BFKL Pomeron and its b dependence

Eugene Levin

Low x WS, May 30 - June 4 , Rehovot-Eilat , 2013

E.L. and S. Tapia: “BFKL Pomeron: modeling confinement”
arXive: 1304.8022

Large b dependence of the BFKL Pomeron

- $$N\,(r_1, r_2; Y, b) \;=\; \int \frac{d\gamma}{2\,\pi\,i}\,\phi^{(0)}_{in}(\nu)\,e^{\omega(\gamma = \frac{1}{2} + i\nu,\,0)\,Y}$$
$$\times\left\{ b_\nu\,(ww^*)^{\frac{1}{2}\,+\,i\nu} \;+\; b_{-\nu}\,(ww^*)^{\frac{1}{2}\,-\,i\nu} \right\} \;\longrightarrow\; \frac{r_1\,r_2}{b^2}\,e^{\omega_0\,Y}$$

- $$w\,w^* = \frac{r_1^2\,r_2^2}{\left(\vec{b} - \frac{1}{2}\,(\,\vec{r}_1\; - \;\vec{r}_2)\right)^2\,\left(\vec{b} \,+\, \frac{1}{2}\,(\,\vec{r}_1\; - \;\vec{r}_2)\right)^2}$$

$$N\,(r_1, r_2; Y, b) \quad \le 1 \quad \textbf{for} \quad b^2 \quad \le \quad r_1\,r_2 e^{\omega_0 Y}$$

Violation of Froissart theorem: (Kovner & Wiedemann)

$$\int d^2b\,N\,(r_1, r_2; Y, b) \;\propto\; s^{\omega_0} \quad \gg \quad Y^2 \;=\; ln^2 s$$

Lessons from numerical solutions and theory considerations:

((Kovner & Wiedemann, McLerran and Iancu,Golec-Biernat & Stasto, Gotsman et al, Berger & Stasto,2011)

- **The confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough);**

- **Suppressing large sizes of the produced dipoles in the decay** $one\ dipole \rightarrow two\ dipoles$ **we reproduce correct** b**-dependence;**

- **Since at large** b **the amplitude is small we do not need to take into account the non-linear corrections;**

Corrections from confinement have to be included in the kernel of the BFKL equation

- $$\frac{\partial N\left(x_{10}, b, Y\right)}{\partial Y} = \bar{\alpha}_S \int d^2 x_{12} K\left(x_{12}, x_{20} | x_{10}\right)$$
$$\times \left\{ 2\, N\left(x_{12}, \vec{b} - \tfrac{1}{2}\vec{x}_{02}; Y\right) \; - \; N\left(x_{10}, b; Y\right) \right\}$$

Modified BFKL kernel:

- $$K\left(x_{12}, x_{20} | x_{10}\right) \; = \; \frac{x^2_{10}}{x^2_{12}\, x^2_{02}}\, e^{-B\left(x^2_{12} + x^2_{02}\right)}$$

Results

1. $N\left(x_{10}, b, Y\right) \xrightarrow{Bb^2 \gg 1} e^{-4B\,b^2}$ $\longleftarrow$ **expected**

2. $N\left(x_{10}, b, Y\right) \xrightarrow{Y \gg 1} e^{\omega_0 Y}$ **with** $\omega_0 = \omega_{\text{BFKL}}$; $\longleftarrow$ **!!!!**

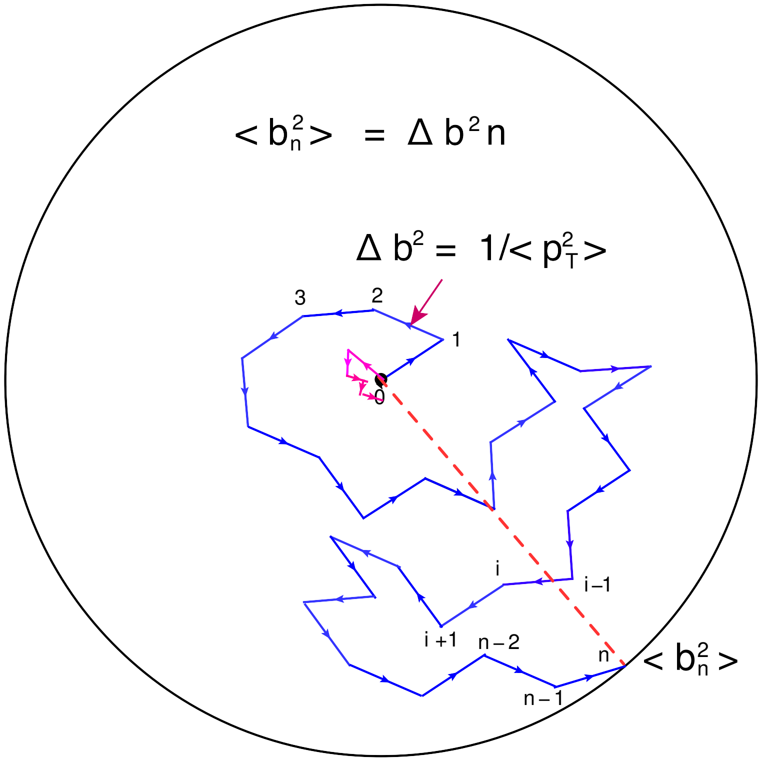
3. $\langle |b^2| \rangle = $ **Constant (Y)** ; $\longleftarrow$ **expected**

4. **Saturation scale** $Q_s^2 \propto e^{\lambda Y}$ **with** $\lambda \ll \lambda_{\text{BFKL}}$ $\longleftarrow$ **!!!!**

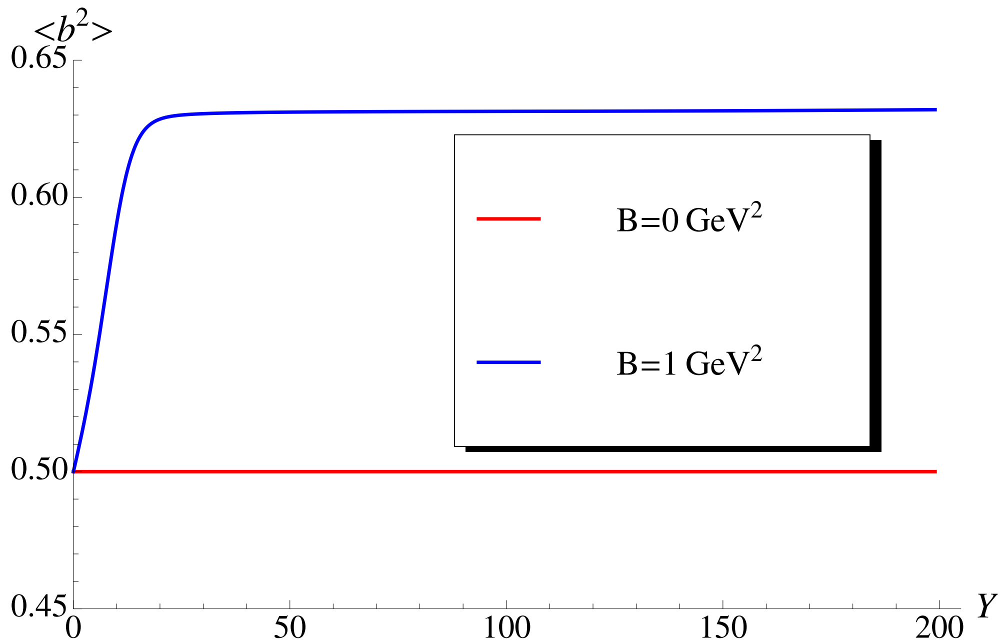
5. **The modified BFKL Pomeron looks similar to the Pomeron in N=4 SYM and high energy phenomenology:** $\Delta_{I\!P} \sim 0.3;\ \alpha'_{I\!P} = 0$ $\longleftarrow$ **!!!!**

$\langle|b^2|\rangle$ versus Y

Gribov's diffusion: $\Delta b \, p_T \sim 1$



Numerical calculations



$$\frac{\partial \widehat{\mathcal{N}}\left(x_{01};Y\right)}{\partial Y} = \frac{\partial}{\partial Y}\int d^2 b\, b^2\, \mathcal{N}\left(x_{01};Y\right) \qquad \left(\langle |b^2| \rangle = \widehat{\mathcal{N}}\left(x_{01};Y\right) \Big/ \mathcal{N}\left(x_{12};Y\right) \right)$$

$$= \bar{\alpha}_S \iint d^2 b'\, d^2 x_{12} \left(\vec{b'} + \tfrac{1}{2}\vec{x}_{12}\right)^2 \frac{1}{x^2_{12}} \left\{ 2\, \widetilde{\mathcal{N}}\left(x_{12}, \vec{b'}; Y\right) \; - \; \frac{x_{02}}{x^2_{12}} \widetilde{\mathcal{N}}\left(x_{01}, b; Y\right) \right\}$$

$$= \; \bar{\alpha}_S \int d^2 x_{12}\, \frac{1}{x^2_{12}} \left\{ 2\, \widehat{\mathcal{N}}^{BFKL}\left(x_{12};Y\right) \; - \; \frac{x_{01}}{x^2_{12}} \widehat{\mathcal{N}}\left(x_{01};Y\right) \right\} \; + \; \tfrac{1}{2}\bar{\alpha}_S \int d^2 x_{12}\, \mathcal{N}\left(x_{12};Y\right)$$

$$+ \left\{ \tfrac{1}{2}\bar{\alpha}_S \iint d^2 b'\, d^2 x_{12}\; \vec{b'} \cdot \vec{x}_{12}\, \frac{1}{x^2_{12}}\, 2\, \widetilde{\mathcal{N}}\left(x_{12}, \vec{b'} \equiv \vec{b} - \tfrac{1}{2}\vec{x}_{12}; Y\right) \; = \; 0 \right\}$$

Pomeron intercept ω_0

- **Remnant of conformal symmetry:** $x_{ik} \to \bar{x}_{ik} = \sqrt{B}\, x_{ik}$

$$\int d^2 x_{12} \frac{x_{10}^2}{x_{12}^2\, x_{02}^2}\, e^{-B\left(x_{12}^2 + x_{02}^2\right)} \to \int d^2 \bar{x}_{12} \frac{\bar{x}_{10}^2}{\bar{x}_{12}^2\, \bar{x}_{02}^2}\, e^{-\left(\bar{x}_{12}^2 + \bar{x}_{02}^2\right)}$$

- $N\ (x_{01}; Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y}\, N_\omega\,(x_{01})$

- $\omega N_\omega\,(x_{01}) = -\,\bar{\alpha}_S \mathcal{H}\ N_\omega\,(x_{01})$ **or** $E\, N_\omega\,(x_{01}) = \mathcal{H}\ N_\omega\,(x_{01})$

- $N_\omega\,(x_{01}) \xrightarrow{x_{01}^2 \ll 1/B} N_\nu^{BFKL}\,(x_{01}) = \left(\frac{1}{x_{01}^2}\right)^{\frac{1}{2} + i\nu}$ $\qquad N_\omega\,(x_{01}) \xrightarrow{x_{01}^2 \gg 1/B}$ **Const**

- $E\,(\nu) = 2\psi\,(1) - \psi\,\left(-\tfrac{1}{2} + i\nu\right) - \psi\,\left(\tfrac{1}{2} - i\nu\right)$

- $\int d^2 x_{01} N_\omega^*\,(x_{01})\, N_{\omega'}\,(x_{01}) < \infty \qquad \int d^2 x_{01} N_\nu^{BFKL*}\,(x_{01})\, N_{\nu'}^{BFKL}\,(x_{01}) = \delta\left(\nu - \nu'\right)$

Variational method

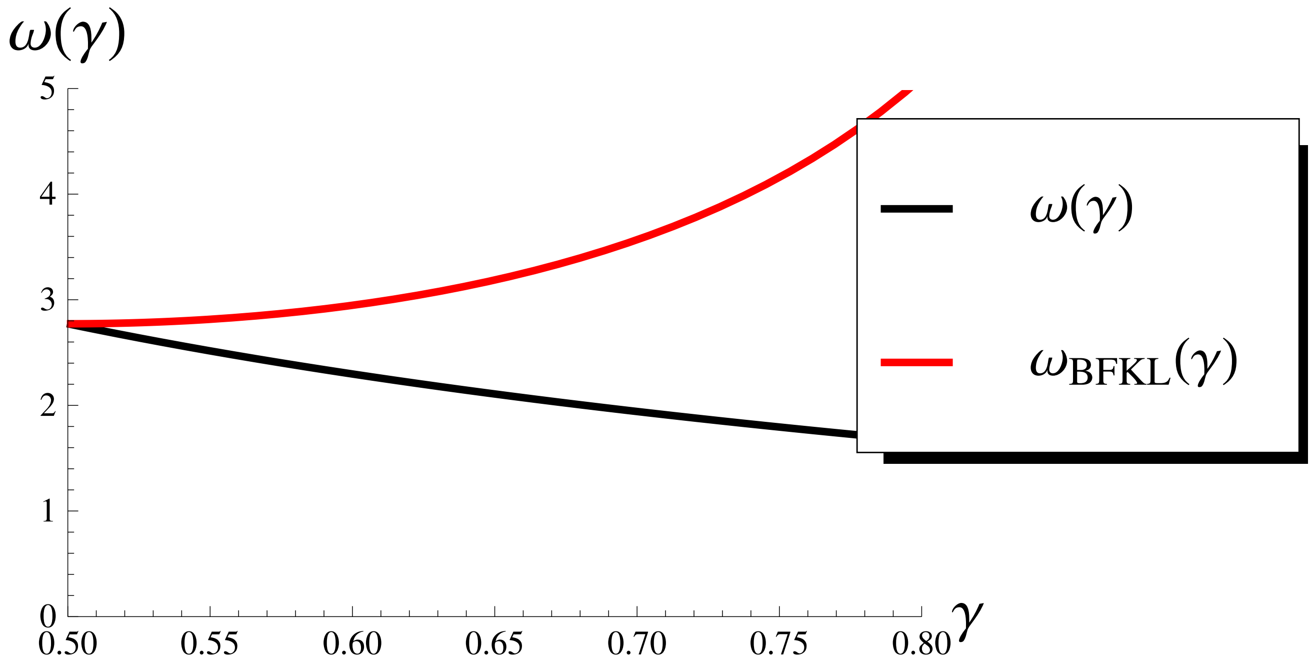
- $E_{\text{ground}} \equiv -\omega_0 \leq F[\{N\}] = \dfrac{\left\langle N^*(x_{01}) \middle| \mathcal{H} \middle| N(x_{01}) \right\rangle}{\left\langle N^*(x_{01}) \middle| N(x_{01}) \right\rangle}$

- Our choice: $\{N\} = \{N^{BFKL}\}$

  - $\mathcal{H}\, N^{\text{BFKL}}_{\gamma=-\frac{1}{2}+i\nu}(x_{10}) = \chi(\gamma, x_{10})\, N^{\text{BFKL}}_{\gamma=-\frac{1}{2}+i\nu}(x_{10})$

- $\chi(\gamma; \tilde{x}_{12}) = \int\limits_0^1 dt\, \dfrac{t^{\gamma-1}}{1-t} + \int\limits_1^{1/\tilde{x}^2_{12}} dt \dfrac{t^{\gamma-1}}{t-1} - \int_0^{1/\tilde{x}^2_{12}} \dfrac{1}{t} \left[\dfrac{1}{|t-1|} - \dfrac{1}{\sqrt{4\,t^2+1}}\right]$

- $\chi(\gamma; \tilde{x}_{12}) = \chi^{\text{BFKL}}(\gamma) + \text{arccsch}\left(2/\tilde{x}^2_{12}\right) - B\left(\tilde{x}_{12}; 1-\gamma, 0\right) - \ln\left(1 - \tilde{x}^2_{01}\right)$



$$\omega \geq \omega_{\rm BFKL}$$

Semi-classical approach

- $N\left(\mathcal{Y};l\right) = e^{S(\mathcal{Y},l)} = e^{\omega(\mathcal{Y},l)\mathcal{Y} \,+\, (\gamma(\mathcal{Y},l)-1|)l}$

where $\omega\left(\mathcal{Y},l\right) = \dfrac{\partial S\left(\mathcal{Y};l\right)}{\partial \mathcal{Y}}; \quad \gamma\left(\mathcal{Y},l\right) - 1 = \dfrac{\partial S|\left(\mathcal{Y};l\right)}{\partial l}$

with smooth functions $\omega\left(\mathcal{Y},l\right)$ and $\gamma\left(\mathcal{Y},l\right)$ and $\mathcal{Y} = \bar{\alpha}_S Y$.

Equation: $\omega\left(\mathcal{Y},l\right) - \chi\left(\gamma, x_{12}^2\right) = 0$

$$F\left(\mathcal{Y}, l, S, \gamma, \omega\right) = 0$$

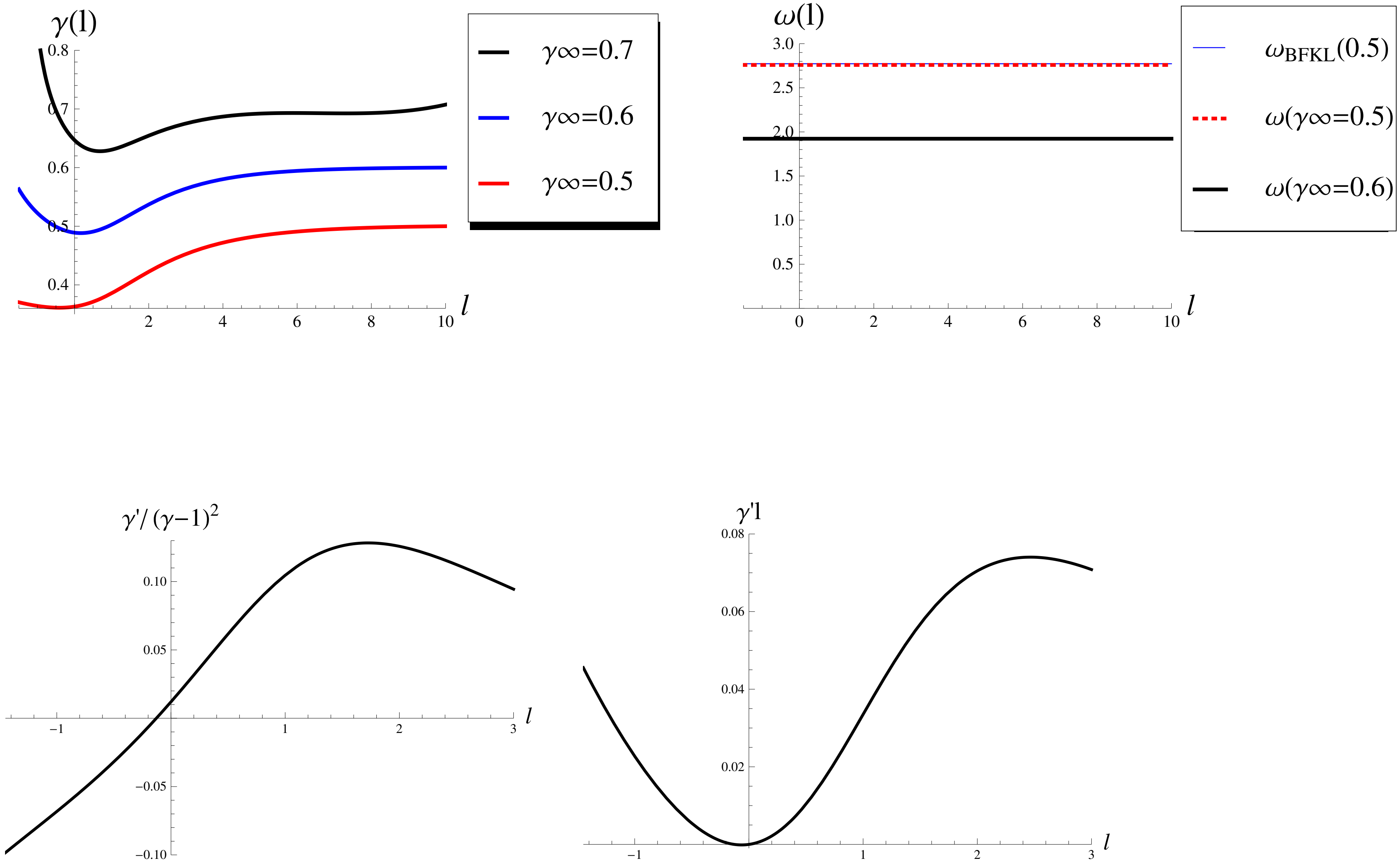
$$(1.)\quad \frac{dl}{d\,t} = F_\gamma = -\frac{d\chi\left(\gamma,0,l\right)}{d\gamma}$$

$$(2.)\quad \frac{d\mathcal{Y}\left(t\right)}{d\,t} = F_\omega = 1$$

$$(3.)\quad \frac{dS}{d\,t} = \gamma\,F_\gamma + \omega\,F_\omega = -\left(\gamma-1\right)\frac{\partial\chi\left(\gamma,0,l\right)}{\partial\gamma} + \omega$$

$$(4.)\quad \frac{d\,\gamma}{d\,t} = -(\,F_l + \gamma\,F_S\,) = \frac{\partial\chi\left(\gamma(t),0,l(t)\right)}{\partial\,l}$$

- $$\frac{d\,\gamma}{d\,l} = -\frac{\frac{\partial\chi(\gamma(t),l(t))}{\partial\,l}}{\frac{d\chi(\gamma,l(t))}{d\gamma}}$$



$$\omega \leq \omega_{\mathrm{BFKL}}$$

Diffusion approximation

$$\bar{N}\left(\mathcal{Y},l\right) \;=\; e^{\frac{1}{2}\,l} N\left(\mathcal{Y},l\right) \;=\; \int_{\epsilon-i\infty}^{\epsilon+i\infty}\frac{d\omega}{2\pi i}e^{\omega\,\mathcal{Y}}\,\bar{n}\left(\omega,l\right)$$

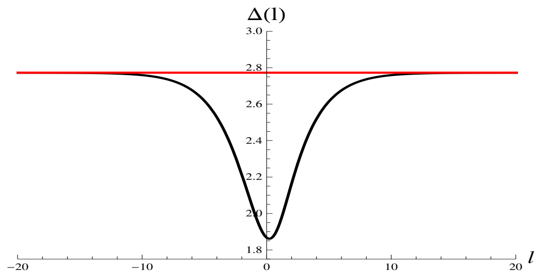
$$=\; \int_{\epsilon-i\infty}^{\epsilon+i\infty}\frac{d\omega}{2\pi i}\int_{i\epsilon-\infty}^{i\epsilon+\infty}\frac{d\nu}{2\pi i}\bar{n}\left(\omega,\nu\right)\,e^{\omega\,\mathcal{Y}+i\nu l}$$

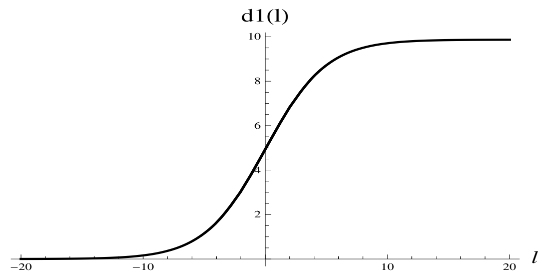
$$\bar{n}\left(\omega,l'\right) \;=\; \bar{n}\left(\omega,l\right) \;+\; \frac{\partial\bar{n}\left(\omega,l'\right)}{\partial l'}|_{l'=l}\left(l'-l\right) \;+\; \frac{1}{2}\frac{\partial^2\bar{n}\left(\omega,l'\right)}{\partial l'^2}|_{l'=l}\left(l-l\right)^2 \;+\;\dots$$

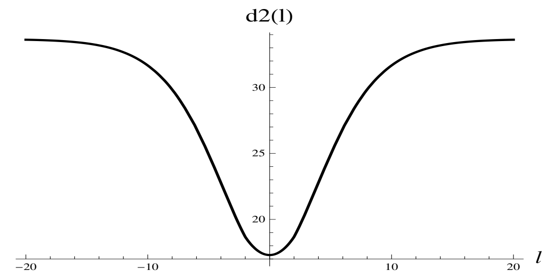
$$\Delta\left(l\right) \;=\; \chi\left(\frac{1}{2},e^{\frac{1}{2}l}\right);\quad d1\left(l\right) \;=\; -\,i\frac{\partial\chi\left(\frac{1}{2}+i\nu,e^{\frac{1}{2}l}\right)}{\partial\nu}|_{\nu=0};$$

$$d2\left(l\right) \;=\; -\frac{\partial^2\chi\left(\frac{1}{2}+i\nu,e^{\frac{1}{2}l}\right)^2}{\partial\nu}\;|_{\nu=0}$$

$$\left(\omega\;-\;\Delta\left(l\right)\right)\bar{n}\left(\omega,l\right)\;-d1\left(l\right)\,\frac{\partial\bar{n}(\omega,l)}{\partial l}\;-\;\frac{1}{2}d2\left(l\right)\,\frac{\partial^2\bar{n}(\omega,l)}{\partial l^2}\;=\;0$$



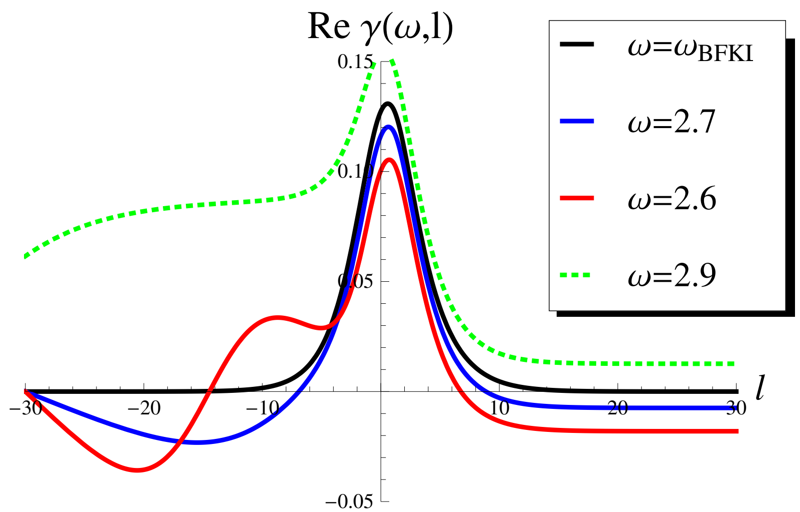


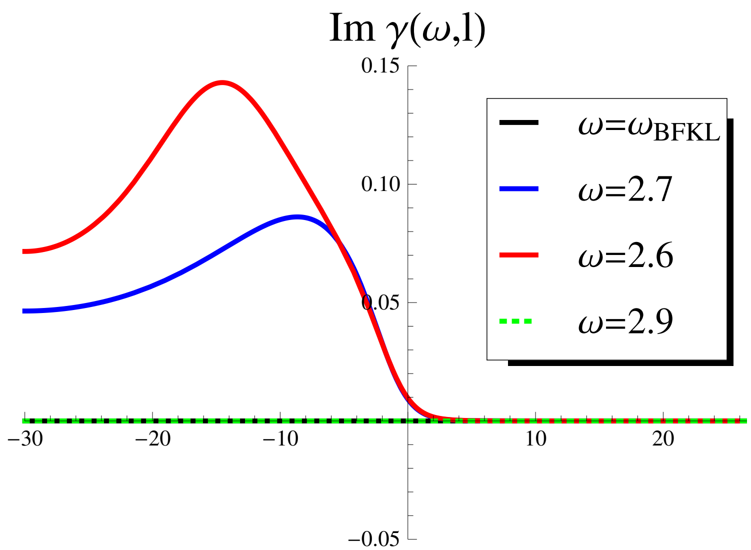


$$\bar{n}(\omega, l) = \exp(\phi(\omega, l)); \quad \text{and} \quad \gamma = \frac{\partial \phi(\omega,l)}{\partial l}$$

$$(\omega - \Delta(l)) = d1(l)\,\gamma(\omega, l) + \frac{1}{2} d2(l) \left(\frac{\partial \gamma(\omega,l)}{\partial l} + \gamma^2(\omega, l) \right)$$

$$\gamma(\omega, l) \xrightarrow{|l| \gg 1} \sqrt{(\omega - \omega_{\mathrm{BFKL}})/(2D_0)}$$





$$\omega \leq \omega_{\mathrm{BFKL}}$$

Numerical calculations of Y dependence

Two problems:

1. **The kernel is not Fredholm type**

- $$\int d^2x_{01} d^2x_{12}\, K\left(x_{12}, x_{02} | x_{01}\right) \Longrightarrow \infty$$

2. **The kernel is singular at** $x_{12} \quad \rightarrow \quad x_{01}$

Checks:

1. Dependence on x_{min} and x_{max};

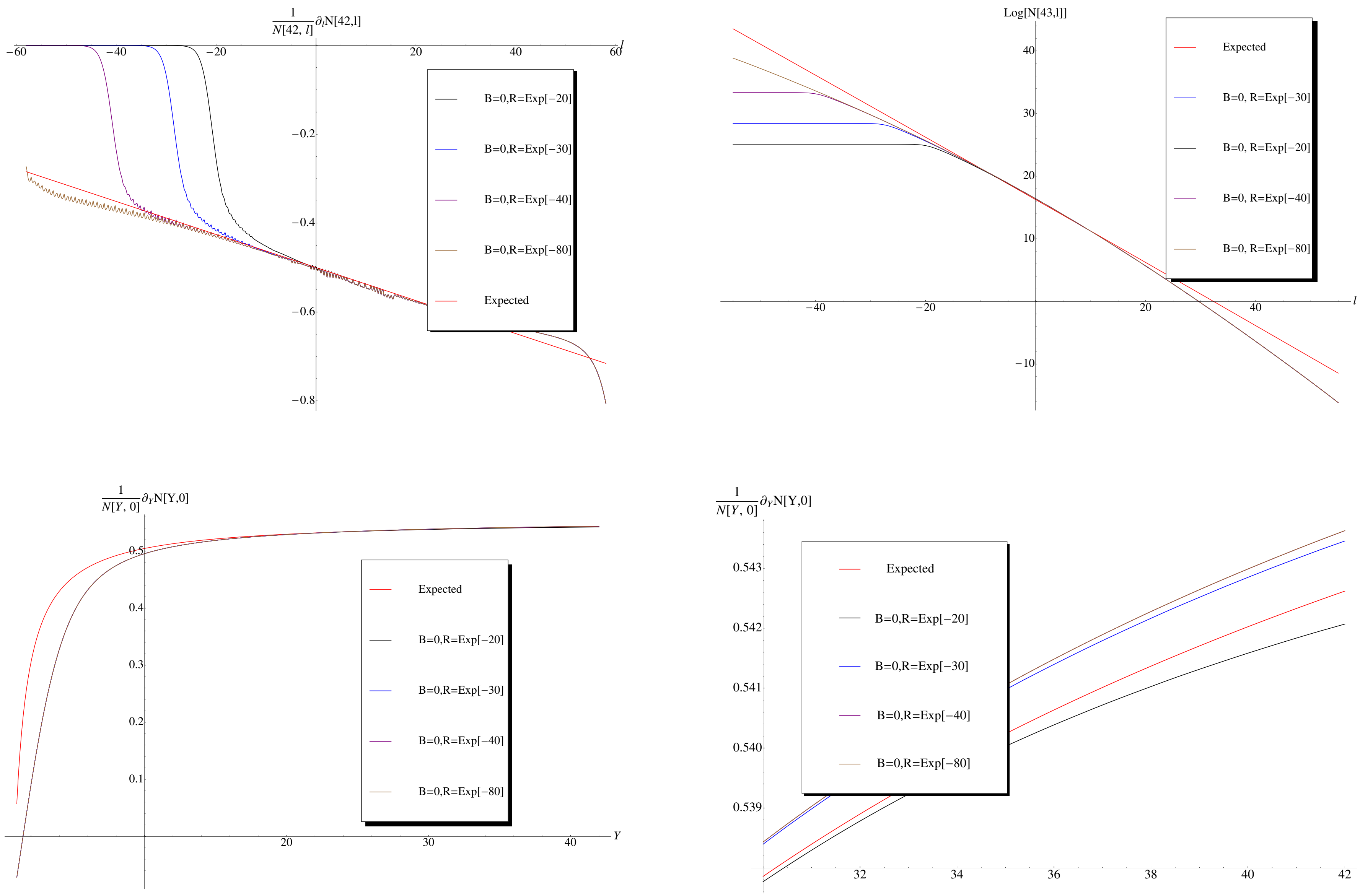
2. Numerical solution to the BFKL equation coincide with the analytic one;

3. Independence on value of the regulator R;

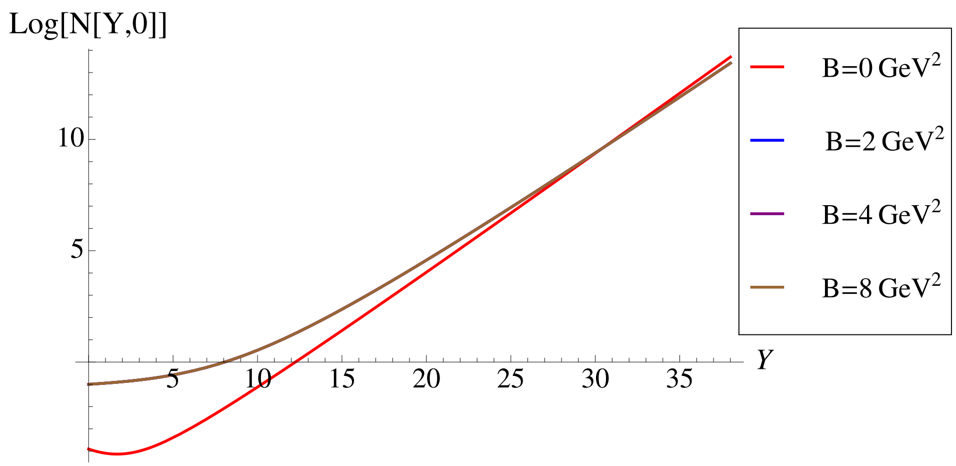
$$\int d^2x_{13} K^B_R\left(x_{13}, x_{32}|x_{12}\right)\mathcal{N}\left(x_{12};Y\right) \equiv$$

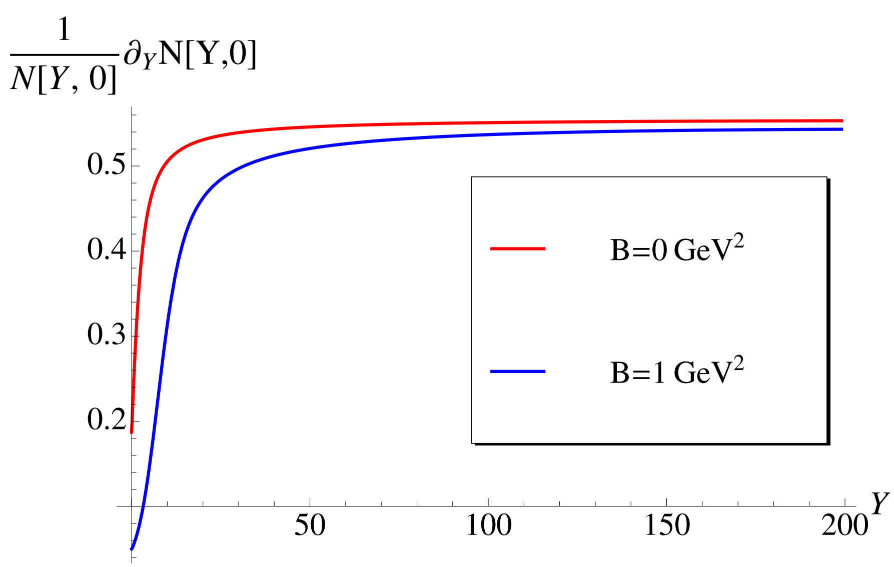
$$\int d^2x_{13}\,\frac{e^{-B\left(x^2_{13}+x^2_{32}\right)}}{x^2_{32}+R^2}\left\{2\,\mathcal{N}\left(x_{13};Y\right) - 2\,\frac{x^2_{12}}{x^2_{13}\,+\,x^2_{23}\,+\,2\,R^2}\mathcal{N}\left(x_{12};Y\right)\right\}$$

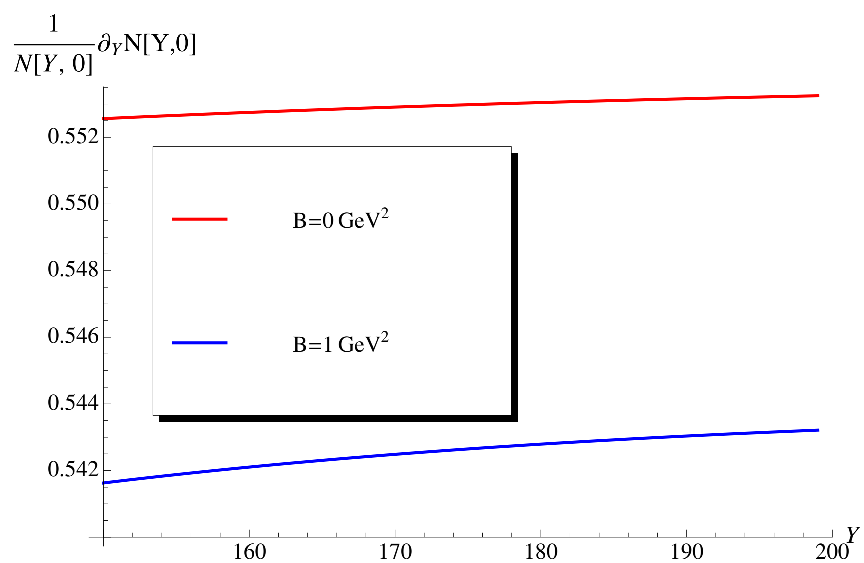
4. Independence on value of B;



1/N[42, l] ∂_l N[42,l]
l
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]
Expected
Log[N[43,l]]
l
Expected
B=0, R=Exp[−30]
B=0, R=Exp[−20]
B=0, R=Exp[−40]
B=0, R=Exp[−80]
1/N[Y, 0] ∂_Y N[Y,0]
Y
Expected
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]
1/N[Y, 0] ∂_Y N[Y,0]
Y
Expected
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]







$$\omega = \omega_{\text{BFKL}}$$

Saturation momentum

Saturation momentum can be found from linear equation:

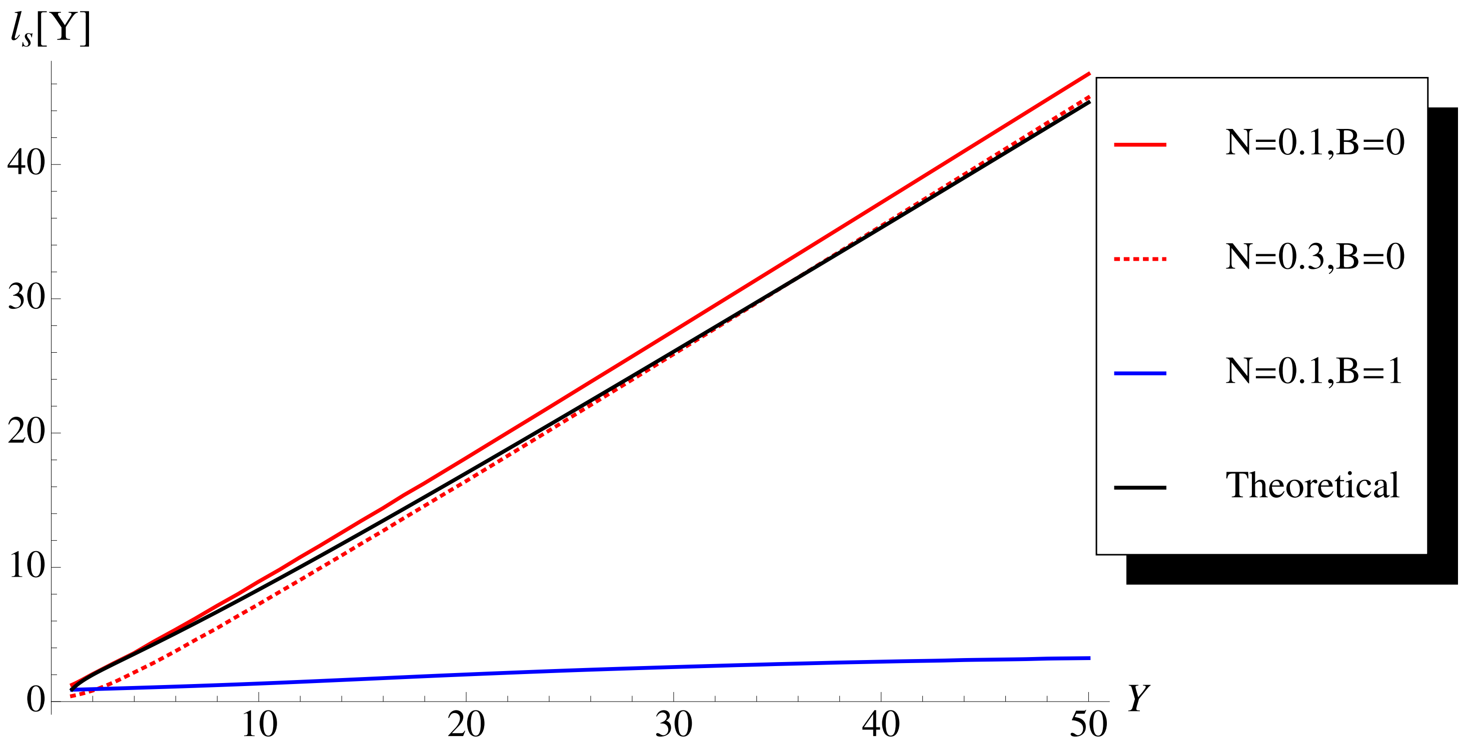
- $$\mathcal{N}^{BFKL}\left(\frac{2}{Q_s\,(Y)};Y\right) = \mathcal{N}_0 \leq 1 \quad \text{where} \qquad \mathcal{N}_0 = \text{Const}$$

For BFKL theoretical prediction:

$$l_s\,(Y) \;\equiv\; \ln\left(Q_s^2(Y)\Big/Q_s^2(Y_0)\right) =$$

$$\frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}\,(Y - Y_0) - \frac{3}{2(1-\gamma_{cr})}\,\ln(Y/Y_0)$$

$$-\frac{3}{(1-\gamma_{cr})^2}\sqrt{\frac{2\,\pi}{\omega''(\gamma_{cr})}}\left(\frac{1}{\sqrt{Y}} - \frac{1}{\sqrt{Y_0}}\right)$$



l_s[Y]
40
30
20
10
0
10
20
30
40
50
Y
N=0.1,B=0
N=0.3,B=0
N=0.1,B=1
Theoretical

Next steps:

- Check that different models for large b dependence in the BFKL kernel lead to $\omega = \omega_{\text{BFKL}}$
  - $K\left(x_{12}, x_{20}|x_{10}\right) = \frac{x_{10}^2}{x_{12}^2\, x_{02}}\, e^{-\mu(x_{12}+x_{02})}$;
  - BFKL in gauge theories with the Higgs mechanism of mass generation (E.L.,L.Lipatov and M.Siddikov)

$$E\phi\left(\kappa\right) = \underbrace{\frac{\kappa+1}{\sqrt{\kappa}\sqrt{\kappa+4}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}\phi\left(\kappa\right)}_{\text{kinetic energy}}$$

$$-\int_0^\infty \frac{d\kappa'\phi\left(\kappa'\right)}{\sqrt{(\kappa-\kappa')^2 + 2(\kappa+\kappa')+1}} + \underbrace{\frac{N_c^2+1}{2N_c^2}\frac{1}{\kappa+1}\int_0^\infty\frac{\phi\left(\kappa'\right)\,d\kappa'}{\kappa'+1}}_{\text{contact term}}$$

- Build Pomeron calculus, based on modified BFKL Pomerons;

- Obtain non-linear equations for amplitude;

Hope:

- **The global features of the BFKL Pomeron does not depend on confinement;**

- **We will be able to build the self-consistent theoretical CGC/saturation approach with correct b behaviour of the scattering amplitude;**

Conclusions back to page 5

THANK YOU