Statistical Thermal Models in High-Energy Nuclear Physics

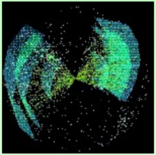
Ludwik Turko

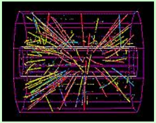
Institute of Theoretical Physics
University of Wrocław, Poland

Particles And Nuclei International Conference

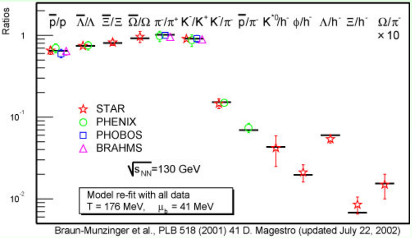
PANIC 2008 in Eilat, 9 – 14 November 2008

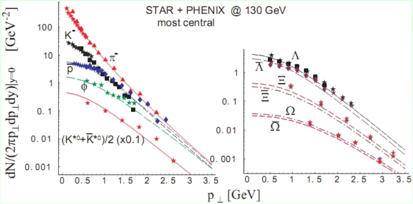
Place for statistical physics





More particles (degrees of freedom) in the process: kinematics tends to dominate the behavior of the system





We cannot solve pre-equilibrium HIC dynamics, we have no good description of hadronization processes. . . Nevertheless, the thermal statistical models work quite well.

Statistical model calculations – in principle

$$\epsilon = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) \int_0^\infty \frac{dp\, p^2 E_j}{\exp\left\{ \frac{E_j - \mu_j}{T} \right\} + \lambda_j} \,,$$

$$n_b = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) b_j \int_0^\infty \frac{dp\, p^2}{\exp\left\{ \frac{E_j - \mu_j}{T} \right\} + \lambda_j} \,,$$

$$n_s = \frac{1}{2\pi^2} \sum_{j=1} (2S_j + 1) s_j \int_0^\infty \frac{dp\, p^2}{\exp\left\{ \frac{E_j - \mu_j}{T} \right\} + \lambda_j} \,,$$

where

$$\mu_j = b_j \mu_b + s_j \mu_s$$

Statistical ensembles of high energy physics

The thermodynamic system of volume V and temperature T composed of charged particles and their antiparticles carrying charge ± 1.
The partition functions of the canonical and grand canonical statistical system

$$\mathcal{Z}_Q^C(V,T) = \mathrm{Tr}_Q\, \mathrm{e}^{-\beta \hat{H}} = \sum_{N_+ - N_- = Q}^{\infty} \frac{z^{N_- + N_+}}{N_-!N_+!} = I_Q(2Vz_0)\,,$$

$$\mathcal{Z}^{GC}(V,T) = \mathrm{Tr}\, \mathrm{e}^{-\beta(\hat{H} - \mu \hat{Q})} = \exp\left(2Vz_0 \cosh\frac{\mu}{T}\right).$$

Vz_0 is the sum over all one-particle partition functions

$$z_0^{(i)}(T) = \frac{1}{V}\frac{V}{(2\pi)^3} g_i \int d^3p\, e^{-\beta\sqrt{p^2+m_i^2}} = \frac{1}{2\pi^2} T g_i\, m_i^2\, K_2\left(\frac{m_i}{T}\right),$$

g_i – the spin degeneracy factor.
N.B. Natural dimensionless unit: VTm^2

The chemical potential μ determines the average charge in the grand canonical ensemble

$$\langle Q \rangle = T \frac{\partial}{\partial \mu} \ln \mathcal{Z}^{GC} \,.$$

This allows to eliminate the chemical potential from further formulae for the grand canonical probabilities distributions

$$\frac{\mu}{T} = \operatorname{arcsinh} \frac{\langle Q \rangle}{2Vz_0} = \ln \frac{\langle Q \rangle + \sqrt{\langle Q \rangle^2 + 4(Vz_0)^2}}{2Vz_0} \,.$$

Probabilities in ensembles

To have N_- negative particles in the canonical ensemble

$$\mathcal{P}_Q^C(N_-, V) = \frac{(Vz_0)^{2N_-+Q}}{N_-!(N_-+Q)!}\frac{1}{I_Q(2Vz_0)} \,.$$

To have N_- negative particles in the grand canonical ensemble

$$\mathcal{P}_{\langle Q\rangle}^{GC}(N_-, V) =$$

$$\frac{1}{N_-!}\left[\frac{2(Vz_0)^2}{\langle Q\rangle + \sqrt{\langle Q\rangle^2 + 4(Vz_0)^2}}\right]^{N_-} \exp\left[-\frac{2(Vz_0)^2}{\langle Q\rangle + \sqrt{\langle Q\rangle^2 + 4(Vz_0)^2}}\right]$$

Technical details

Cleymans J., Redlich K., and Turko L. Phys. Rev. C **71** 047902 (2005)
Cleymans J., Redlich K., and Turko L. J. Phys. G **31** 1421 (2005)

The thermodynamic limit

The thermodynamic limit is understood as a limit $V \to \infty$ such that densities of the system remain constant.
The canonical ensemble

$$Q, N_- \to \infty; \; \frac{Q}{V} = q \, ; \; \frac{N_-}{V} = n_-$$

The grand canonical ensemble.

$$\langle Q \rangle, N_- \to \infty; \; \frac{\langle Q \rangle}{V} = \langle q \rangle \, ; \; \frac{N_-}{V} = n_-$$

To formulate correctly the thermodynamic limit of quantities involving densities, one defines probabilities for densities

$$\begin{aligned} \mathbf{P}_q^C(n_-, V) &:= V \mathcal{P}_{Vq}^C(Vn_-, V) \,, \\ \mathbf{P}_{\langle q \rangle}^{GC}(q, V) &:= V \mathcal{P}_{V\langle q \rangle}^{GC}(Vq, V) \,. \end{aligned}$$

An average limiting density of (negatively) charged particles

$$\langle n_- \rangle_\infty = \left. \frac{\sqrt{q^2 + 4z_0^2} - q}{2} \right|_{q=\langle q \rangle}$$

Probability distribution of the canonical ensemble

$$\mathbf{P}_q^C(n_-; V) = \delta\left(n_- - \langle n_- \rangle_\infty\right) + \frac{1}{V} \frac{\langle n_- \rangle_\infty (q + \langle n_- \rangle_\infty)}{(2q + \langle n_- \rangle_\infty)^2}\, \delta'\left(n_- - \langle n_- \rangle_\infty\right) + \frac{1}{2V} \frac{\langle n_- \rangle_\infty (q + \langle n_- \rangle_\infty)}{2q + \langle n_- \rangle_\infty}\, \delta''\left(n_- - \langle n_- \rangle_\infty\right) + \mathcal{O}(1/V^2)\,,$$

Probability distribution of the grand canonical ensemble

$$\mathbf{P}_{\langle q \rangle}^{GC}(n_-, V) = \delta\left(n_- - \langle n_- \rangle_\infty\right) + \frac{\langle n_- \rangle_\infty}{2V}\, \delta''\left(n_- - \langle n_- \rangle_\infty\right) + \mathcal{O}(1/V^2)\,.$$

Moments in the thermodynamic limit

The canonical ensemble

$$\langle n_-^k \rangle^C \simeq \langle n_- \rangle_\infty^k - \frac{k}{V} \frac{q + \langle n_- \rangle_\infty}{(q + 2\langle n_- \rangle_\infty)^2} \langle n_- \rangle_\infty^k + \frac{k(k-1)}{2V} \frac{q + \langle n_- \rangle_\infty}{q + 2\langle n_- \rangle_\infty} \langle n_- \rangle_\infty^{k-1} .$$

The grand canonical ensemble

$$\langle n_-^k \rangle^{GC} \simeq \langle n_- \rangle_\infty^k + \frac{k(k-1)}{2V} \langle n_- \rangle_\infty^{k-1} .$$

Canonical suppression factor

Canonical suppression factor for densities

$$\frac{\langle n_-^k \rangle_q^C}{\langle n_-^k \rangle_{\langle q \rangle}^{GC}} \simeq 1 - \frac{1}{V} \frac{k(k+1)\langle q \rangle + 2k^2 \langle n_- \rangle_\infty}{2(2\langle n_- \rangle_\infty + \langle q \rangle)^2}$$

Canonical suppression factor for particles

$$\frac{\langle N_-^k \rangle_q^C}{\langle N_-^k \rangle_{\langle q \rangle}^{GC}} \simeq 1 - \frac{1}{V} \frac{k(k+1)\langle q \rangle + 2k^2 \langle n_- \rangle_\infty}{2(2\langle n_- \rangle_\infty + \langle q \rangle)^2}$$

Conclusions

- In the thermodynamic limit relevant probabilities are density distributions.
- Density probability distributions obtained from different statistical ensembles have the same thermodynamical limit.
- Finite volume effect more relevant for higher moments.
- Canonical suppression factor for particles depends on densities.