Inclusive cross-section for gluon production in deuteron-deuteron collisions in the BFKL approach

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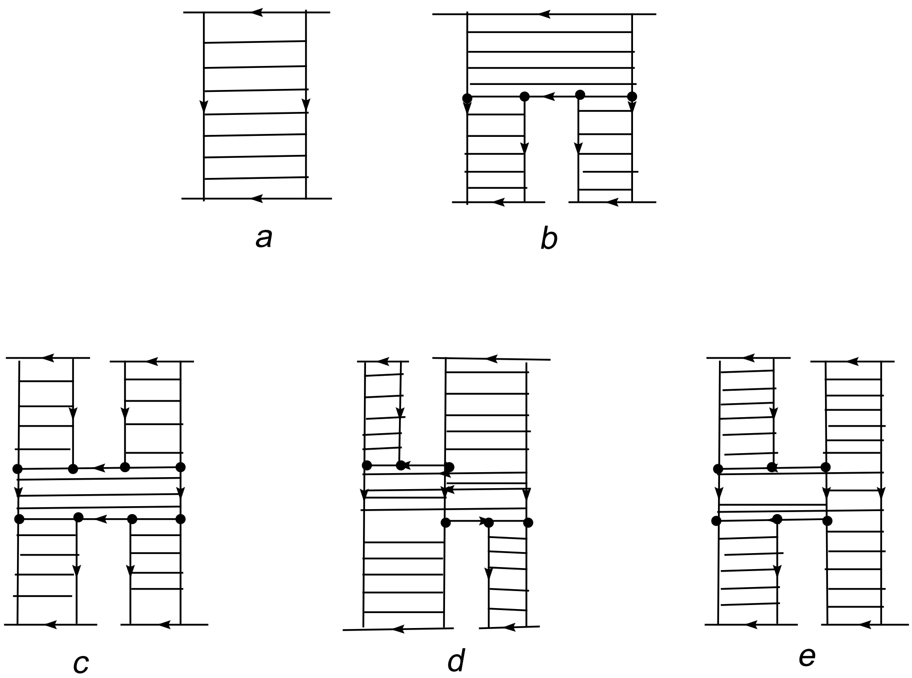
I. Orders of magintude



Diagrams for collisions between two pairs of nucleons in the leading order in N_c

Orders of maginute can be estimated from the lowest order in α_s: in higher orders the leading order will be multiplied by powers of $\alpha_s N_c y \sim 1$.

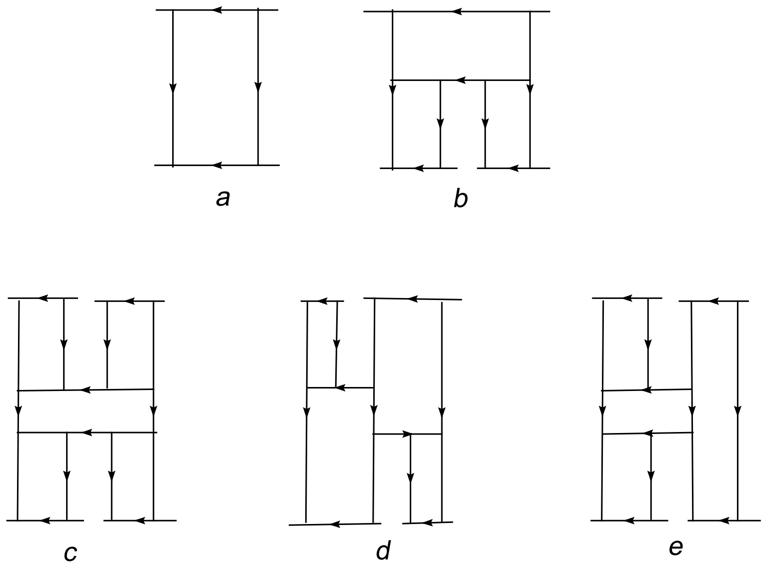
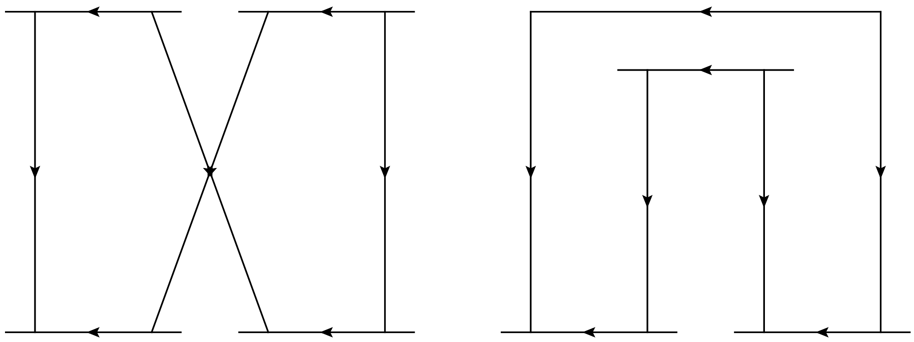


Fig. 1. Diagrams for collisions between two pairs of nucleons in the leading order in N_c and α_s

The dominant is the double gluon exchange: suppressing couplings inside the participants it is $\propto N_c^2$ and does not depend on α_s. Relative order of the rest diagrams:

Diagram b is $\propto \alpha_s^2 N_c^2$.
Diagram c is $\propto \alpha_s^4 N_c^4$.
Diagrams d and e are $\propto \alpha_s^3 N_c^3$.

Connected diagrams subdominant in $1/N_c^2$ in the lowest order (with redistribution of colour) have the same order as the double gluon exchange, so that their relative order is unity.



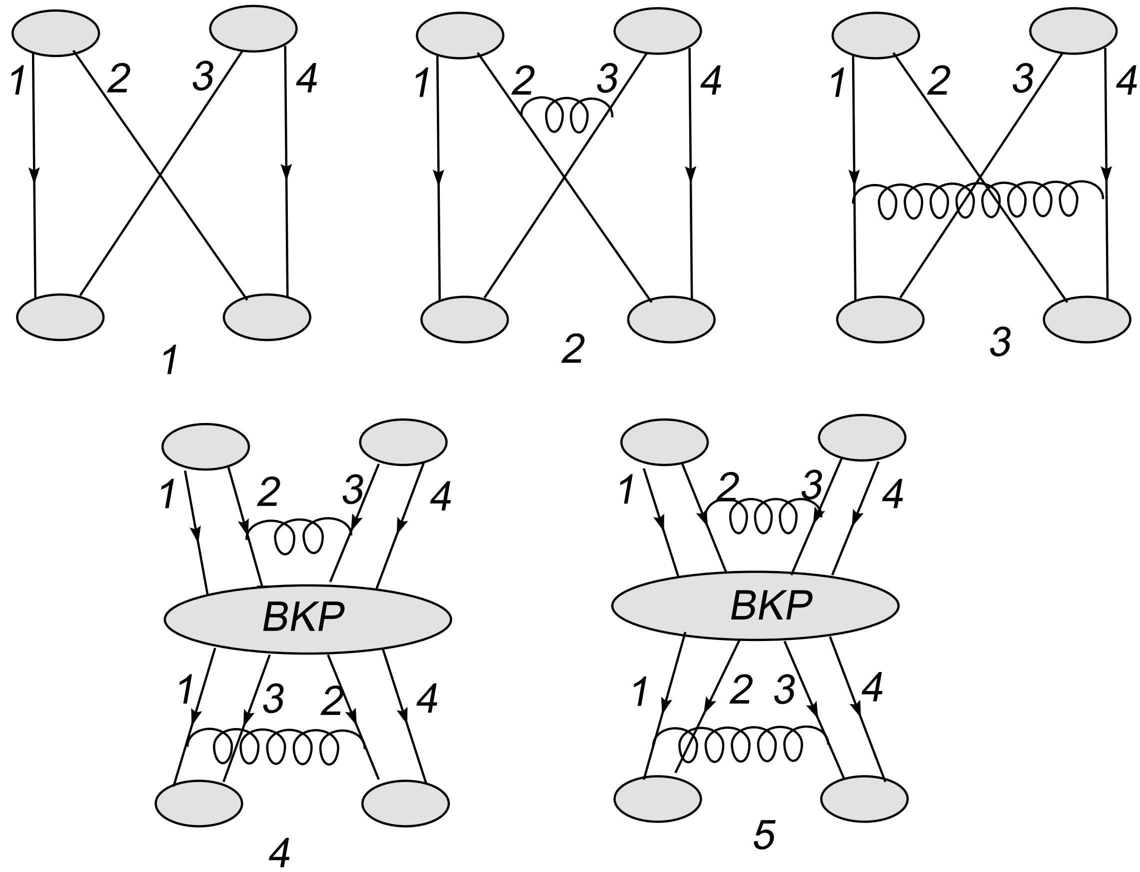
a *b*

Taking into account factors Y from integrations over rapidities and BFKL kinematics conditions $\alpha_s N_c y \sim 1$, $\alpha_s N_c << 1$ final orders are

Diagram b, d and $e \propto \alpha_s N_c$
Diagram $c \propto (\alpha_s N_c)^2$

Conclusion: apart from the double gluon exschange the dominant contribution comes from the diagrams of the relative order $1/N_c^2$ (with colour redisitribution or without).

In higher orders they are



Connected diagrams subdominant in $1/N_c^2$

Total cross-sections

The high-energy part H of the forward scattering amplitude in AB scattering

$$H = -i\pi^2 \delta(\kappa_+)\delta(q_-) N_c^2 s^2 D.$$

κ and q are the transferred momenta,

The total cross-section for double scattering

$$\sigma_{AB}^{(2)}(Y,b) = \frac{1}{4} A(A-1)B(B-1)T_{AB}^{(2)}(b)D(Y),$$

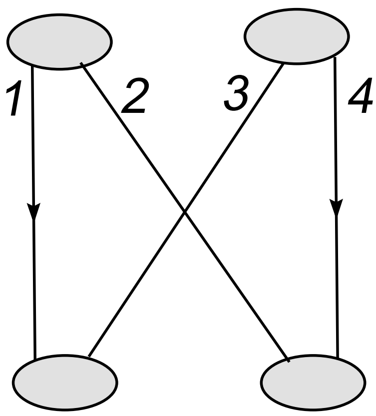
where

$$T_{AB}^{(2)}(b) = \int d^2 b_A T_A^2(b_A) T_B^2(b-b_A)$$

For dd scattering

$$\sigma_{dd}(Y) = \frac{1}{4}\left\langle \frac{1}{2\pi r^2}\right\rangle_A \left\langle \frac{1}{2\pi r^2}\right\rangle_B D(Y).$$

II.1. Pomerons sewed with the redistribution of color.



In the multirapidity (multienergy) formalism the pomeron as a function of two energies is given by

$$\mathcal{P}_{EK}(\epsilon_1 k_1) = \Delta_1 \Delta_2 \hat{P}_{EK}(k_1).$$

where $E = \epsilon_1 + \epsilon_2,\ K = k_1 + k_2,\ \Delta_i = \epsilon_i + \omega_i$ where $\omega_{1,2}$ are the gluon Regge trajectories and the "amputated" pomeron is $\hat{P} = (E + \omega_1 + \omega_2)P$. Using $\mathcal{P}_{EK}$ the diagram is found to be

$$D_1(E_i) = \int \frac{d\epsilon_1 d^2 q}{(2\pi)^3} \prod_{i=1}^{4} \Delta_i \hat{P}_{E_i}(q)$$

where ϵ_i are expressed via ϵ_1, $E_i = E_{12}, E_{34}, E_{13}, E_{14}$ and all ω's are assumed to have a small positive imaginary part. Integration over relative energies gives

$$D_1(E_i) = -i \int \frac{d^2q}{(2\pi)^2}$$

$$\times \Big(E_{12} + E_{34} + 4\omega(q)\Big) \prod_{i=1}^{4} P_{E_i}(q)$$

Passing to rapidities

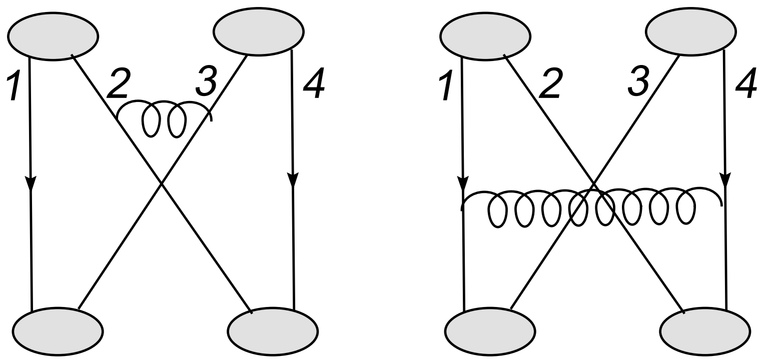
$$D_1(y) = -\int \frac{d^2q}{(2\pi)^2}\Big(4\omega(q) - \frac{\partial}{\partial y}\Big)$$

$$\times \int_0^y dy_1 P^2(y_{01}, q) P^2(y_1, q).$$

Here and in the following $y_0 \equiv y$ and $y_{ik} = y_i - y_k$.

Obvious infrared divergency.

II.2. One interaction between the pomerons



In terms of amputated pomerons

$$D_{2a} = i \int \frac{d\epsilon_1 d\epsilon_4 d^2q_1 d^2q_4}{(2\pi)^6} V(q_1, q_4|q_4, q_1)$$

$$\times \frac{\widehat{P}_{E_{12}}(q_1)\widehat{P}_{E_{13}}(q_1)}{(\epsilon_1+\omega_1)(E_{12}-\epsilon_1+\omega_1)(E_{13}-\epsilon_1+\omega_1)}$$

$$\times \frac{\widehat{P}_{E_{34}}(q_4)\widehat{P}_{E_{24}}(q_4)}{(\epsilon_4+\omega_4)(E_{34}-\epsilon_4+\omega_4)(E_{24}-\epsilon_4+\omega_4)}$$

where V is the BFKL interaction between the reggeized gluons.

After integration over the relative energies

$$D_{2a} = -i\int \frac{d^2q_1 d^2q_4}{(2\pi)^4} V(q_1,q_4|q_4,q_1)$$

$$\times P_{E_{12}}(q_1)P_{E_{34}}(q_4)P_{E_{13}}(q_1)P_{E_{24}}(q_4)$$

Separating the infrared stable and divergent parts

$$D_{2a} = i\int \frac{d^2q_1 d^2q_4}{(2\pi)^4} < q_1,q_4|H^P|q_4,q_1 >$$

$$\times P_{E_{12}}(q_1)P_{E_{34}}(q_4)P_{E_{13}}(q_1)P_{E_{24}}(q_4)$$

$$+2i\int \frac{d^2q}{(2\pi)^2}\omega(q)P_{E_{12}}(q)P_{E_{34}}(q)P_{E_{13}}(q)P_{E_{24}}(q)$$

where H^P is the infrared stable pomeron Hamiltonian.

Similar results follow for the second diagram.

In the sum with the direct sewing of pomerons D_1 infrared divergent terms with ω cancel and we get the infrared stable contributions

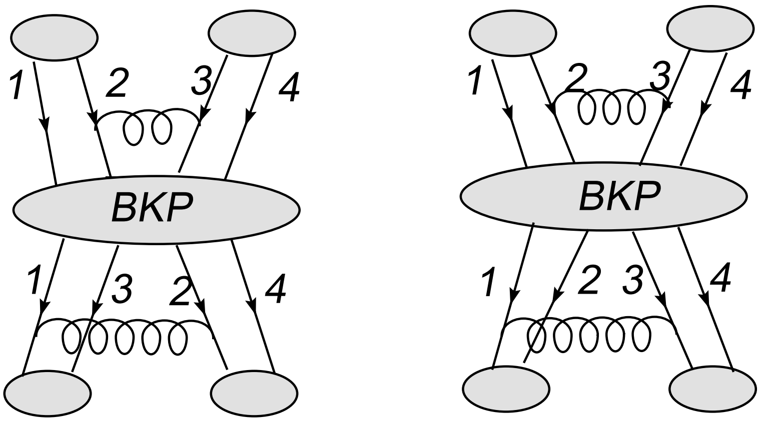
$$D_1 = \frac{\partial}{\partial y}\int_0^y dy_1 \int \frac{d^2q}{(2\pi)^2} P^2(y_{01},q)P^2(y_1,q)$$

$$D_2 = 2\int_0^y dy_1 \int \frac{d^2q d^2q'}{(2\pi)^4} < q,q'|H|q',q >$$

$$\times P(y_{01},q)P(y_{01},q')P(y_1,q)P(y_1,q').$$

II.3. Two interactions and a BKP state between the pomerons.

Transitions both with the redistribution of colour, $|(12)(34)>\to|(13)(24)>$ (a), and without, $|(12)(34)>\to|(12)(34)>$)(b).



a *b*

In case (a) the pomerons will be connected by operator

$$M_E^{(a)}=\frac{1}{4}\Big(V_{13}+V_{24}-V_{23}-V_{14}\Big)$$

$$\times[G_E^{(1243)}+G_E^{(1342)}]\Big(V_{12}+V_{34}-V_{23}-V_{14}\Big),$$

Here $G_E^{(1243)}$ is the BKP Green function for gluons ordered as 1234 which satisfies

$$(E - H^{(1243)})G_E^{1243} = 1$$

with

$$H^{(1243)} = -\sum_{i=1}^{4} \omega_i - \frac{1}{2}(V_{12} + V_{24} + V_{43} + V_{31}).$$

$M_E^{(a)}$ can also be rewritten in terms of H^P

$$M_E^{(a)} = \frac{1}{4}\Big(H_{13}^P + H_{24}^P - H_{23}^P - H_{14}^P\Big)$$

$$\times[G_E^{(1243)} + G_E^{(1324)}]\Big(H_{12}^P + H_{34}^P - H_{23}^P - H_{14}^P\Big)$$

and is infrared safe.

Diagram (a) gives

$$D_{3a}=-\int\frac{dE_1}{2\pi}\frac{dE_1'}{2\pi}\int d\tau_\perp < q|M_E^{(a)}|q' >$$

$$\times P_{E-E_1}(q_1)P_{E_1}(q_4)P_{E-E_1'}(q_1')P_{E_1'}(q_4').$$

where $q=\{q_1,q_2,q_3,q_4\}$ with $q_2=-q_1$, $q_3=-q_4$ and similarly for primed momenta and $d\tau_\perp$ corresponds to interations over $q_{1,4}$ and $q'_{1,4}$. Passing to rapidities

$$D_{3a}=\int_0^y dy_1\int_0^{y_1}dy_2\int d\tau_\perp$$

$$\times < q|M^{(a)}(y_{12})|q' >$$

$$\times P_{12}(y_{01},q_1)P_{34}(y_{01},q_4)P_{13}(y_2,q_1')P_{24}(y_2,q_4')$$

Here P_{ik} is made of reggeiztd gluons i and k.

Similar calculations give for (b)

$$D_{3b} = \int_0^y dy_1 \int_0^{y_1} dy_2 \int d\tau_\perp$$

$$\times < q|M^{(b)}(y_{12})q' >$$

$$\times P_{12}(y_{01}, q_1) P_{34}(y_{01}, q_4) P_{12}(y_2, q_1') P_{34}(y_2, q_4'),$$

where

$$M_E^{(b)} = \frac{1}{4}\Big(H_{13}^P + H_{24}^P - H_{23}^P - H_{14}^P \Big)$$

$$\times [G_E^{(1234)} + G_E^{(1432)} + G_E^{(1243)} + G_E^{(1342)}]$$

$$\times \Big(H_{13}^P + H_{24}^P - H_{23}^P - H_{14}^P \Big).$$

III. Inclusive cross-sections

The high-energy part

$$H(Y,y,k) = -i\pi^2\delta(\kappa_+)\delta(q_-)N_c^2 s^2 F(Y,y,k),$$

y and k are the rapidity and transverse momentum of the observed gluon. The inclusive cross-section for double AB scattering

$$I^{(2)}_{AB}(Y,y,k,b) \equiv \frac{(2\pi)^2 d\sigma_{AB}}{d^2k\,dy\,d^2b}$$

$$= \frac{1}{4}A(A-1)B(B-1)T^{(2)}_{AB}(b)F(Y,y,k))$$

and for dd scattering

$$I_{dd}(Y,y,k) = \frac{1}{4}\Big\langle\frac{1}{2\pi r^2}\Big\rangle_A\Big\langle\frac{1}{2\pi r^2}\Big\rangle_B F(Y,y,k).$$

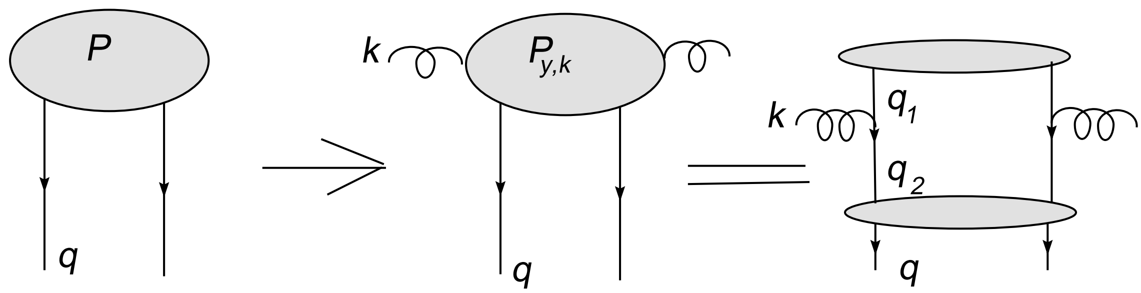
III.1. Emission from pomerons.

Cuts in the forward amplitude with the observed gluon rapidity y and momentum k fixed in the cut.

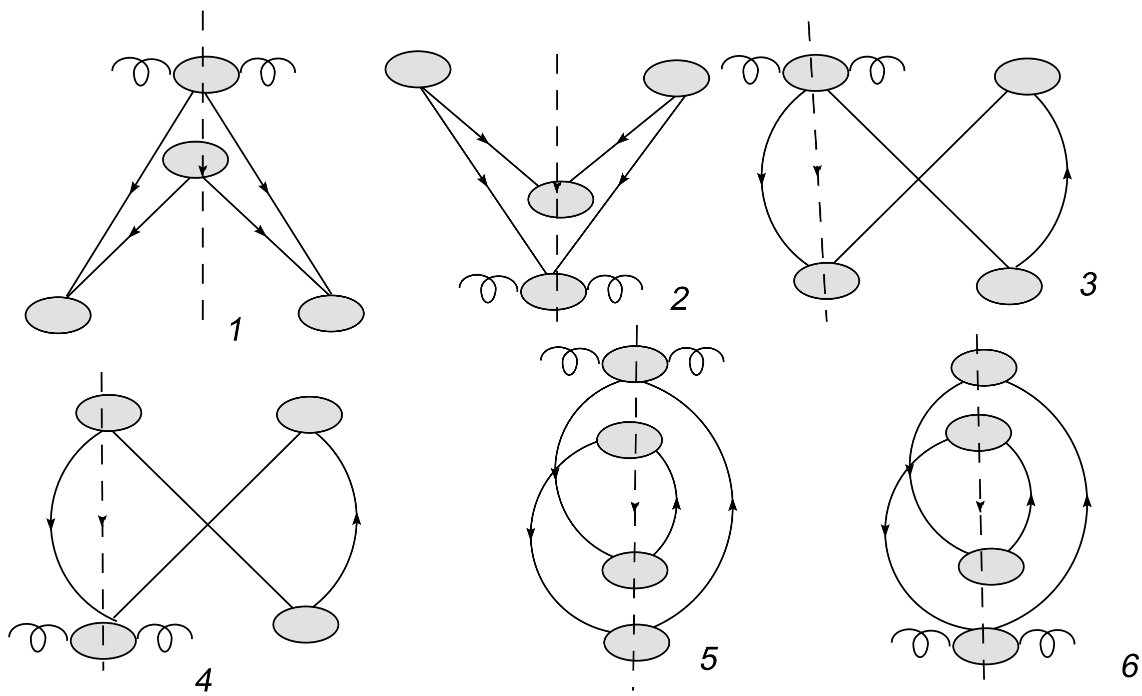
Diffractive with respect to the targets DT: both targets uncut and both projectiles cut
Diffractive with respect to the projectiles DP: both targets cut and both projectiles uncut
Single cut (S): one target and one projectile cut others uncut
Double cut (DC): both targets and both projectiles cut

• Integrated cut and uncut BFKL interactions give the same contribution.

Emission from the pomeron corresponds to "opening" the pomeron chain



An example: direct sewing of pomerons with redistribution of colour. Various cuts in the amplitude



The corresponding contributions are

$$F^{DT} = 4\frac{\partial}{\partial Y}\int_0^Y dy' \int \frac{d^2q}{(2\pi)^2}$$

$$\times P_{y,k}(Y, y', q)P^(Y - y', q)P^2(y', q).$$

$$F^{DP} = 4\frac{\partial}{\partial Y}\int_0^Y dy' \int \frac{d^2q}{(2\pi)^2}$$

$$\times P^2(Y - y', q)P_{y,k}(y', 0, q)P(y', q).$$

$$F^S = -F^{DT} - F^{DP}, \quad F^{DC} = F^{DT}F^{DP}$$

In the sum the S and DC contributions cancel $F_1^P = F^{DT} + F^{DP}$.

Inclusion of other interactions in between does not change the form of the result, which is obtained from the forward amplitude by $P(Y-y',q)\to P_{y,k}(Y,y',q)$

So with a single interaction between the pomerons

$$F_2^P = 8\int_0^Y dy_1\int\frac{d^2q d^2q'}{(2\pi)^4} < q,q'|H|q',q >$$

$$\times\Big[P_{y,k}(Y,y_1,q)P(y_{01},q')P(y',q)P(y',q')$$

$$+\Big(y_1\to y_{01},(Y,y_1)\to(y_1,0)\Big)\Big].$$

where $y_0\equiv Y$

With two interactions and the BKP state between the pomerons, with redistribution of color

$$F_3^P = 2\int_0^Y dy_1 \int_0^{y_1} dy_2 \int d\tau_\perp$$

$$\times < q|M^{(a)}(y_{12})|q' > P(y_{01}, q_4)P(y_2, q_4')$$

$$\times\Big(P_{y,k}(y_{01}, q_1)P(y_2, q_1')$$

$$+P(y_{01}, q_1)P_{y,k}(y_2, q_1')\Big).$$

Without redistribution of color

$$F_4^P = F_3^P(M^{(a)} \to 2M^{(b)})$$

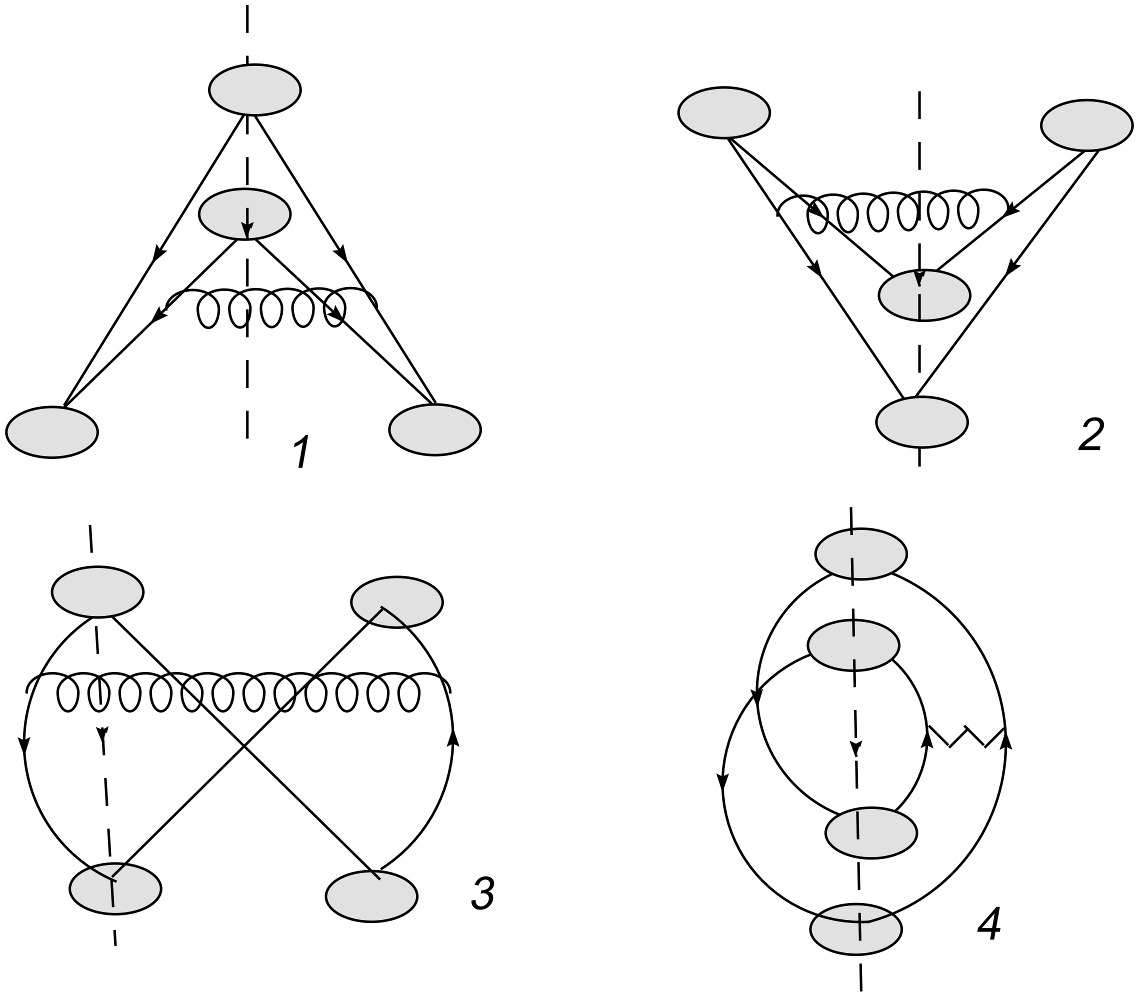
III.2. Emission from one interaction between the pomerons

Substitution in the corresponding forward amplitude

$$< q, q'|H|q', q >$$

$$\to < q, q'|V|q', q > (2\pi)^2\delta^2(q - q' - k)$$

Various cuts



Each can be presented in the form

$$F = 4\kappa \int_0^y dy' \int \frac{d^2q d^2q'}{(2\pi)^2}$$

$$\times < q, q'|V|q', q > \delta^2(q - q' - k)$$

$$\times P(y - y', q)P(y - y', q')P(y', q)P(y', q')$$

where $\kappa^{DT} = \kappa^{DP} = 1, \quad \kappa^{DC} = 0, \quad \kappa^{S} = -1.$
In the sum

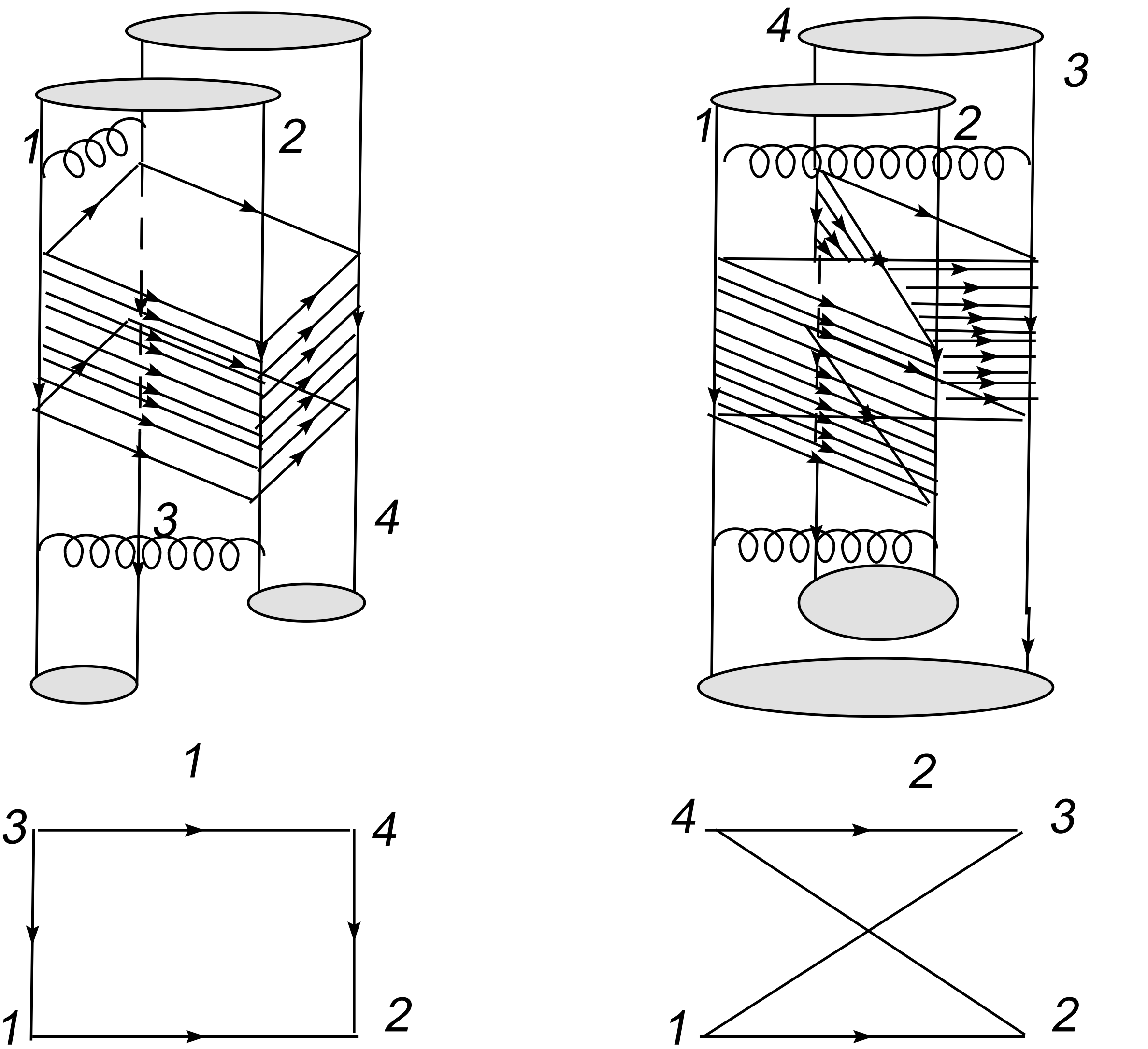
$$F^V = 4 \int_0^y dy' \int \frac{d^2q d^2q'}{(2\pi)^2}$$

$$\times < q, q'|V|q', q > \delta^2(q - q' - k)$$

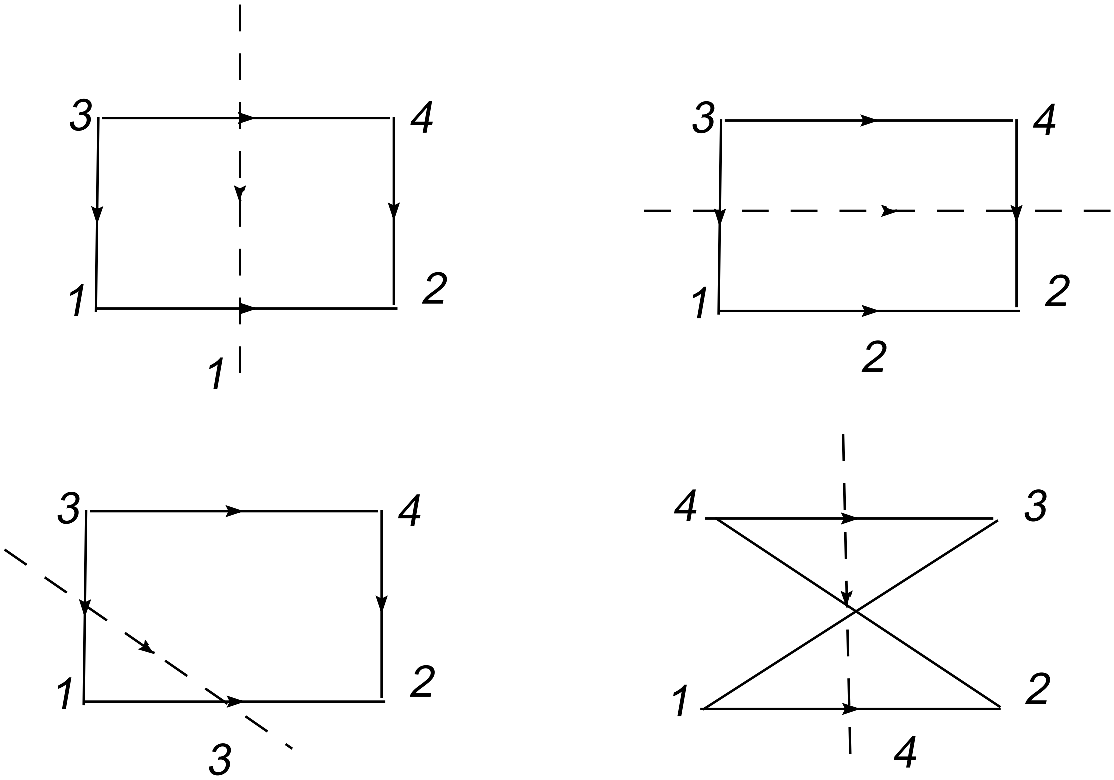
$$\times P(y - y', q) P(y - y', q') P(y', q) P(y', q')$$

III.3. Emission from two interactions between the pomerons with redistribution of colour

The position the cut through the BKP state is uniquely determined by the cut passing through the pomerons.
Example:
the BKP state appears between v_{13} and v_{12} in G^{1243} (in two different forms with gluons on the surface of a cylinder (1) and in the natural order 1234 (2)).



The schemes below show interactions betwen gluons 1,2,3 and 4. The cutting plane goes as shown in the next figure. Its position in configurations DT,DP,S and DC are shown in 1,2,3 and 4 respectively.



Since cut and uncut interactions give the same contribution, in all cases the BKP state appears as a whole between the interactions which generate the observed gluon. So it is sufficient to first take $G^{1243}(y) \to 1$, and introduce this Green function between the interactions afterwards.

Diagrams corresponding to DT,DP,S and DC configurations without the BKP state are shown below in Figs. 10 - 15.

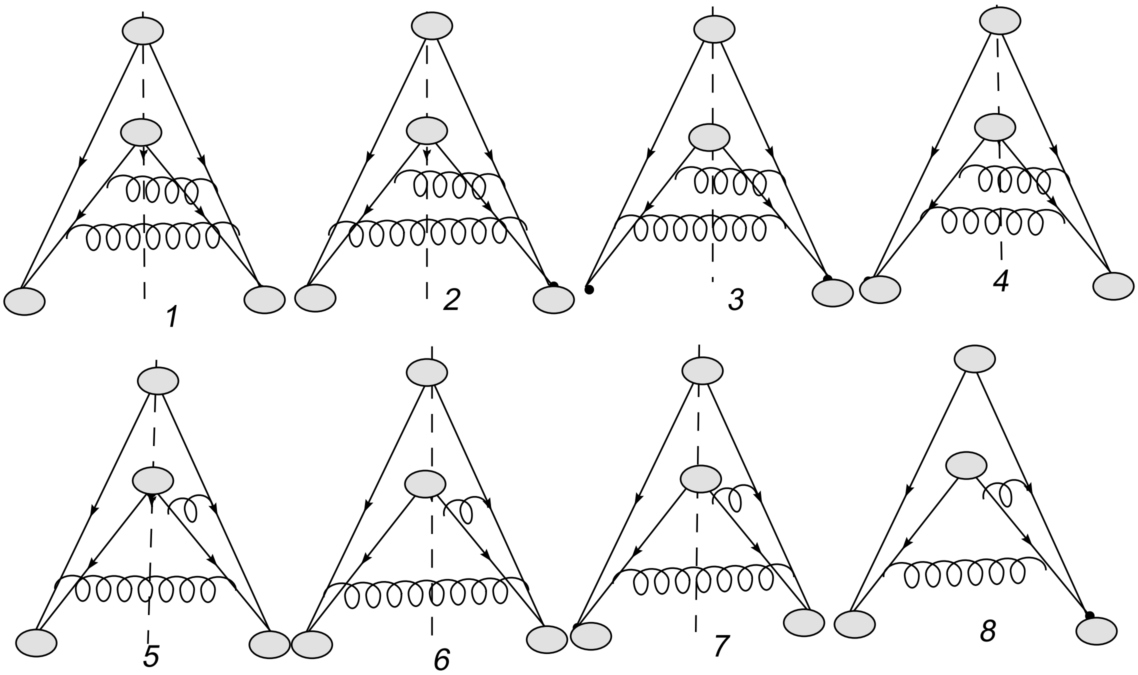


Fig. 10. DT configuration

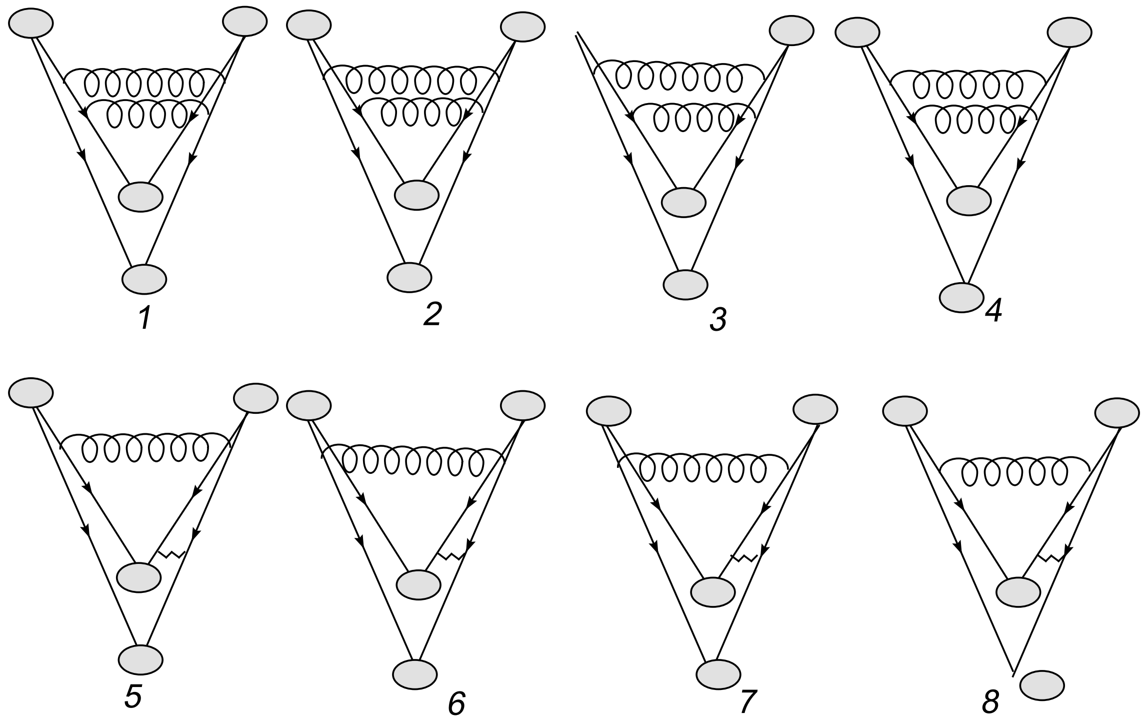


Fig. 11. DP configuration

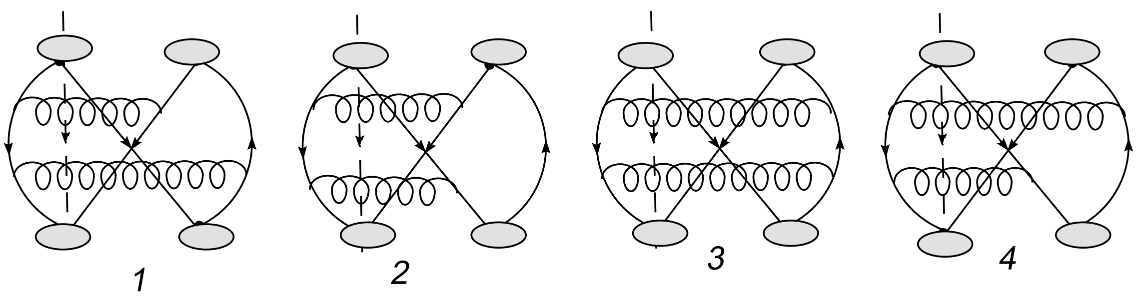


Fig.12. S configuration with 2 gluons in the intermediate state

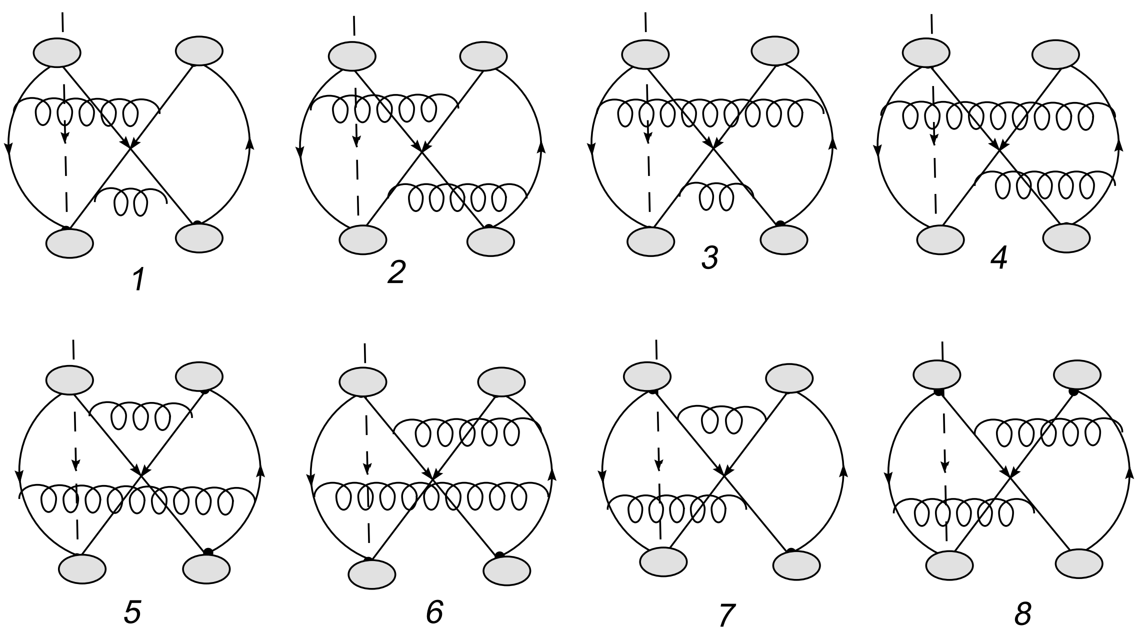


Fig. 13. S configuration with one gluon in the intermediate state

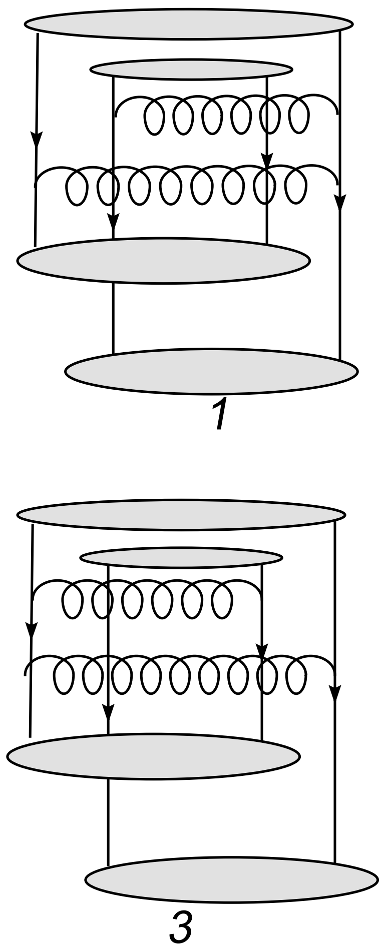


Fig. 14. DC configuration with 2 gluons in the intermediate state

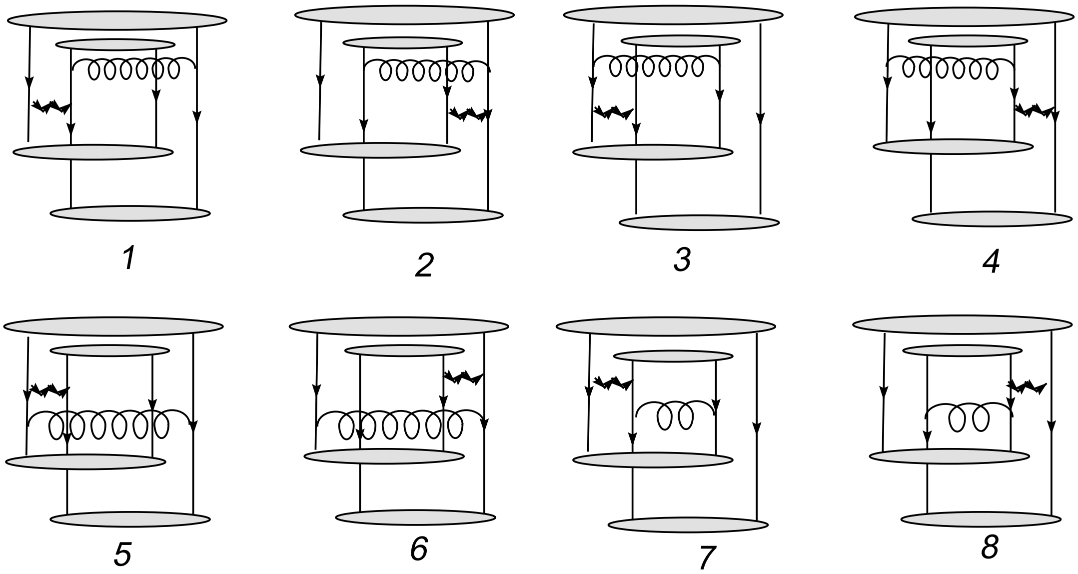


Fig.15. DC configuration with one gluon in the intermediate state

Presenting the result as

$$F^{2V} = \int_0^Y dy_1 \int_0^{y_1} dy_2 d\tau_\perp^2 f$$

$$\times P_{12}(y_{01} P_{34}(y_{01}) P_{13}(y_2) P_{24}(y_2)$$

we find after summation over all cuts

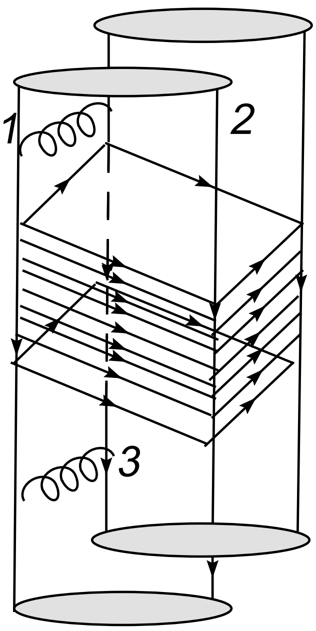
$$f = \frac{1}{2}(V_{14} + V_{23} - V_{13} - V_{24})$$

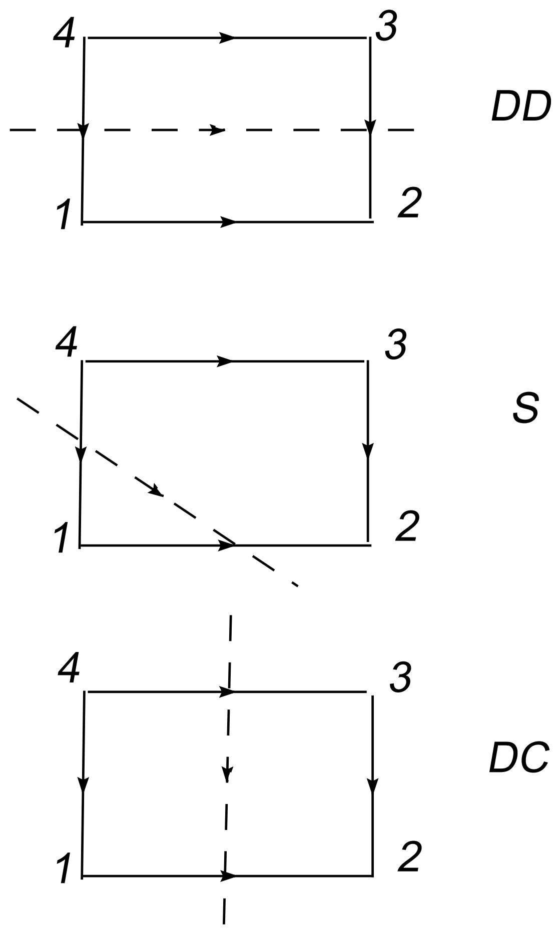
$$\times G^{1234}(y_{12})(V_{14} + V_{23} - V_{12} - V_{34}),$$

The observed gluon is to be located in one of the 4 interactions either on the left or on the right. Corespondingly either $y' = y$ or $y'' = y$

III.4. Emission from two interactions between the pomerons without redistribution of colour

The BKP state and position of the cuts are again uniquely fixed by the configuration.





Cut diagrams for different configurations are shown in Figs. 17 - 21 (with the BKP state suppressed)

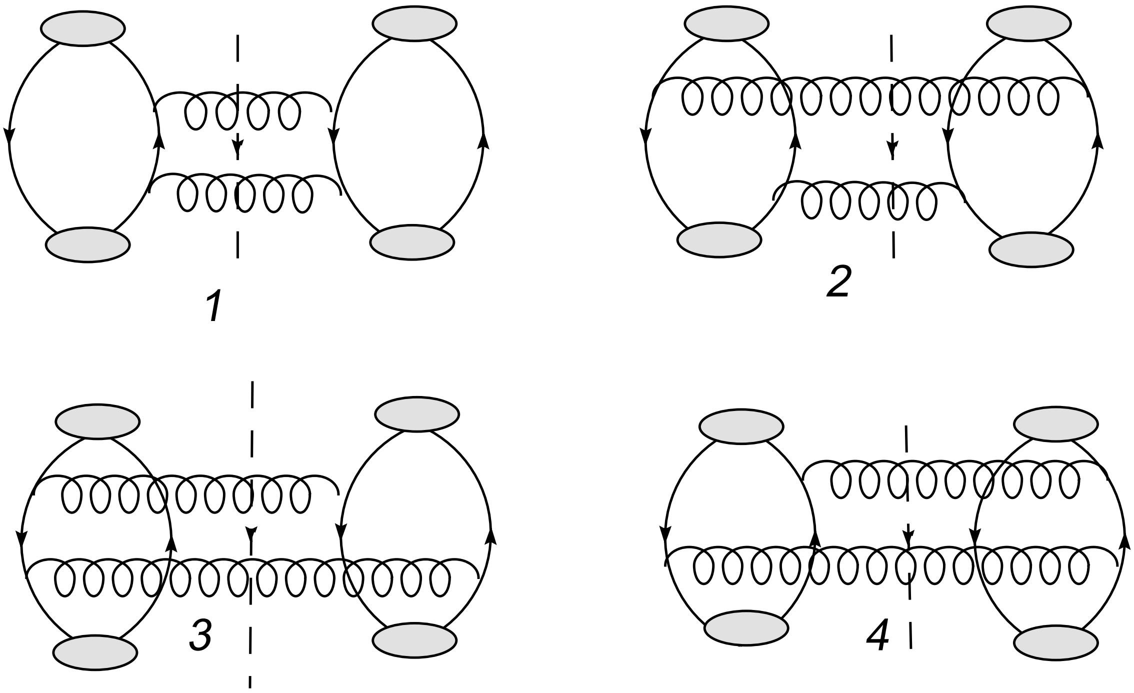


Fig. 17. Double diffractive configuration.

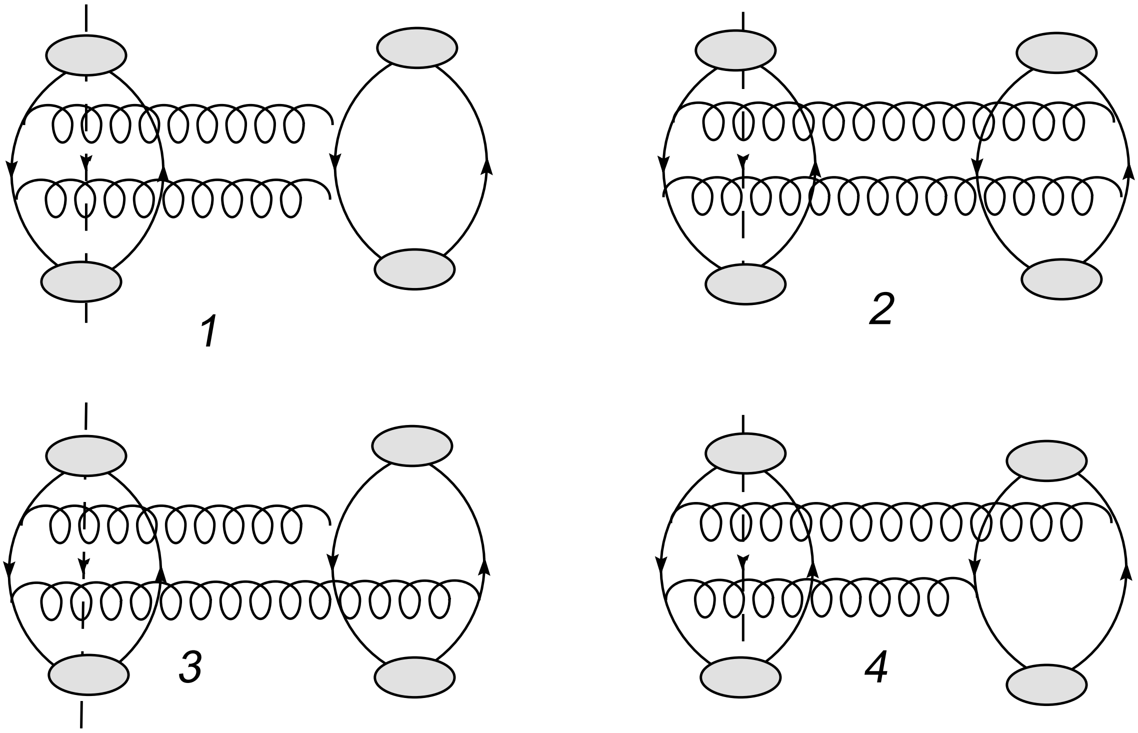


Fig. 18. S configuration with 2 gluons in the intermediate state

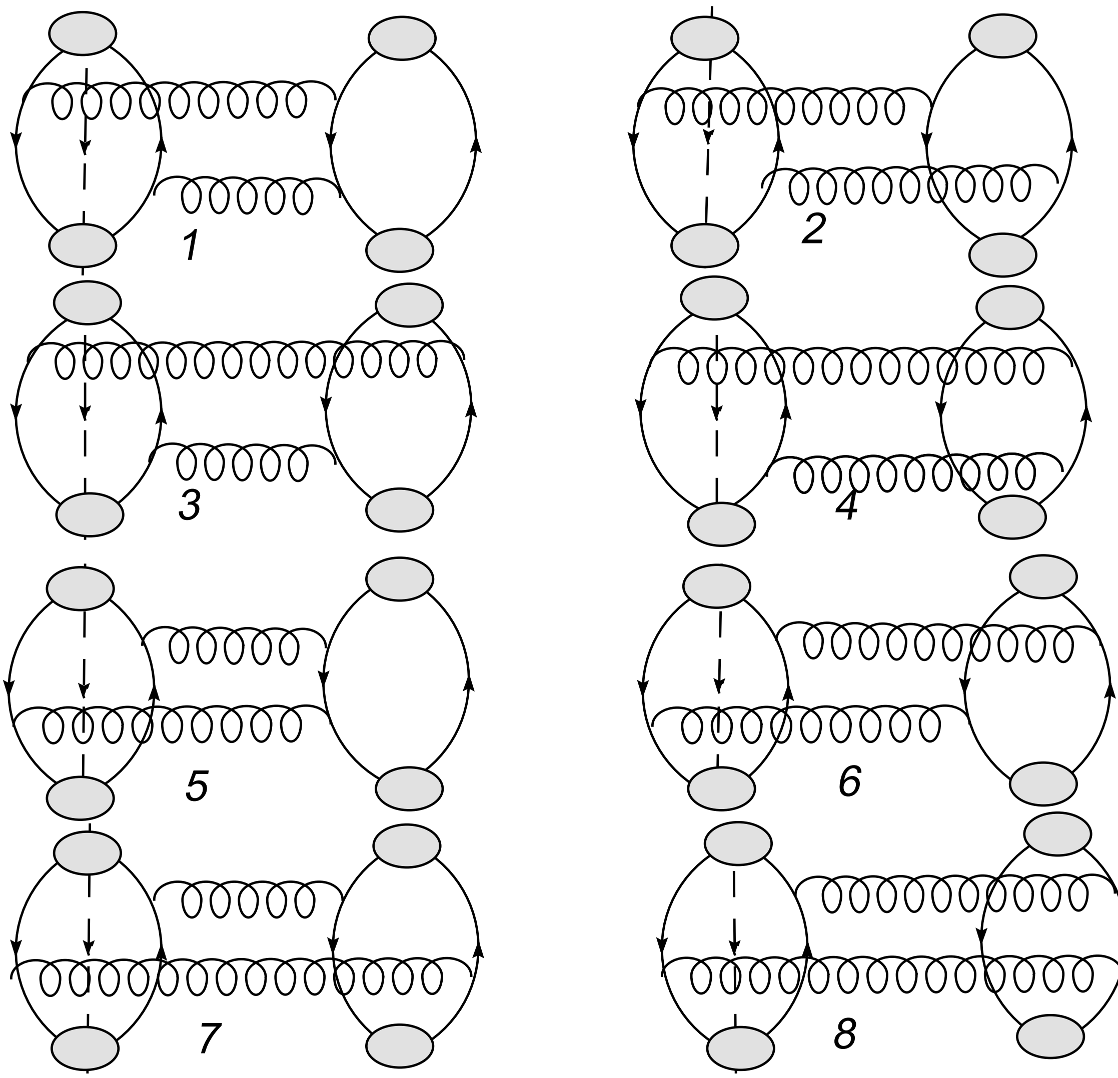


Fig. 19. S configuration with one gluon in the intermediate state

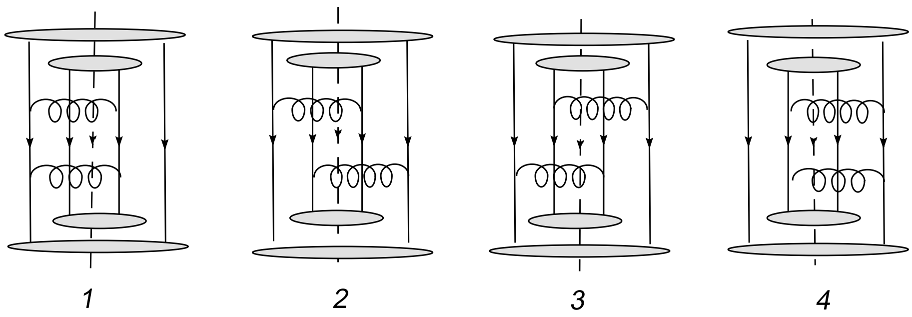


Fig. 20. DC configuration with 2 gluons in the intermediate state

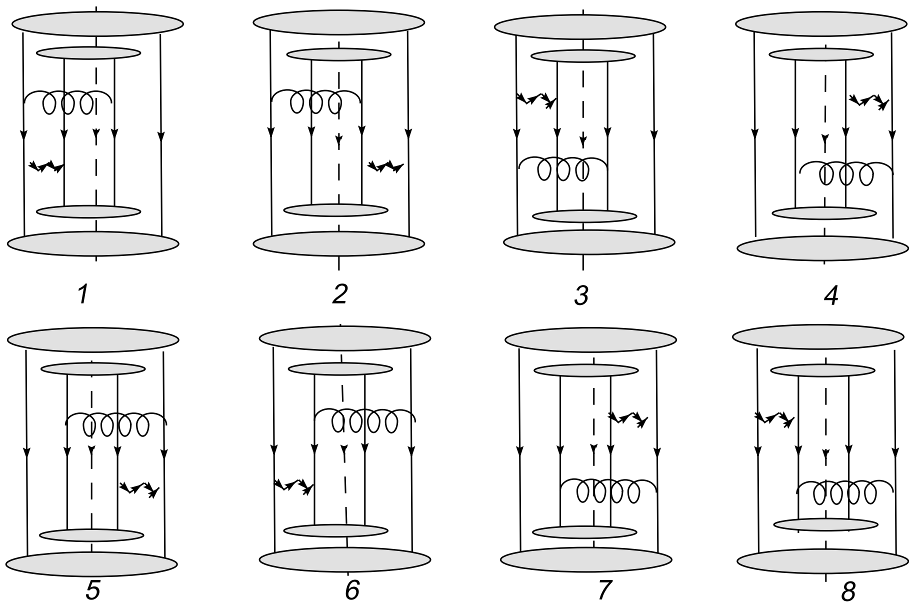


Fig. 21. DC configuration with one gluon in the intermediate state

Presenting the result as

$$\tilde{F}^{2V} = \int_0^y dy_1 \int_0^{y_1} dy_2 d\tau_\perp^2 \tilde{f}$$

$$\times P_{12}(y_{01}) P_{34}(y_{01}) P_{13}(y_2) P_{24}(y_2)$$

we find after summation over all cuts

$$\tilde{f} = \frac{1}{2}(V_{14} + V_{23} - V_{13} - V_{24})$$

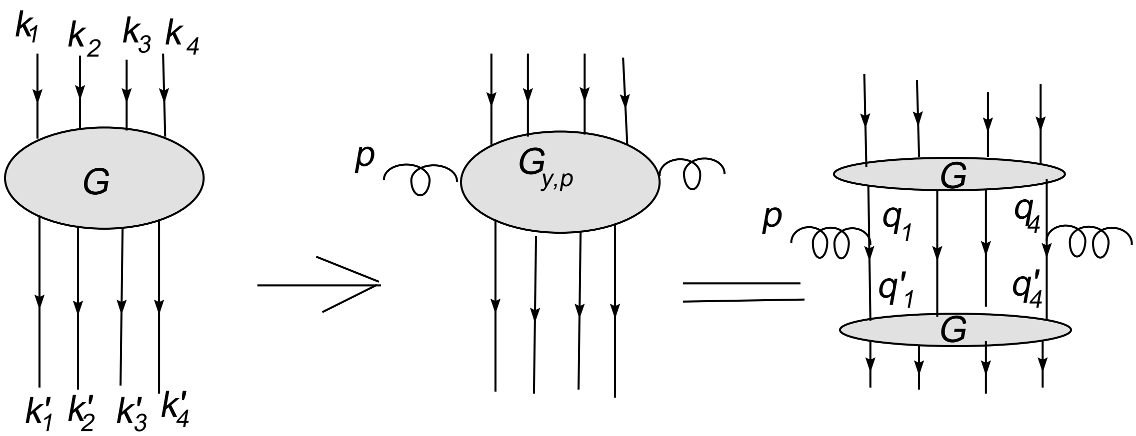
$$\times G^{1234}(y_{12})(V_{14} + V_{23} - V_{13} - V_{24}),$$

Again observed gluon is to be located in one of the 4 interactions either on the left or on the right. Corespondingly either $y' = y$ or $y'' = y$

III.4. Emission from the BKP state

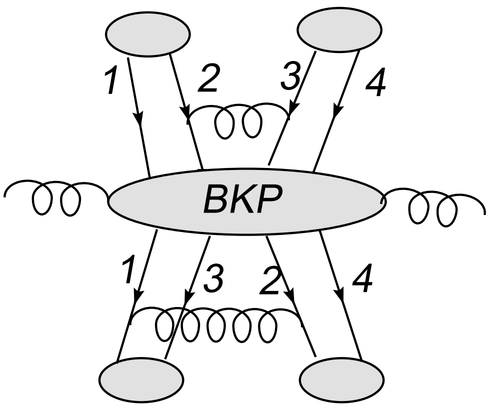
To fix one of the intermediate real gluons inside the BKP state one has to 'open' it similarly to opening the BFKL pomeron

$$G^{1234}(y',y'') \to G^{1234}_{y,k}(y',y'')$$

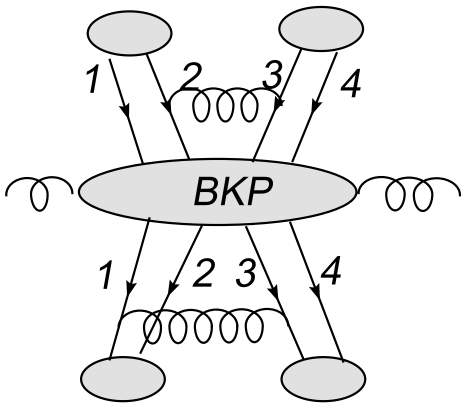


The difference is that the cutting plane always passes through 2 or 4 interactions connecting different gluons inside the BKP state. So the contribution from the BKP state has to be multiplied by 2 or 4 depending on the number of crossed interactions: 4 for DC configuration with colour redisitribution and 2 in all other cases.

Graphically emission from the BKP state is



1



2

The result for colour redisiribution is presented as

$$F^{BKP} = \int_0^Y dy_1 \int_0^{y_1} dy_2 d\tau_\perp^2 f$$

$$\times P_{12}(y_{01}) P_{34}(y_{01}) P_{13}(y_2) P_{24}(y_2).$$

The D contribution:

$$f_D = (V_{23} + V_{14} - V_{13} - V_{24})$$

$$\times G^{1243}_{y,k}(y_1, y_2)(V_{23} + V_{14} - V_{12} - V_{34}).$$

and S and DC contributions: $f_{DC} = -F_S = f_D$. So the total $f = f_D$

For the case without colour redistribution $f \to \tilde{f}$. In the double diffractive configuration

$$\tilde{f}_{DD} = (v_{23} + v_{14} - v_{13} - v_{24})$$

$$\times G^{1234}_{y,k}(y_1, y_2)(v_{23} + v_{14} - v_{13} - v_{24}).$$

and in the S and DC configurations $\tilde{f}_{DC} = (1/2)\tilde{f}_{DD}$ and $\tilde{f}_S = -\tilde{f}_{DD}$. So the total $\tilde{f} = (1/2)\tilde{f}_{DD}$

Conclusions

•. In the BFKL kinematics for non-trivial interaction of two projectiles with two targets the dominant contribution comes from diagrams of the relative order $1/N_c^2$ which include contributions with redistribution of color.

•. Both for the total and inclusive cross-sections there are contributions from intermediate BKP states made of 4 reggeized gluons.

•. With the pure BFKL pomerons attached to the participants contributions grow as the square of the pomeron, the BKP state providing corrections growing as powers of rapidity.

•. With unitarized pomerons the growth will be deiermined by the BKP state $\sim \exp 0.243\Delta_{BFKL}Y$, which can be traced experimentally.