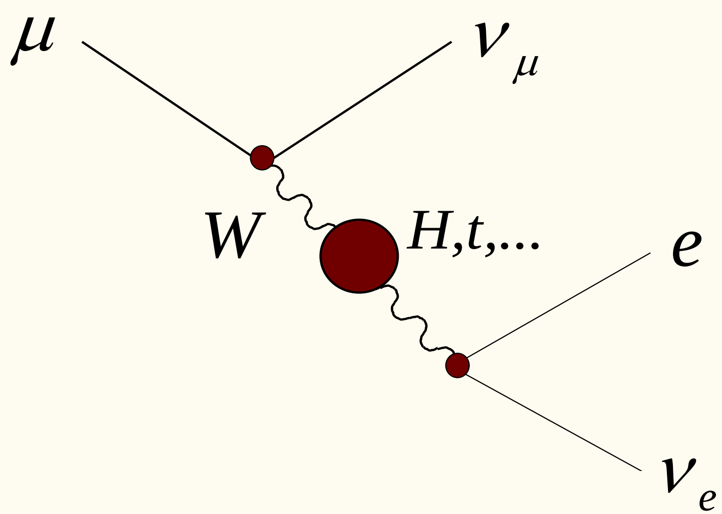
Mass Effects in Semileptonic b→c and Muon Decays



PANIC 2008

Eilat, November 2008

Andrzej Czarnecki   University of Alberta

Outline

(Focus on the muon; b → c analogous at the perturbative level.)

Muon decay and its new measurements

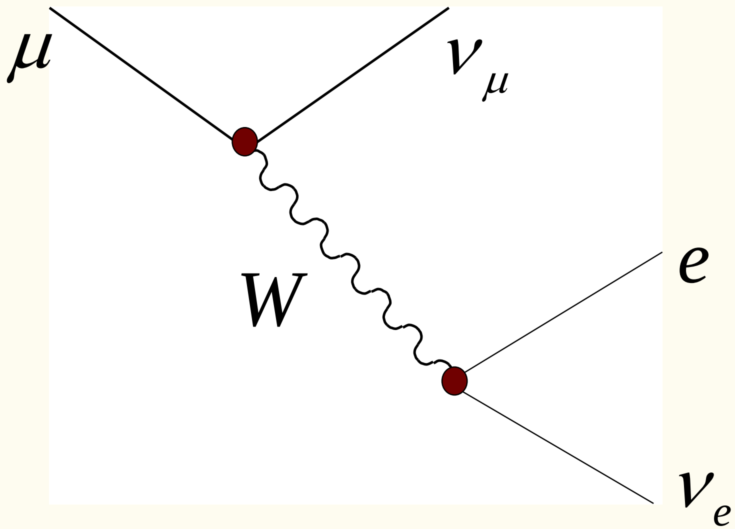
QED radiative corrections

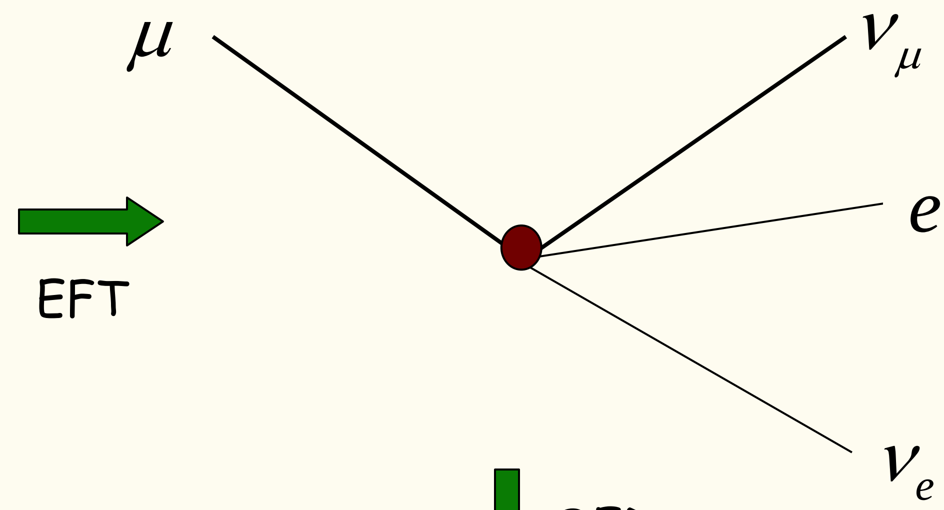
* massless-electron limit
* effect of the finite m_e (new)

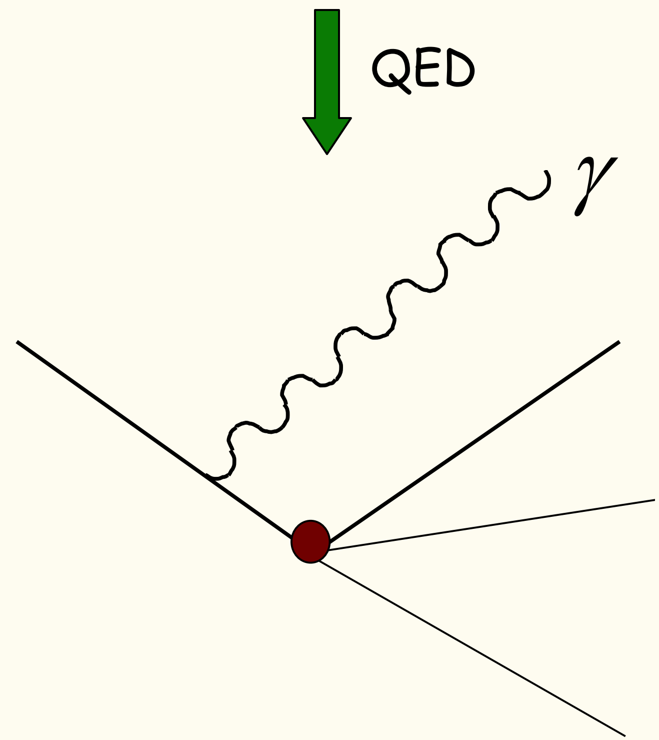
What accuracy can be achieved?

Correction for $\mu \to e\gamma$ (analogous to $b \to s\gamma$)

Effective theory for the muon decay







$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[1+\Delta q\right]$$

Finite m_e and QED corrections
in the four-fermion EFT

Fermi constant: progress in muon lifetime measurements

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[1+\cdots\right]$$

MuLan:

G_F = 1.166371(6) × 10^{-5} GeV^{-2} (5 ppm)

Chitwood et al., arXiv:0704.1981; PRL

Future: 2006 run: 10^{12} events → 1 ppm.

FAST:

G_F = 1.166353(9) × 10^{-5} GeV^{-2} (8 ppm)

Barczyk et al., arXiv:0707.3904; PLB

Similar future goal, 1 ppm

QED radiative corrections

1956: one-photon, with m_e — Behrends, Finkelstein, Sirlin

1999: two-photon, m_e=0 — van Ritbergen and Stuart

2008: two-photon, with m_e — Pak, AC

Related work:
Numerical tests of the $O(\alpha^2)$ result (not able to determine the m_e effect):
Chetyrkin, Harlander, Seidensticker, Steinhauser (1999);
Blokland, AC, Ślusarczyk,Tkachov (2004)

2005, Anastasiou, Melnikov, Petriello: $O(\alpha^2)$ electron spectrum

Muon lifetime in Fermi theory, with QED

$$\Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[X_0 + \frac{\alpha}{\pi}X_1 + \left(\frac{\alpha}{\pi}\right)^2 X_2 + \ldots\right]$$

$$X_0 = 1 - 8\rho^2 - 24\rho^4 \ln\rho + 8\rho^6 - \rho^8$$

$$\rho \equiv \frac{m_e}{m_\mu}$$

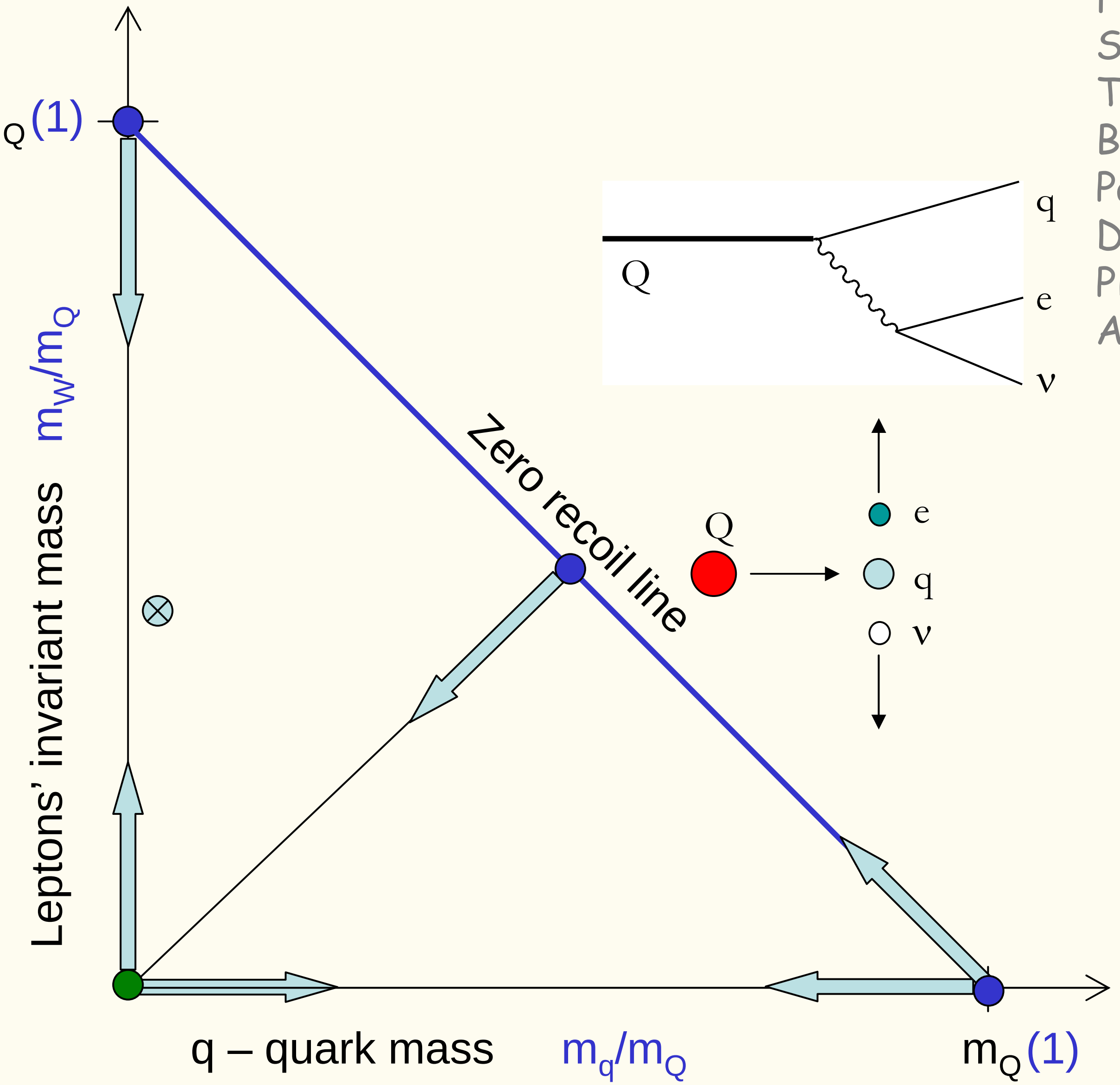
$$X_1 = \frac{25}{8} - \frac{\pi^2}{2} - (34 + 24\ln\rho)\rho^2 + 16\pi^2\rho^3 - \left(\frac{273}{2} - 36\ln\rho + 72\ln^2\rho + 8\pi^2\right)\rho^4 + \ldots$$

$$X_2 = X_2(\rho = 0) - \frac{5}{4}\pi^2\rho + \ldots$$

Note appearance of higher powers of logs and odd powers of ρ

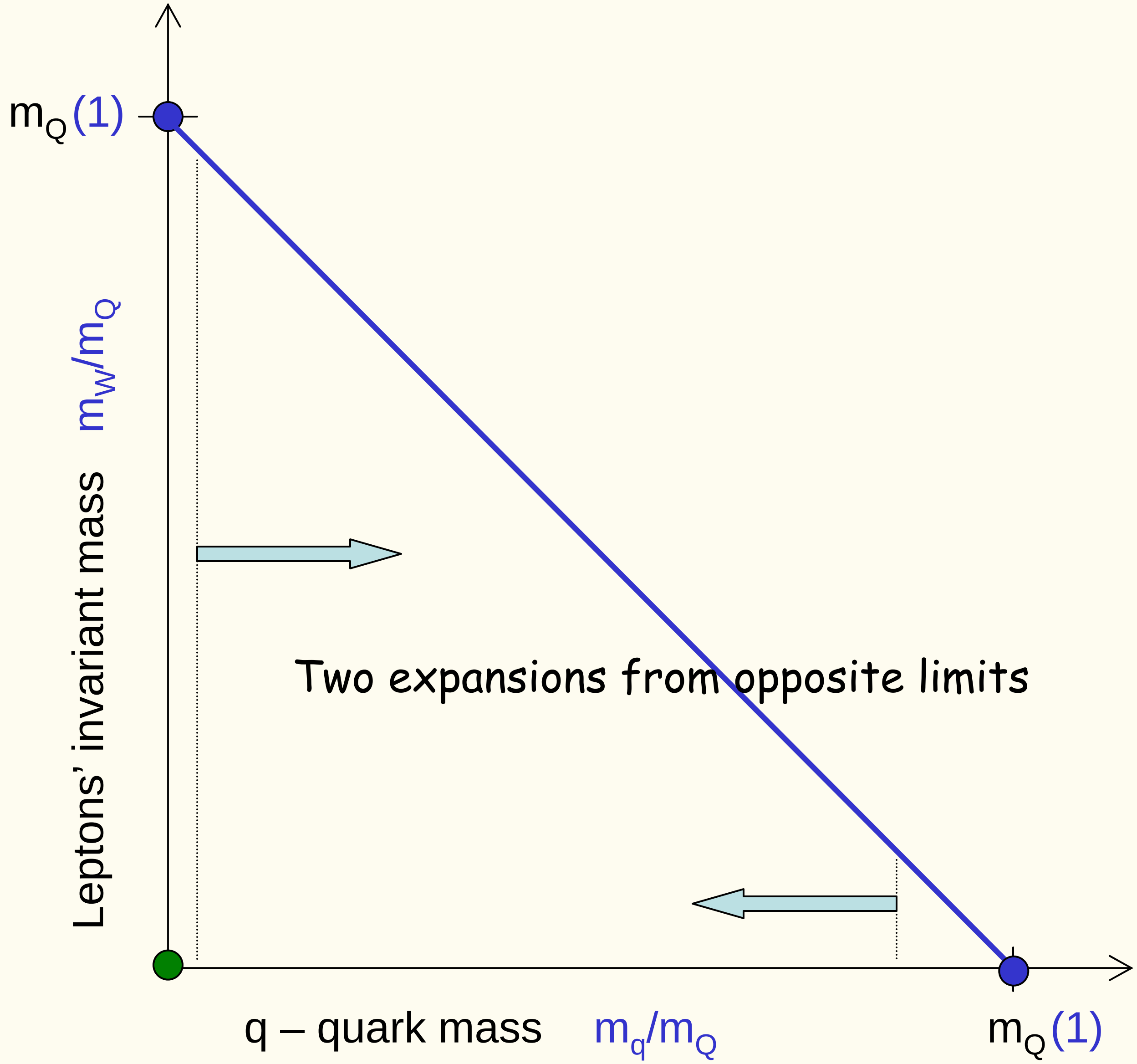
Calculation of X_2 – end of a long road

Since 1996,
expansion techniques developed;
decays expanded in quark mass difference,
for various kinematic configurations;
focus on $d\Gamma/dq^2$



Melnikov,
Tausk,
Franzkowski,
Slusarczyk,
Tkachov,
Blokland,
Pak,
Dowling,
Piclum,
AC

Calculation of X_2: integration over W^* mass (2008)



Example: electron vacuum polarization

$$X_C = -\frac{1009}{288} + \frac{8\zeta_3}{3} + \frac{77\pi^2}{216}$$

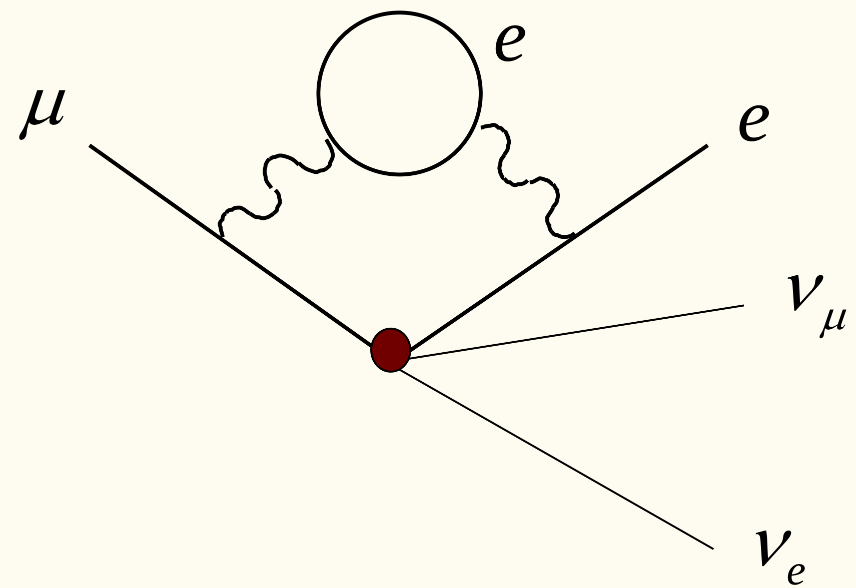
$$-\frac{5}{4}\pi^2\rho$$

$$+\left[\frac{145}{3} + \frac{52}{3}\ln\rho - 8\ln^2\rho + \frac{16\pi^2}{3}\right]\rho^2$$

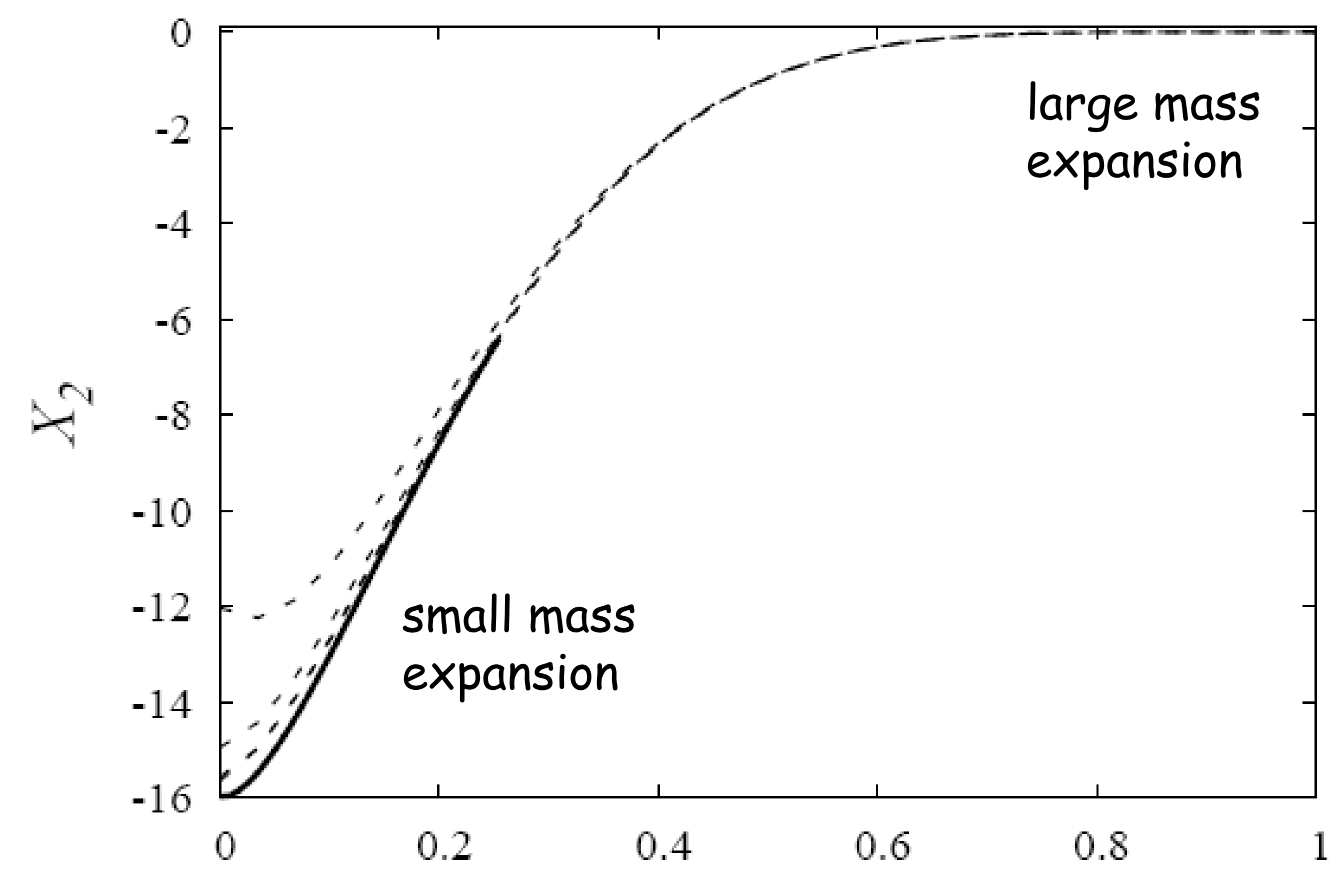
$$+\left[\frac{569}{36} + \frac{64}{3}\ln\rho\right]\pi^2\rho^3 + \ldots$$

Series in powers and logs of the mass ratio, $\rho = m_e / m_\mu$

$$\Delta q(m_e) \simeq -0.43 \cdot 10^{-6}$$



Results



"Electron to muon" mass ratio

From Dowling, Piclum, AC

Note: the plot actually for QCD.
QED given by a subset of QCD results.

Example: electron vacuum polarization

$$X_C = -\frac{1009}{288} + \frac{8\zeta_3}{3} + \frac{77\pi^2}{216}$$

Massless result confirms van Ritbergen and Stuart

$$-\frac{5}{4}\pi^2\rho$$

$$+\left[\frac{145}{3} + \frac{52}{3}\ln\rho - 8\ln^2\rho + \frac{16\pi^2}{3}\right]\rho^2$$

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Example: electron vacuum polarization

$$X_C = -\frac{1009}{288} + \frac{8\zeta_3}{3} + \frac{77\pi^2}{216}$$

$$-\frac{5}{4}\pi^2\rho$$

Linear correction missed in previous studies

$$+\left[\frac{145}{3} + \frac{52}{3}\ln\rho - 8\ln^2\rho + \frac{16\pi^2}{3}\right]\rho^2$$

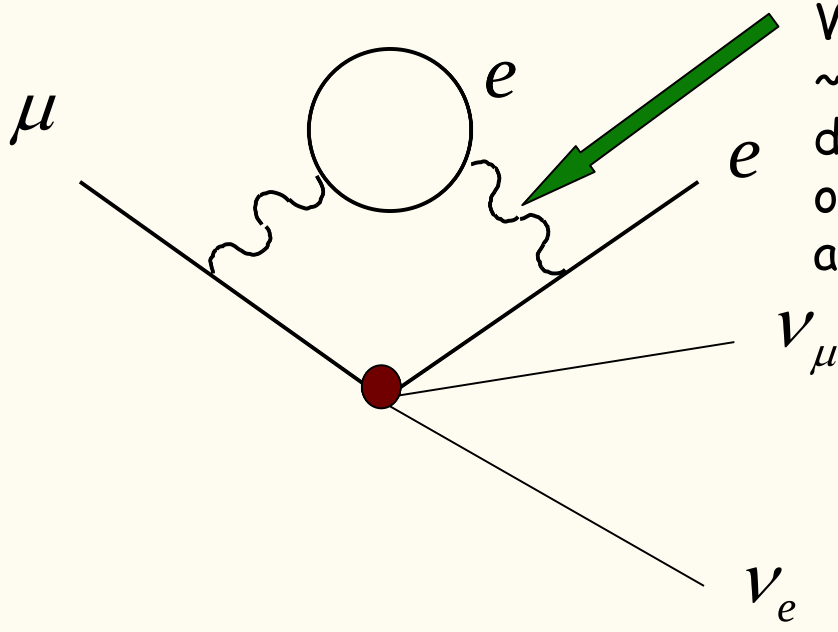
$$+\left[\frac{569}{36} + \frac{64}{3}\ln\rho\right]\pi^2\rho^3 + \ldots$$

Linear electron mass effects at $O(\alpha^2)$

$$\Gamma(\mu \to e\bar{\nu}\nu) = \Gamma_0 \left[X_0 + \frac{\alpha}{\pi} X_1 + \left(\frac{\alpha}{\pi}\right)^2 X_2 + \ldots \right]$$

$$X_2 = X_2(\rho \to 0) - \frac{5}{4}\pi^2 \rho + \ldots$$

The origin of the linear correction is in the muon mass definition



When this loop momentum is ~m_e it can be ignored in the daughter electron and modifies only the muon leg; this can be absorbed in the muon mass definition

Future achievable accuracy

$$\Gamma = \Gamma_0 \left[1 + \frac{\alpha}{\pi} x_1 + \left(\frac{\alpha}{\pi}\right)^2 \left(x_2 - \frac{x_1}{3} \ln \rho^2 \right) \right],$$

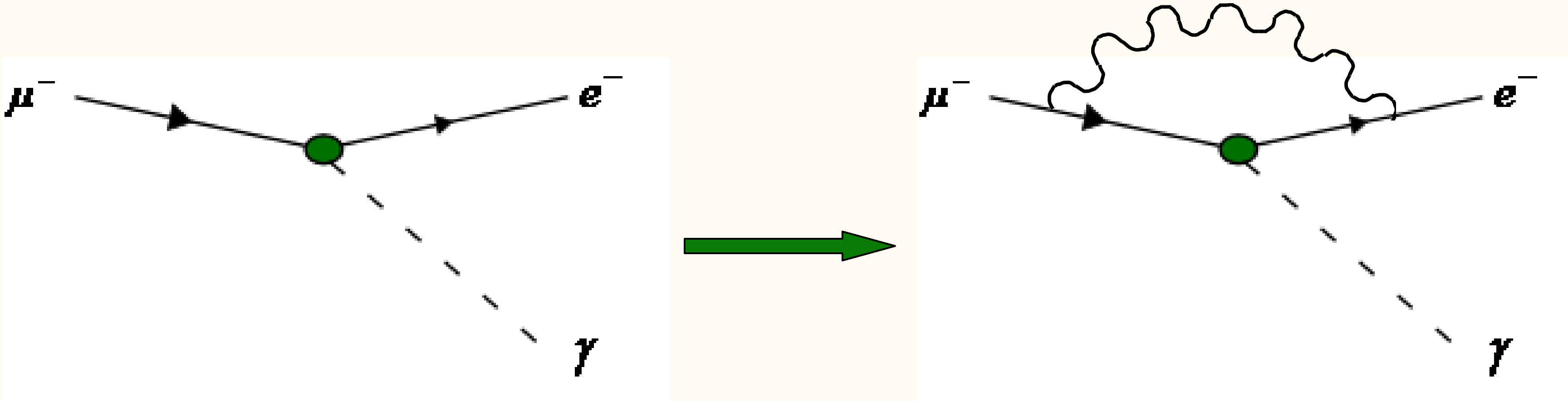
$$x_1 = \frac{25}{8} - \frac{\pi^2}{2} \simeq -1.81,$$

$$x_2 = \frac{156815}{5184} - \frac{1036}{27}\zeta_2 - \frac{895}{36}\zeta_3 + \frac{67}{8}\zeta_4 + 53\zeta_2 \ln 2 \simeq 6.74.$$

If x_3 is ~40, it gives rise to Δq~0.5 ppm.

Other sources of uncertainty:
m_μ: 0.2 ppm
hadronic loops: 0.02 ppm

QED suppression of the decay $\mu\rightarrow e\gamma$



$$\sigma_{\alpha\beta} q^{\beta} \left(E - M\gamma_5 \right) A^{\alpha}$$

$$\times \left(1 - \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu} \right)$$

AC and Jankowski,
PRD 65, 113004 (2002)

This is the largest known QED correction to a decay rate;
15 percent for $\Lambda \sim 250$ GeV. In general, $\sim 2\ln(\Lambda/m_\mu)$ percent.

For comparison, correction for the normal muon decay is 0.4 percent.

Summary

Muon lifetime prediction is now accurate to below ppm.

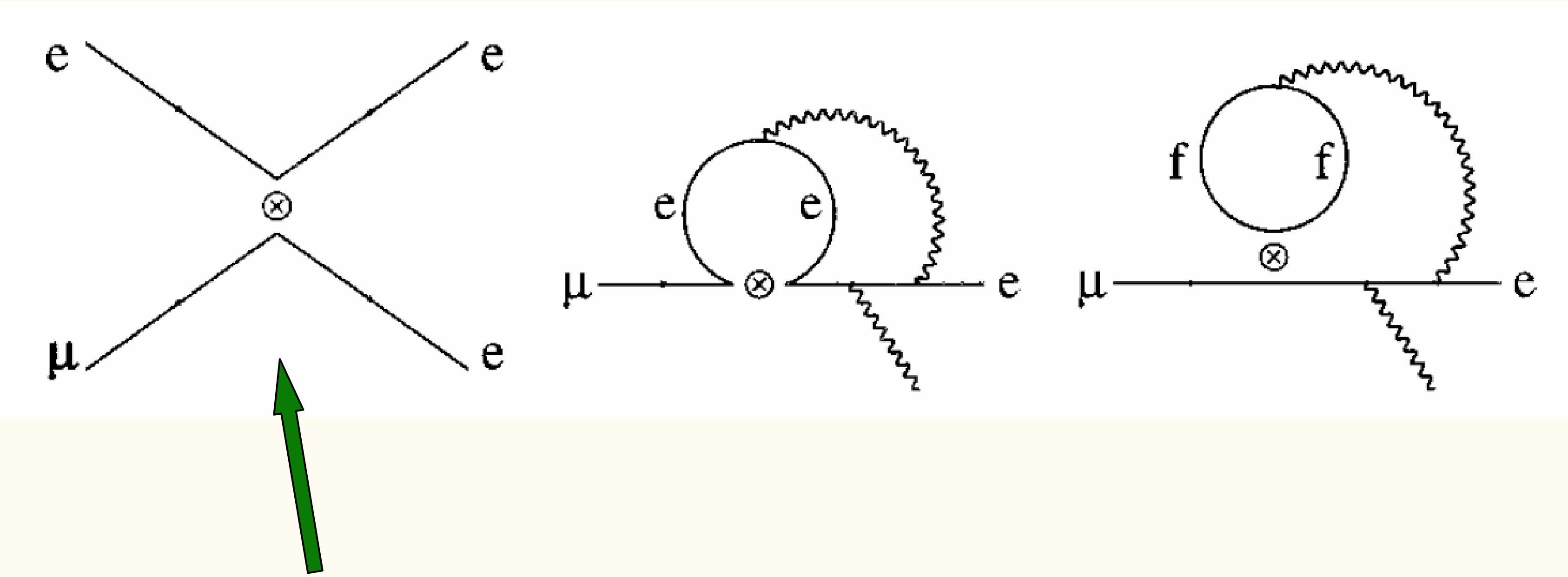
Linear electron-mass effect at $O(\alpha^2)$ due to the on-shell muon mass definition.

Further significant improvements require better understanding of the perturbative coefficients.

Extra slides

The rate suppression $\times\left(1-\frac{8\alpha}{\pi}\ln\frac{\Lambda}{m_\mu}\right)$ **is universal (independent of the mechanism of LFV)**

This is because the non-dipole operators (four-fermion) which would have a different scaling, contribute little:



Wilson coefficient of this operator
is constrained by direct searches (SINDRUM),

$$\frac{\Gamma(\mu\to eee)}{\Gamma(\mu\to e\nu\nu)}<10^{-12}$$

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Starting date:	10-Nov-2008 14:35 (Asia/Jerusalem)
Duration:	15'