^{\dagger}_2 \cdot U^{\dagger}_3$

$$B^{+}_{123} = B^{\eta}_{123} + B^{\eta}_{\bar{1}\bar{2}\bar{3}} - 12, \quad B^{-}_{123} = B^{\eta}_{123} - B^{\eta}_{\bar{1}\bar{2}\bar{3}}$$

$$B^{+}_{123} = \frac{1}{2}(B^{+}_{133} + B^{+}_{211} + B^{+}_{322}) + \tilde{B}^{+}_{123},$$

where $\tilde{B}^{+}_{123}$ works from the 4-gluon exchange. In SU(3)

$$B_{iij} = 2tr(U_j U^{\dagger}_i)$$

$\rightarrow B^{+}_{123}$ splits into 3 LO C-even BK Green functions and one NLO contribution. cf. Bartels and Motyka 2007 ?

C-odd case

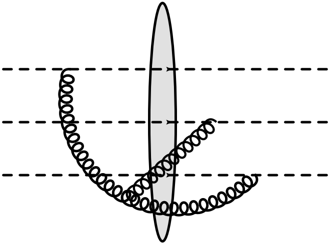
$$\frac{\partial B^-_{123}}{\partial \eta} = \frac{\alpha_s 3}{4\pi^2} \int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}} \Big[B^-_{423} + B^-_{143} - B^-_{123}$$

$$-B^-_{124} - B^-_{443} + B^-_{424} + B^-_{144} + \frac{1}{12}\left(B^+_{144}B^-_{324} + B^+_{244}B^-_{314} - B^+_{344}B^-_{214}\right)$$

$$+ \frac{1}{12}\left(B^-_{144}B^+_{324} + B^-_{244}B^+_{314} - B^-_{344}B^+_{214}\right)\Big] + (2 \leftrightarrow 3) + (1 \leftrightarrow 3).$$

The linear part of this result coincides with the linear result of Hatta, Iancu, Itakura, McLerran 2005, which they proved to coincide with the BKP equation.

NLO corrections: connected part



$$K^{conn}_{NLO}\otimes B^{\eta}_{123} = \frac{\alpha_s^2}{8\pi^3}\int d\vec{z}_0 \left[\frac{(\vec{z}_{10}\vec{z}_{20})}{\vec{z}_{10}^{\,2}\vec{z}_{20}^{\,2}} - \frac{(\vec{z}_{30}\vec{z}_{20})}{\vec{z}_{30}^{\,2}\vec{z}_{20}^{\,2}}\right] \ln\frac{\vec{z}_{30}^{\,2}}{\vec{z}_{31}^{\,2}} \ln\frac{\vec{z}_{10}^{\,2}}{\vec{z}_{31}^{\,2}} \left(B_{100}B_{320} - B_{300}B_{210}\right)$$

$$+\frac{\alpha_s^2}{4\pi^4}\int d\vec{z}_0 d\vec{z}_4 \{(U_2U_0^{\dagger}U_1)\cdot U_4\cdot(U_0U_4^{\dagger}U_3)+(U_3U_4^{\dagger}U_0)\cdot U_4\cdot(U_1U_0^{\dagger}U_2)-(1,4\leftrightarrow 3,0)\}$$

$$\times\left[\frac{1}{2\vec{z}_{04}^{\,2}}\frac{(\vec{z}_{10}\vec{z}_{34})}{\vec{z}_{10}^{\,2}\vec{z}_{34}^{\,2}} + \frac{(\vec{z}_{10}\vec{z}_{40})}{\vec{z}_{10}^{\,2}\vec{z}_{40}^{\,2}}\frac{(\vec{z}_{24}\vec{z}_{34})}{\vec{z}_{24}^{\,2}\vec{z}_{34}^{\,2}} + \frac{(\vec{z}_{04}\vec{z}_{34})}{\vec{z}_{04}^{\,2}\vec{z}_{34}^{\,2}}\frac{(\vec{z}_{10}\vec{z}_{20})}{\vec{z}_{10}^{\,2}\vec{z}_{20}^{\,2}} - \frac{(\vec{z}_{20}\vec{z}_{10})}{\vec{z}_{02}^{\,2}\vec{z}_{01}^{\,2}}\frac{(\vec{z}_{24}\vec{z}_{34})}{\vec{z}_{24}^{\,2}\vec{z}_{34}^{\,2}}\right]\ln\frac{\vec{z}_{02}^{\,2}}{\vec{z}_{24}^{\,2}}$$

$$+(2\leftrightarrow 1)+(2\leftrightarrow 3).$$

NLO corrections: connected part

- For C-odd case the connected part of the linearized kernel for the 3-quark Wilson loop should be equivalent to the connected $3 \to 3$ NLO odderon kernel calculated by J. Bartels, V.S.Fadin, L.N.Lipatov, G.P.Vacca 2012.
- Taking Fourier transform one sees that it is not the case.
- Therefore the kernels must be connected by an equivalence transformation $K \to K + [K_{LO}, O]$.

Summary

LO

- The nonlinear LO evolution equation (closed in color space) for a 3QWL operator.
- The nonlinear LO evolution equation for the NLO C-even baryon Green function starting from the 4-gluon exchange. (Absent in the dipole picture).
- LO results in R.E. Gerasimov, A.V. Grabovsky JHEP 1304 (2013) 102.

NLO

- connected part of the 3QWL NLO kernel.

Work in progress and plans

- Calculation of the disconnected part of the NLO 3QWL kernel.
- Comparison to BKP kernel.

Thank you for your attention