Mueller-Navelet small-cone jets

Beatrice Murdaca[1]

[1] Università della Calabria & INFN - Cosenza

in collaboration with
F. Caporale[1], D.Yu. Ivanov[2] and A. Papa[1]
[2]Sobolev Institute of Mathematics, Novosibirsk

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Outline

1. Introduction
   - The Mueller-Navelet jet production process
   - Theoretical set up: BFKL and collinear factorization
   - Jet Vertex
   - State of the art
2. Results
   - Ingredients
   - Our analysis
   - Some results
   - Summary

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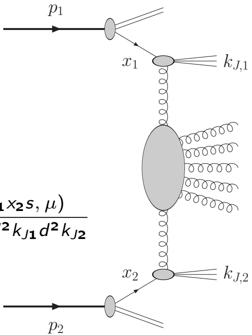
Mueller-Navelet jets



$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_1) + \text{jet}_2(k_2) + X$$

$$\frac{d\sigma}{dx_{J1}dx_{J2}d^2k_{J1}d^2k_{J2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu)}{dx_{J1}dx_{J2}d^2k_{J1}d^2k_{J2}}$$

Sudakov decomposition:

$$k_{J,1} = x_{J,1}p_1 + \frac{\vec{k}_{J,1}^2}{x_{J,1}s}p_2 + k_{J,1\perp}\,, \quad k_{J,1\perp}^2 = -\vec{k}_{J,1}^2\,, \quad s = 2p_1 \cdot p_2$$

$$k_{J,2} = x_{J,2}p_2 + \frac{\vec{k}_{J,2}^2}{x_{J,2}s}p_1 + k_{J,2\perp}\,, \quad k_{J,2\perp}^2 = -\vec{k}_{J,2}^2$$

- large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{\rm QCD}^2$ $\longrightarrow$ perturbative QCD applicable
- large rapidity gap between jets, $\Delta y = \ln \frac{x_{J,1}x_{J,2}s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$, $\longrightarrow$ $s = 2p_1 \cdot p_2 \gg \vec{k}_{J,1\,2}^2$

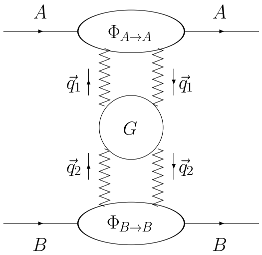
$\longrightarrow$ BFKL resummation: $\sum_n c_n\, \alpha_s^n \ln^n s + d_n\, \alpha_s^n \ln^{n-1} s$

Outline

The BFKL approach

Total cross section $A+B\to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im_s}\mathcal{A}}{s}$

- **Regge limit** ($s\to\infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s\mathcal{A}$:
  convolution of the **Green's function** of two interacting Reggeized gluons and of the **impact factors** of the colliding particles.

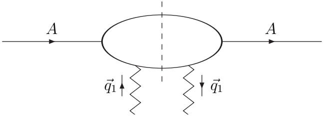


- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s\mathcal{A} = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A(\vec{q}_1, s_0)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B(-\vec{q}_2, s_0)\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1,\vec{q}_2)$$

- The **Green's function** is **process-independent** and is determined through the **BFKL equation**. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1,\vec{q}_2) = \delta^{D-2}(\vec{q}_1-\vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_1)\ .$$

- **Impact factors** are **process-dependent**; 

only very few have been calculated in the NLA:

- colliding partons

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

- $\gamma^* \longrightarrow V$, with $V = \rho^0$, ω, ϕ, forward case

  [D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- forward jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri (2012)]
  (small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]

- forward identified hadron production

  [D.Yu. Ivanov, A. Papa (2012)]

- $\gamma^* \longrightarrow \gamma^*$

  [J. Bartels *et al.* (2001) $\rightarrow$]
  [I. Balitsky, G.A. Chirilli (2011)-(2013)]

Solution of the BFKL equation

- The Green's function obeys the BFKL equation

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega\, G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$$

- Transverse momentum space definition

$$\hat{\vec{q}}\, |\vec{q}_i\rangle = \vec{q}_i |\vec{q}_i\rangle \; , \quad \langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2) \; , \quad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A^*(\vec{k}) B(\vec{k})$$

- The kernel operator $\hat{K}$ is

$$K(\vec{q}_2, \vec{q}_1) = \langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle$$

- Straightforward solution in the transverse momentum space

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega \quad \longrightarrow \quad \hat{G}_\omega = (\omega - \hat{K})^{-1} \; , \qquad \hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1 \; , \quad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

- Solution for the $\hat{G}_\omega$ operator with NLA accuracy

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1\right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2 \hat{K}^1\right)^2\right]$$

- Basis of eigenfunctions of the LO kernel

$$\hat{K}^0|n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle\,, \qquad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$\langle\vec{q}\,|n,\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} e^{in\theta}\,, \qquad \cos\theta \equiv q_x$$

$$\langle n',\nu'|n,\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} e^{i(n-n')\theta} = \delta(\nu-\nu')\,\delta_{nn'}$$

- The action of the full NLO BFKL kernel on these functions may be expressed as follows:

$$\hat{K}|n,\nu\rangle = \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle$$

$$+ \quad \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle\,,$$

Impact factors

- Projection of the impact factors $\phi_i(\vec{q})$ onto the eigenfunctions of LO BFKL kernel, i.e. the transfer to the (n,ν)-representation

$$c_i(n,\nu) = \int d^2\vec{q}\,\frac{\phi_i(\vec{q})}{\vec{q}^{\,2}}\frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} e^{in\phi}\,.$$

Here ϕ is the azimuthal angle of the vector $\vec{q}$ counted from some fixed direction in the transverse space.

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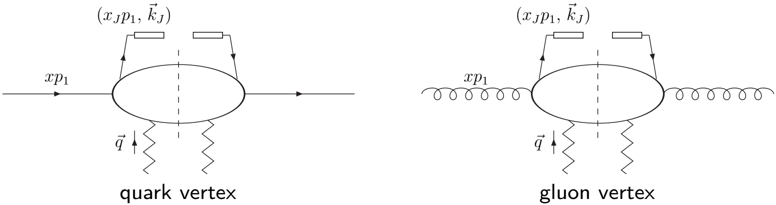
Outline

Jet Vertex

- **Step 1**: "open" one of the integrations over the phase space of the final state to allow one parton to form a jet



- **Step 2**: take the convolution with the PDFs and the jet function

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes S_J \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes S_J$$

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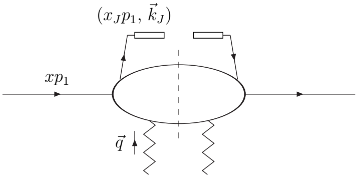
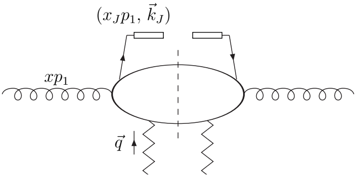


quark vertex



gluon vertex

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Jet definition

LO: one-particle final state.

The kinematics of the produced parton *a* is completely fixed by the jet kinematics.

NLO, virtual corrections: one-particle final state (see Fig. 1).

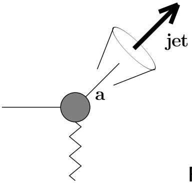

Fig. 1

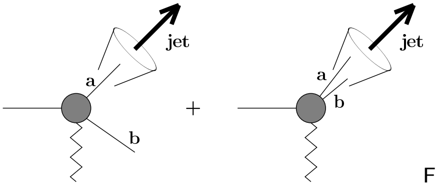

Fig. 2

NLO, real corrections: two-particle final state (see Fig. 2).

Outline

State of the art

Observables of interest:

- Differential cross section
- Moments of the azimuthal decorrelation of the jets

So far...

- Convolution of the NLA Green's function with the LO jet vertices

[A. Sabio Vera (2006)]
[A. Sabio Vera, F. Schwennsen (2007)]
[C. Marquet, C. Royon (2007)]

- Full NLO calculation

[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
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Ingredients

- Small Cone Approximation (**SCA**): small jet cone aperture in the rapidity-azimuthal angle plane. [D.Yu. Ivanov, A. Papa (2012)]
    - Easily implementable in numerical calculations.
    - Particularly suitable for a semi-analytical cross-check of the numerical approaches which treat the cone size exactly.

- Principle of Minimal Sensitivity (**PMS**): optimized method. [P.M. Stevenson (1981)]

Cross section

Moments of the azimuthal decorrelations

$$\langle \cos\left[n\left(\phi_{J_1}-\phi_{J_2}-\pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0},$$

$$\mathcal{C}_n = \int_0^{2\pi} d\phi_{J_1} \int_0^{2\pi} d\phi_{J_2}\ \cos\left[n\left(\phi_{J_1}-\phi_{J_2}-\pi\right)\right] \frac{d\sigma_n}{dJ_1\ dJ_2}$$

with $dJ_i = dx_{J_i} dk_{J_i}$

and

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^{\mathbf{2}}(\mu_R)k^{(\mathbf{1})}(n,\nu)} \alpha_s^2\left(\mu_R\right) c_1\left(n,\nu\right) c_2\left(n,\nu\right)$$

$$\times \left[1+\bar{\alpha}_s\left(\mu_R\right)\left(\frac{c_1^{(1)}\left(n,\nu\right)}{c_1\left(n,\nu\right)}+\frac{c_2^{(1)}\left(n,\nu\right)}{c_2\left(n,\nu\right)}\right)\right]$$

with

$$k^{(1)}\left(n,\nu\right) = \bar{\chi}\left(n,\nu\right)+\frac{\beta_0}{8N_c}\chi\left(n,\nu\right)\left(-\chi\left(n,\nu\right)+\frac{10}{3}+\imath\frac{d}{d\nu}\ln\left(\frac{c_1\left(n,\nu\right)}{c_2\left(n,\nu\right)}\right)+2\ln\left(\mu_R^2\right)\right)$$

Outline

Kinematical settings

- In order to compare our predictions with the **forthcoming LHC data** we fix

$$R = 0.5$$

$$\sqrt{s} = 14 \text{ TeV} \qquad \text{and} \qquad \sqrt{s} = 7 \text{ TeV}$$

- Following a CMS study [CMS Collaboration, S. Cerci, D. d'Enterria (2009)], we restrict the rapidities of the jets to the region

$$3 \leq |y_J| \leq 5 \, , \qquad \text{with steps} \qquad \Delta y_J = 0.5$$

- Symmetric case:
  - $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV $\longrightarrow$ comparison with previous results.
    [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
    [B. Ducloué, L. Szymanowski, S. Wallon (2013)]
  - $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV $\longrightarrow$ more undetected gluons radiated in the final state.

Our analysis

- We study the cross section and the azimuthal correlations versus the relative rapidity $Y = y_{J_1} - y_{J_2}$.
- Unavoidable terms beyond NLA depend on μ_R and s_0, whose numerical impact can be non-negligible $\Longrightarrow$ **optimization needed!**

PMS

We take as optimal choices for μ_R and s_0 those values for which the physical observable under exam exhibits the minimal sensitivity to changes of both these scales. [P.M. Stevenson (1981)]

The complete resummation of the perturbative series would not depend on μ_R and s_0
$\Longrightarrow$ PMS is supposed to mimic the effect of the most relevant unknown subleading terms.

$$Y_0 = \ln \frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} = 0, 1, 2, \ldots \qquad \mu_R = N\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \qquad N = 1, 2, 3, \ldots$$

$\Longrightarrow$ For each fixed value of Y we look for a stationary point in the $N - Y_0$ plane.

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Outline

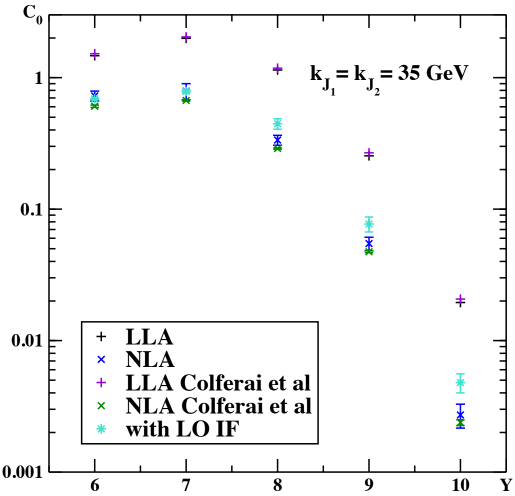
Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV

Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$
6	1	2
7	2	2
8	2	2
9	3	2
10	4	2

Values of the parameters corresponding to our "optimal" values.

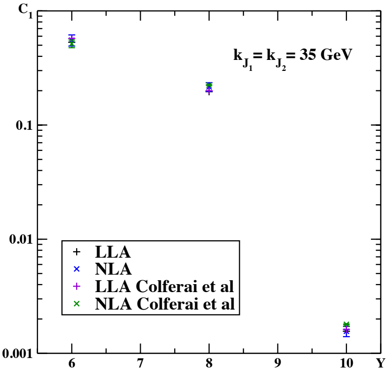
LLA: $\mu_R^2 = s_0 = |\vec{k}_{J_1}||\vec{k}_{J_2}|$

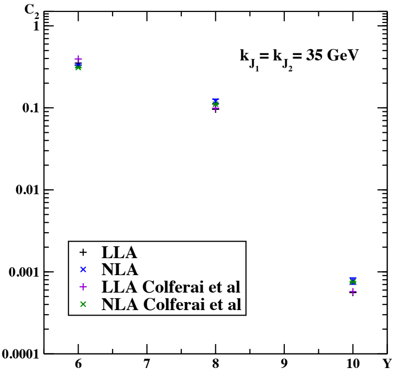
- We have found wide regions of stability.



- The uncertainty comes from the resolution of our grid in the N–Y_0 plane.
- Our predictions are compatible with previous calculations $\Longrightarrow$ reliability of the SCA.

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV





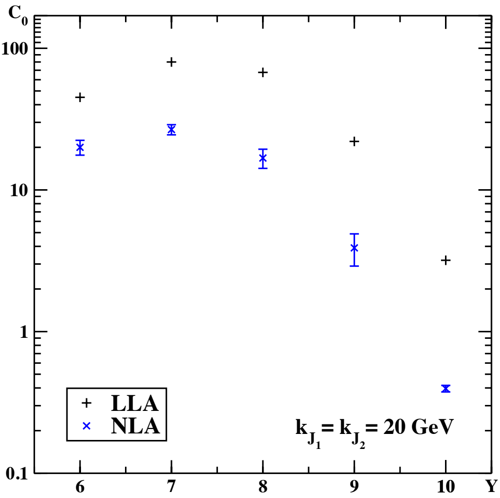
Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$
6	1	2
8	2	2
10	3	2

Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$
6	0	1.5
8	2	2.5
10	4	4

- It is possible to calculate $\langle \cos\phi \rangle$ and $\langle \cos(2\phi) \rangle$.
- Our predictions are compatible with previous calculations $\Longrightarrow$ reliability of the SCA.

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV

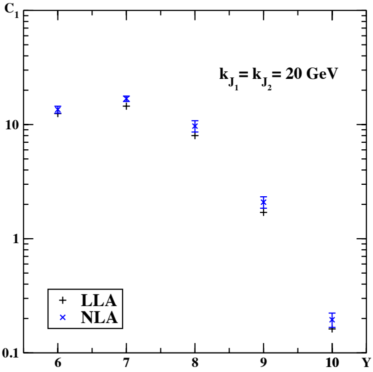
Y	Y_0	$\mu_R / \sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$
6	1	1
7	1	1
8	2	1
9	3	1
10	3	2



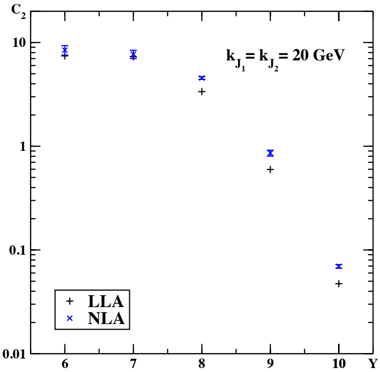
- We have found wide regions of stability.

- Smaller transverse momentum $\Longrightarrow$ more undetected gluons $\Longrightarrow$ possible discrimination between BFKL and fixed-order approach.

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV



Y	Y_0	$\mu_R/\sqrt{\vert\vec{k}_{J_1}\vert\vert\vec{k}_{J_2}\vert}$
6	0	1
7	0.5	1
8	1	1
9	1	1
10	1	1

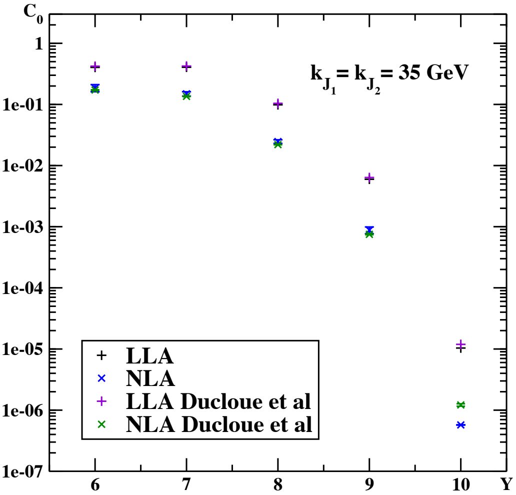


Y	Y_0	$\mu_R/\sqrt{\vert\vec{k}_{J_1}\vert\vert\vec{k}_{J_2}\vert}$
6	1	3
7	0.5	1
8	2.5	4
9	3	5
10	3.5	3

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV, $\sqrt{s} = 7$ TeV

Y	Y_0	$\mu_R/\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$
6	2	2
7	2	3
8	3	3
9	4	3
10	5.5	7

- We have found wide regions of stability.
- With our approach we reproduced the same results of previous calculations.



Outline

Summary

- We have studied the Y-dependence of the differential cross section and of the azimuthal decorrelation of the Mueller-Navelet jets in full NLO BFKL using the jet vertex in the small cone approximation.

- We did not fix the energy scale and the argument of the running coupling
$\Longrightarrow$ we used an optimization method.

- Our results for $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV with $\sqrt{s} = 14$ TeV and $\sqrt{s} = 7$ TeV are compatible with those performed by D. Colferai *et al* and B. Ducloué *et al*, respectively.
$\Longrightarrow$ Confirmation of their results.
$\Longrightarrow$ Reliability of the small cone approximation.

- We considered also smaller transverse momenta w.r.t. D. Colferai *et al.* $\longrightarrow$ we expect more discrimination between BFKL and DGLAP dynamics.

- In asymmetric case we do not find a stability region $\Longrightarrow$ Collinear improvement
[C. Caporale, B. M., A. Sabio Vera, C. Salas, arXiv:1305.4620]