

Threshold Pion Production at Large Momentum Transfers

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based on

V.M. Braun, D. Ivanov, A. Lenz and A. Peters, Phys.Rev.D75:014021,2007

V.M. Braun, D. Ivanov and A. Peters, Phys.Rev.D77:034016,2008

PANIC 2008, Eilat, 13.11.08

Electroproduction with Q^2 in a few GeV^2 range:

- Tradition: excitation of nucleon resonances (transition form factors)

$$e(l) + p(P) \rightarrow e(l') + \Delta(1232)(P')$$

$$e(l) + p(P) \rightarrow e(l') + N(1440)(P')$$

- Proposal: pion electroproduction close to threshold $W \rightarrow W_{\text{th}}$

$$e(l) + p(P) \rightarrow e(l') + \pi^+(k) + n(P')$$

$$e(l) + p(P) \rightarrow e(l') + \pi^0(k) + p(P')$$

$$W^2 = (P' + k)^2$$

$$W_{\text{th}} = m_N + m_\pi$$

$$Q^2 = -q^2 = -(\ell - \ell')^2$$

Hard ($pQCD$) and soft ($ChPT$) physics meet together !

Generalized Form Factors = S-wave Multipoles at Threshold

at the threshold

$$\langle \pi N | j_\mu^{\text{em}} | p \rangle = -\frac{i}{f_\pi} \bar{N}(P_2) \gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

related to S-wave multipoles in the PWA, e.g. for $m_\pi = 0$

$$E_{0+}^{\pi N}(Q^2, W_{\text{th}}) = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N}(Q^2, W_{\text{th}}) = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

e.g. the differential cross section at threshold is given by

$$\left. \frac{d\sigma_{\gamma^*}}{d\Omega_\pi} \right|_{\text{th}} = \frac{2|\vec{k}_f|W}{W^2 - m_N^2} \left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\text{th}})^2} (L_{0+}^{\pi N})^2 \right]$$

Chiral rotation

Spontaneous Breaking of Chiral Symmetry

- In the chiral limit, $m_\pi/m_N \rightarrow 0$, the pion can be “rotated” away:

$$\begin{aligned}
 |p \uparrow\rangle &= \frac{\phi_s(x)}{\sqrt{6}} |2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
 |p \uparrow \pi^0\rangle &= \frac{\phi_s(x)}{2\sqrt{6}f_\pi} |6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
 |n \uparrow \pi^+\rangle &= \frac{\phi_s(x)}{\sqrt{12}f_\pi} |2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle
 \end{aligned}$$

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

- allows one to “look” at the proton from a different “angle”

The relevant degrees of freedom change with Q^2

⇒ rich physics

⇒ rich theory

$Q^2 < 0.1 \text{ GeV}^2$: Chiral Perturbation Theory

- local effective low-energy theory
- systematic expansion in powers of m_π and $|q|$
- applicable for $|q| \sim m_\pi < 300 \text{ MeV}(\?)$

Bernard, Kaiser, Meissner; IJMP, E4 (1995)193
 Drechsel, Tiator; J. Phys. G **18** (1992) 449

Kroll, Ruderman

$$\mu = m_\pi/m_N \simeq 1/7$$

$$E_{0+}^{\pi^+n}(q^2 = 0, W_{\text{th}}) = \sqrt{2} \frac{eg_{\pi N}}{8\pi m_N} \left[1 - \frac{3}{2}\mu + O(\mu^2 \ln \mu^2) \right] = 26.6 \cdot 10^{-3}/m_\pi$$

exp: $27.9 \pm 0.5; 28.8 \pm 0.7$

$$E_{0+}^{\pi^0p}(q^2 = 0, W_{\text{th}}) = -\frac{eg_{\pi N}}{8\pi m_N} \mu \left\{ 1 - \mu \left[\frac{1}{2}(3 + \kappa_p) + \left(\frac{m_N}{4f_\pi} \right)^2 \right] \right\}$$

- **Subtlety:** $q \rightarrow 0$ and $m_\pi \rightarrow 0$ limits do not commute

Bernard, Kaiser, Meissner; PRL69 (1992)1877

Nambu, Lurié, Shrauner

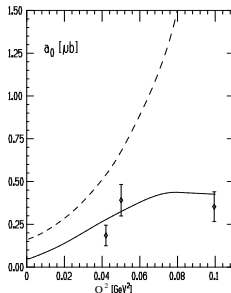
$$E_{0+}^{(-)}(m_\pi = 0, q^2) = \frac{eg_A}{8\pi f_\pi} \left\{ 1 + \frac{q^2}{6} r_A^2 + \frac{q^2}{4m_N^2} \left(\kappa_v + \frac{1}{2} \right) + \frac{q^2}{128f_\pi^2} \left(1 - \frac{12}{\pi^2} \right) \right\}$$

$$G_A(q^2) = g_A \left(1 + \frac{q^2}{6} r_A^2 + \dots \right)$$

Experiment: $r_A = 0.65 \pm 0.03$ (elastic ep); $r_A = 0.59 + 0.04 \pm 0.05$ (pion el.prod)**S-wave cross section**

$\gamma^* p \rightarrow \pi^0 p$

$W = 1074, \epsilon = 0.58$



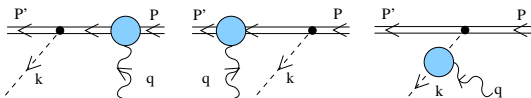
$Q^2 \ll m_N^3/m_\pi$: Low-Energy Theorems

- predate ChPT and QCD

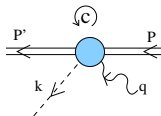
Chiral symmetry:

- 1 pion mass $m_\pi \rightarrow 0$
- 2 pion coupling $\sim |k| \rightarrow 0$

- Pion emission from external legs



- Chiral Rotation



$$\langle \pi^a N | j_\mu^{\text{em}} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j_\mu^{\text{em}}, Q_5^a] | N \rangle$$

Kroll, Ruderman '54

Low-Energy Theorems – *continued*

PCAC + current algebra:

Vainshtein, Zakharov, NPB36(1972)589

Scherer, Koch, NPA534(1991)461

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p, \quad G_2^{\pi^0 p} = \frac{2g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A, \quad G_2^{\pi^+ n} = \frac{2\sqrt{2}g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n,$$

Derivation does not imply $Q^2 \sim m_\pi^2$!

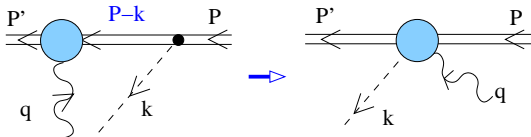
- Threshold photoproduction of π^0 is suppressed compared to π^+
- The π^0/π^+ -ratio is rapidly increasing with Q^2

- ◇ The $\mathcal{O}(m_\pi)$ corrections can be added
- ◇ but, no systematic way to treat $\mathcal{O}(m_\pi^2)$ terms (ChPT)

Low-Energy Theorems — continued (II)

- expected to fail for $Q^2 \sim \frac{m_N^3}{m_\pi}$

since π cannot have small momentum w.r.t. the initial and final state protons simultaneously



at threshold

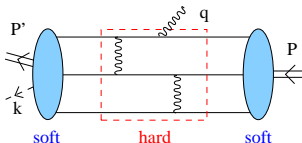
$$m_N^2 - (P - k)^2 = \frac{m_\pi}{m_N} [Q^2 + 2m_N^2]$$

- ⇒ phenomenological Lagrangians to take into account nucleon resonances
- ⇒ or go over to quark-gluon description

$Q^2 \gg m_N^3/m_\pi$: Perturbative QCD

QCD factorization for $Q^2 \gg \Lambda_{\text{QCD}}^3/m_\pi$

Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:



$$|p \uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}} |2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

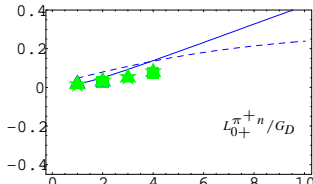
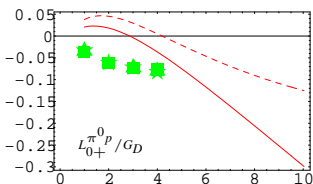
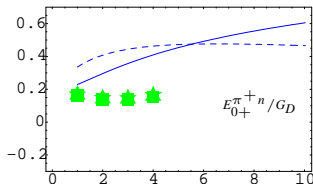
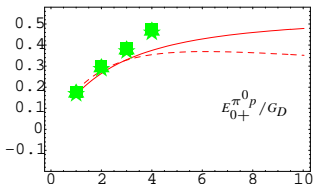
$$|p \uparrow \pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi} |6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n \uparrow \pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi} |2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

- ◇ Only for $G_1^{\pi N} (E_{0+})$
- ◇ Probably unrealistic at reachable momentum transfers

$Q^2 \sim m_N^3/m_\pi$: Light-Cone Sum Rules: $Q^2 \sim 1 - 10 \text{ GeV}^2$

- ◇ normalized to the dipole formula $G_D = 1/(1 + Q^2/0.71)^2$



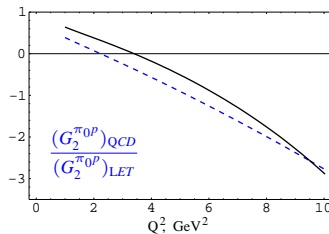
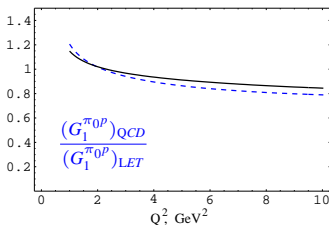
green dots: MAID; ■: $W = 1074 \text{ MeV}$, ▲: $W = 1084 \text{ MeV}$, ★: $W = 1094 \text{ MeV}$

solid curves: LCSRs using experimental elastic EM formfactors as input

dashed curves: pure LCSRs, no experimental input

Light Cone Sum Rules —continued

Deviation from LET:

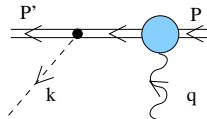


- ♥ Reproduce LET for $Q^2 \sim 1 \text{ GeV}^2$
- ♥ Reproduce pQCD for $Q^2 \rightarrow \infty$ (part of the NNLO α_s^2 contribution)
- ♥ No double counting of “soft” and “hard” contributions
- ♥ Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors

Moving away from threshold

◇ Higher partial waves

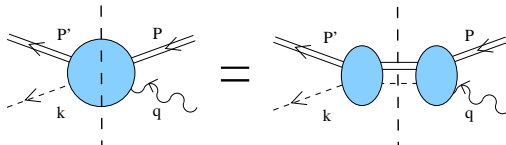
- P -wave dominated by pion emission from the final state



◇ Energy dependence

$E_{0+}(W)$, $L_{0+}(W)$, etc.

- due to final state interactions



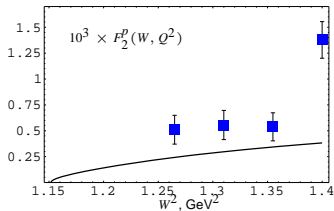
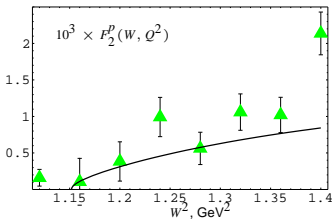
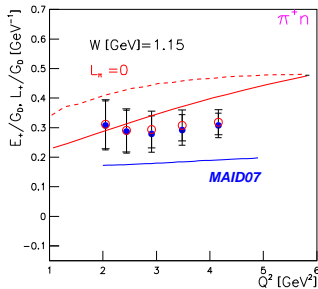
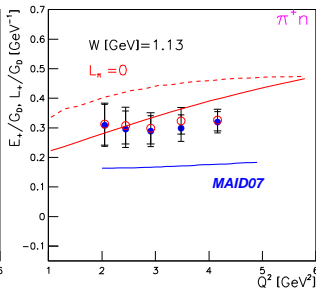
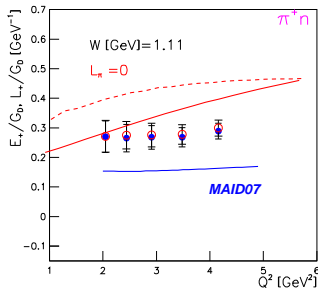
P. E. Bosted *et al.*; PRD49(1994)3091


Figure: The structure function $F_2^p(W, Q^2)$ as a function of W^2 scaled by a factor 10^3 compared to the SLAC E136 data at the average value $Q^2 = 7.14 \text{ GeV}^2$ (left panel) and $Q^2 = 9.43 \text{ GeV}^2$ (right panel).

CLAS (preliminary) $ep \rightarrow e\pi^+n$



Blue circle = assumption of $G_E^n = 0$
 Red circle = $G_E^n(Q^2) = -a\mu_n\tau G_0(Q^2)/(1+b\tau)$
 with assumption $\text{Im}(G_2^{nn^*}) = 0$

Stat. Err. ONLY in data

Red Solid line =
 LCSR using experimental
 EM form factors as input

Red Dash line = pure LCSR

Kijun Park; DNP 2008
 Oakland, CA (Oct. 23-26)

Summary

◇ Use pion as a handle to “rotate” the nucleon wave function

- a novel object: generalized form factor; an overlap between usual and rotated WF
- check Low Energy Theorems (Nambu, ...) and transition to QCD
- new scale in QCD: $Q^2 \sim m_N^3/m_\pi$
- measure nucleon axial form factor (requires π^+)
- theory progress feasible, large community (ChPT, pQCD, PWA)
- an (almost) untouched terrain...

! no data at $Q^2 \sim 0.1 - 1 \text{ GeV}^2$ MAMI?

Perfectly suited for the JLab 12 GeV upgrade physics program

Supplementary Material

$Q^2 \sim 1 - 10 \text{ GeV}^2$: Light Cone Sum Rules

Balitsky, V.B., Kolesnichenko '86-'88

- consider

$$T_\nu^{\pi N}(P', q) = i \int d^4x e^{-iqx} \langle N(P') \pi^a(k) | T \{ j_\nu^{\text{em}}(x) \bar{\eta}_p(0) \} | 0 \rangle$$

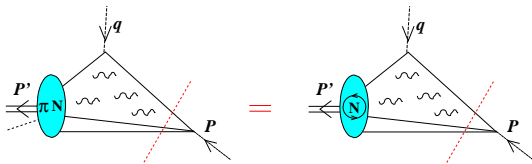
$$\eta_p(x) = \epsilon^{ijk} \left[u^i(x) C \gamma_\mu u^j(x) \right] \gamma_5 \gamma^\mu d^k(x), \quad \langle 0 | \eta_p | N(P) \rangle = \lambda_p m_N N(P)$$

- take $P = P' + q - k$, $P^2 \sim -1 \text{ GeV}^2$ and make a matching between

(a) The Operator Product Expansion in terms of pion-nucleon DAs

$$\langle N(P') \pi^a(k) | T \{ j_\nu^{\text{em}}(x) \bar{\eta}_p(0) \} | 0 \rangle = \sum_{\text{twist}} H_\nu(x^2, px) \otimes \langle 0 | q(x_1) q(x_2) q(x_3) | N(P') \pi^a(k) \rangle^\dagger$$

(b) The dispersion representation in terms of hadronic states



- ◇ Borel transformation to improve convergence

Light Cone Sum Rules — *continued*

Good things:

- ♥ Reproduce LET for $Q^2 \sim 1 \text{ GeV}^2$
- ♥ Reproduce pQCD for $Q^2 \rightarrow \infty$ (part of the NNLO α_s^2 contribution)
- ♥ No double counting of “soft” and “hard” contributions
- ♥ Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors

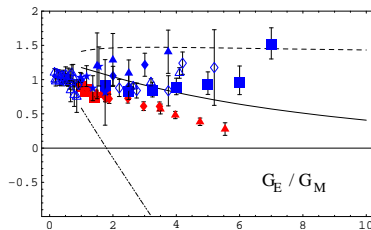
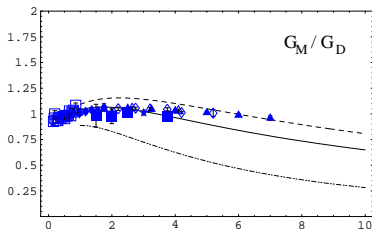
Not-so-good things:

- ◇ Use nucleon distribution amplitudes as input — not so well known
- ◇ Calculation rather demanding, especially in NLO

Bad things:

- ♠ Approximation for the continuum contribution not improvable
— irreducible error of order 20% for all Q^2

Light-Cone Sum Rules: Nucleon Electromagnetic form factors



choice of nucleon DAs:

solid: BLW model
long dashes: asymptotic
short dashes: CZ model

Braun, Lenz, Wittmann; PRD73(2006)094019

A. Lenz; arXiv:0708.0633v1

Light-Cone Sum Rules: Pion-Nucleon Intermediate States

- **New: Semidisconnected pion-nucleon contributions in the intermediate state**

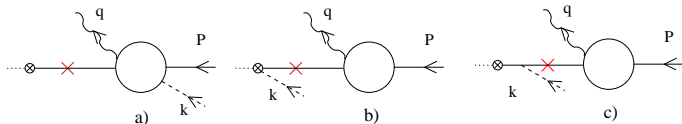


Figure: Schematic structure of the pole terms in the correlation function

- **b) and c) correspond to πN coupling to the Ioffe current**

$$\langle 0 | \eta_p(0) | N(P' - k) \pi(k) \rangle = \frac{i\lambda_1^p m_N}{2f_\pi} \left[1 - \frac{g_A}{P'^2 - m_N^2} (\not{P}' - \not{k} + m_N) \not{k} \right] \gamma_5 N(P' - k).$$

In the threshold kinematics, with $\delta = m_\pi/m_N$

$$\begin{aligned} T_\nu^{\pi^0 p}(P, q) = & \frac{i\lambda_1^p m_N}{f_\pi} \left\{ \frac{((1 + \delta) \not{P} - \not{q} + m_N) \gamma_5}{m_N^2 - P'^2} \left[(\gamma_\nu q^2 - q_\nu \not{q}) \frac{G_1^{\pi^0 p}}{m_N^2} - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} G_2^{\pi^0 p} \right] \right. \\ & + \frac{1}{2} \frac{(1 + \delta) \gamma_5 (\not{P} - \not{q} + m_N)}{[m_N^2(1 + \delta)^2 + \delta Q^2] - P'^2} \left[\gamma_\nu F_1^p - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} F_2^p \right] \\ & \left. - \frac{(1 + \delta) g_A (\not{P} - \not{q} + m_N) \gamma_5}{[m_N^2(1 + \delta)^2 + \delta Q^2] - P'^2} \left[(\gamma_\nu q^2 - q_\nu \not{q}) G_M^p - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} 4m_N^2 G_E^p \right] \right\} N(P) \end{aligned}$$

Light-Cone Sum Rules: Pion-Nucleon Intermediate States — *cont.*

- The semidisconnected πN contributions can be included in the continuum if

$$m_\pi Q^2 > m_N(s_0 - m_N^2) \Rightarrow Q^2 > 7 \text{ GeV}^2 \quad [\sim \Lambda_{\text{QCD}}^3/m_\pi]$$

- Otherwise they have to be taken into account explicitly

$$\begin{aligned} \frac{Q^2}{m_N^2} G_1^{\pi^0 p} &= \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{A}^{\pi^0 p}](M^2, Q^2) - \frac{1}{2} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^p(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^p(Q^2) \right] \\ G_2^{\pi^0 p} &= -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{B}^{\pi^0 p}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{2} F_2^p(Q^2) + \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p(Q^2) \right] \\ \frac{Q^2}{m_N^2} G_1^{\pi^+ n} &= \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{A}^{\pi^+ n}](M^2, Q^2) - \frac{1}{\sqrt{2}} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^n(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^n(Q^2) \right] \\ G_2^{\pi^+ n} &= -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{B}^{\pi^+ n}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{\sqrt{2}} F_2^n(Q^2) + \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n(Q^2) \right] \end{aligned}$$

where $\mathcal{A}(P'^2, Q^2)$ and $\mathcal{B}(P'^2, Q^2)$ are the invariant functions defined as

$$z^\nu \Lambda^+ T_\nu^{\pi N}(P, q) = \frac{i}{f_\pi} (pz + kz) \gamma_5 \left\{ m_N \mathcal{A}(P'^2, Q^2) + \not{q}_\perp \mathcal{B}(P'^2, Q^2) \right\} N^+(P)$$

πN scattering phases

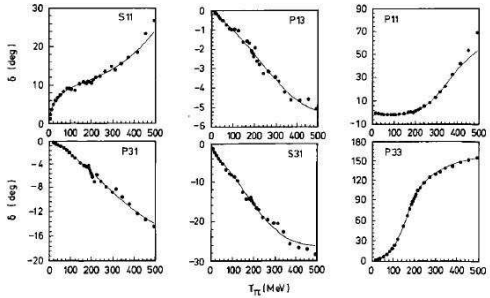


Figure from Nozawa *et al.*, PRC41(1990)213

Including P-waves:

for $W - W_{\text{th}} \ll m_\pi$ accept

$$\begin{aligned}
 & \langle N(P') \pi(k) | j_\mu^{\text{em}}(0) | P(P) \rangle = \\
 & = -\frac{i}{f_\pi} \bar{N}(P') \gamma_5 \left\{ \left(\gamma_\mu Q^2 - q_\mu \not{Q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P) \\
 & + \frac{ic_\pi g_A}{2f_\pi [(P'+k)^2] - m_N^2} \bar{N}(P') \not{k} \gamma_5 (\not{P}' + m_N) \left\{ F_1^p(Q^2) \left(\gamma_\mu - \frac{q_\mu \not{Q}}{Q^2} \right) + \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} F_2^p(Q^2) \right\} N(P)
 \end{aligned}$$

- ◇ **S-wave: generalized form factors from LCSR**
- ◇ **P-wave: pion emission from the final state nucleon; exact in chiral limit**
- ◇ **Eventually can take into account the final state interactions**

$$G_1^{\pi N}(Q^2) \rightarrow G_1^{\pi N}(Q^2, W) \equiv G_1^{\pi N}(Q^2) [1 + i t_{\pi N}]$$

Structure Functions at $x_B \rightarrow 1$

$$F_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2 + 4m_N^2}{2m_N^4} |Q^2 G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 \right\}$$

$$F_2(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{m_N^4} \left(|Q^2 G_1^{\pi N}|^2 + \frac{m_N^2}{4} Q^2 |G_2^{\pi N}|^2 \right) + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W) Q^2 m_N^2}{4(W^2 - m_N^2)^2} \left(\frac{Q^2 G_M^2 + 4m_N^2 G_E^2}{Q^2 + 4m_N^2} \right) \right\}$$

$$g_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 - m_N^2 \text{Re}(Q^2 G_1^{\pi N} G_2^{*, \pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M F_1^p \right\}$$

$$g_2(W, Q^2) = -\frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 + \frac{1}{4} Q^2 \text{Re}(Q^2 G_1^{\pi N} G_2^{*, \pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{32(W^2 - m_N^2)^2} Q^4 G_M F_2^p \right\}$$

$$\beta(W) = \frac{|\vec{k}_f|}{W},$$

$$x_B = \frac{Q^2}{Q^2 + W^2 - m_N^2}$$

Differential Cross Section

For unpolarized protons, the virtual photon cross section is

$$d\sigma_{\gamma^*} = \frac{\alpha_{\text{em}}}{8\pi} \frac{k_f}{W} \frac{d\Omega_\pi}{W^2 - m_N^2} |\mathcal{M}_{\gamma^*}|^2$$

with

$$|\mathcal{M}_{\gamma^*}|^2 = M_T + \epsilon M_L + \sqrt{2\epsilon(1+\epsilon)} M_{LT} \cos(\phi_\pi) + \epsilon M_{TT} \cos(2\phi_\pi) + \lambda \sqrt{2\epsilon(1-\epsilon)} M'_{LT} \sin(\phi_\pi)$$

$$f_\pi^2 M_T = \frac{4\vec{k}_i^2 Q^2}{m_N^2} |G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 + \cos\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} 4Q^2 G_M \text{Re } G_1^{\pi N}$$

$$f_\pi^2 M_L = \vec{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m_N^2)^2} m_N^4 G_E^2 - \cos\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} 4m_N^2 G_E \text{Re } G_2^{\pi N}$$

$$f_\pi^2 M_{LT} = -\sin\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N [G_M \text{Re } G_2^{\pi N} + 4G_E \text{Re } G_1^{\pi N}]$$

$$f_\pi^2 M_{TT} = 0,$$

$$f_\pi^2 M'_{LT} = -\sin\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N [G_M \text{Im } G_2^{\pi N} - 4G_E \text{Im } G_1^{\pi N}]$$

- ◇ $M_{TT} = 0$: no D-wave; tests quality of the approximation
- ◇ M'_{LT} : single-spin asymmetry; arises because of FSI, calculable

Miscellaneous Results

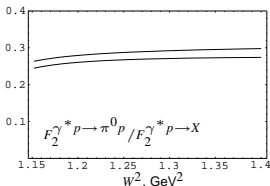


Figure: The fraction of $\pi^0 p$ in $F_2^p(W, Q^2)$ for $Q^2 = 3 \text{ GeV}^2$ (upper curve) and $Q^2 = 9 \text{ GeV}^2$ (lower curve)

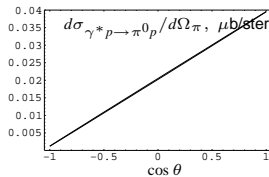


Figure: Differential cross section $d\sigma_{\gamma^* p \to \pi^0 p} / d\Omega_\pi$ for $\phi_\pi = 135 \text{ grad}$, $Q^2 = 4.2 \text{ GeV}^2$ and $W = 1.11 \text{ GeV}$

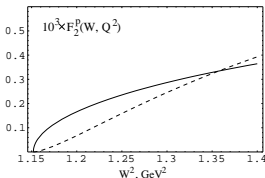


Figure: S-wave (solid) vs. P-wave (dashed) for $F_2^p(W, Q^2)$ at $Q^2 = 7.14 \text{ GeV}^2$

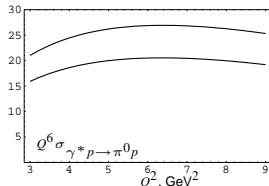


Figure: Integrated cross section $Q^6 \sigma_{\gamma^* p \to \pi^0 p}$ for $W = 1.11 \text{ GeV}$ (lower curve) and $W = 1.15 \text{ GeV}$ (upper curve)