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**On behalf of the NA48/2 collaboration:**

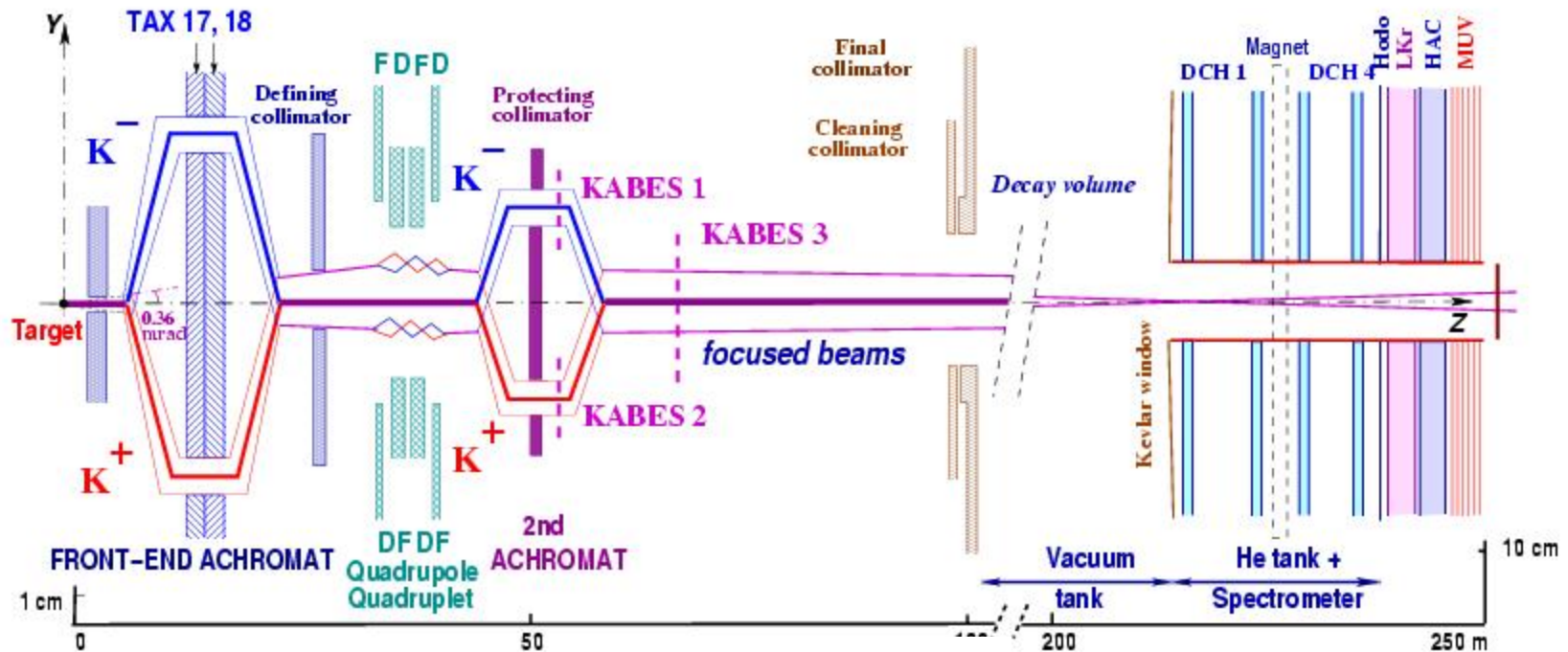
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze,  
Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

## outline

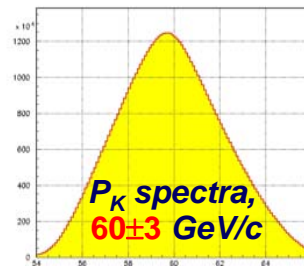
- Introduction to NA48/2
- QCD tests from study of Kaon decays
- Ke4 decays ( $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu$ ) :  
Form Factors and  $\pi\pi$  scattering lengths
- K3 $\pi$  decays ( $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ ) : the "cusp" effect  
Dalitz plot parameters and  $\pi\pi$  scattering lengths
- Summary

# The NA48/2 experiment at the CERN-SPS : primarily designed for CP violating charge asymmetries studies in $K^3\pi$ decays

2003 run: ~ 50 days + 2004 run: ~ 60 days



Simultaneous  $K^+$  and  $K^-$  beams:  
large charge symmetrization of  
experimental conditions



Beams coincide within ~1mm  
all along the 114m decay volume  
flux ratio  $K^+/K^-$  ~1.8

# The NA48/2 experiment: detector and performances

## Magnetic spectrometer :

4 high-resolution DCH's + dipole magnet

$$\Delta p/p = (1.0 \oplus 0.044 p)\% \quad (p \text{ in } \text{GeV}/c)$$

Very good resolution for charged invariant masses:  $\sigma (M3\pi^\pm) = 1.7 \text{ MeV}/c^2$

## Hodoscope for charged fast trigger

$$\sigma\tau = 150 \text{ ps}$$

## LKr electromagnetic calorimeter :

quasi-homogenous and high granularity

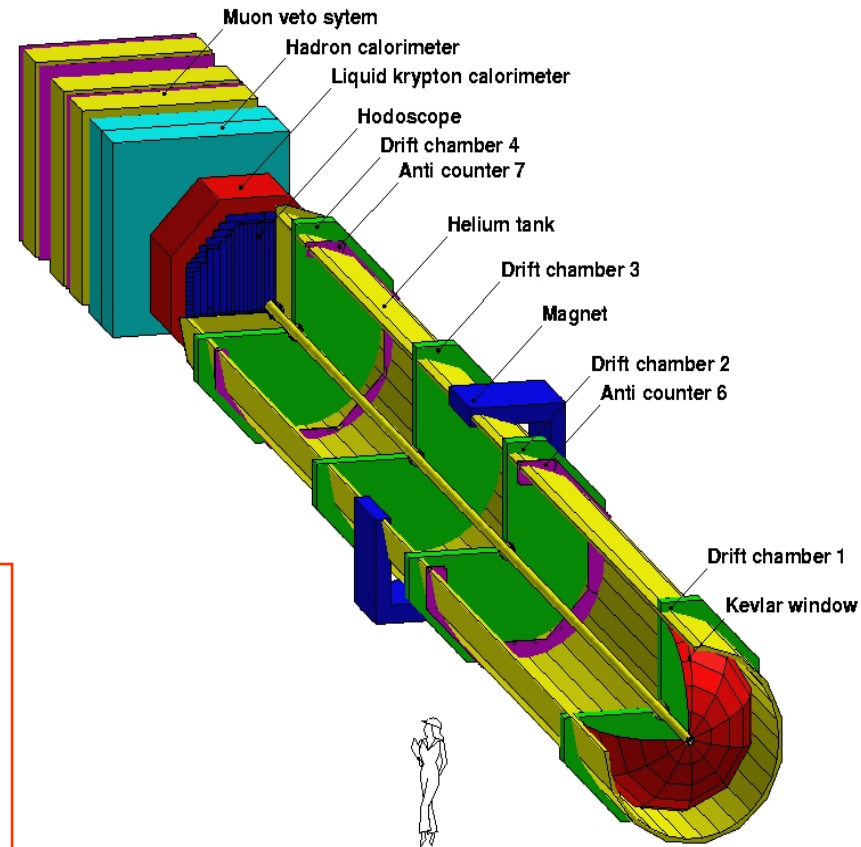
$$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\% \quad (E \text{ in } \text{GeV})$$

$$\sigma_x = \sigma_y \sim 1.5 \text{ mm for } E=10 \text{ GeV}$$

Very good resolution for neutrals ( $\pi^0 \rightarrow \gamma\gamma$ )

$$\sigma (M\pi\pi^0\pi^0) = 1.4 \text{ MeV}/c^2$$

$E/p$  ratio used for  $e/\pi$  discrimination



# Kaon decays : what can be learned on QCD @ Low Energy ?

Hadronic decay modes into 3 pions:

- large Br's :  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$  (1.7 %) and  $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$  (5.6 %),

**60 Millions events now analyzed (PRL B633 (2006) partial sample)**

- three pions  $\rightarrow \pi^0 \pi^0$  system + nearby hadron
- accessible  $M_{\pi\pi}$  range from threshold to  $(M_K - M_\pi)$

Semileptonic decay mode Ke4:

- small Br's :  $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$  ( $4.1 \cdot 10^{-5}$ ),

**1.1 Million events now analyzed (EPJC 54 (2008) partial sample)**

- only two  $\pi^+ \pi^-$  pions, very clean environment
- accessible  $M_{\pi\pi}$  range from threshold to  $(M_K - M_e) = M_K$

Two different but complementary approaches to  $\pi\pi$  scattering near threshold  $\rightarrow$  extract s-wave scattering lengths ( $a_0, a_2$ ) for Isospin  $I = 0$  and  $I = 2$

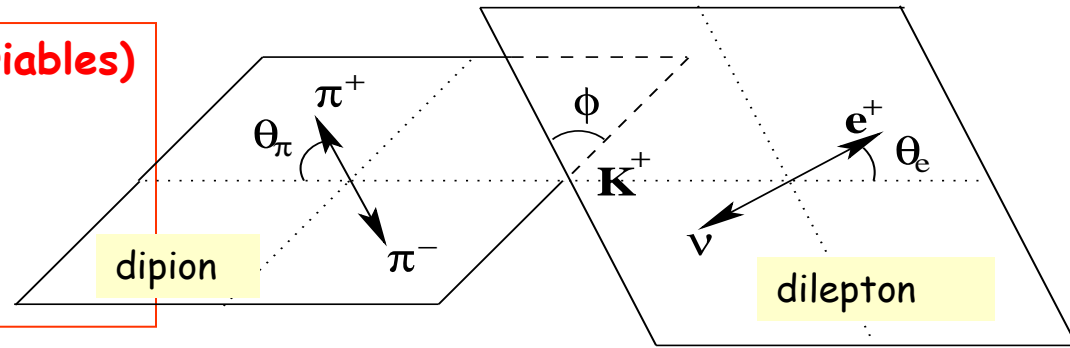
# Ke4 decays : formalism

## Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

$$S_\pi (M^2_{\pi\pi}), S_e (M^2_{e\nu}),$$

$$\cos\theta_\pi, \cos\theta_e \text{ and } \phi.$$



## Partial Wave expansion of the amplitude

(Pais-Treiman 1968) into s and p waves

**F, G = 2 Axial Form Factors**

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi$$

$$G = G_p e^{i\delta_g}$$

**H = 1 Vector Form Factor**

$$H = H_p e^{i\delta_h}$$

Map the five-dimensional space of the Ca.Ma. variables with 4 Form factors and one phase shift, assuming identical phases for the p-wave Form Factors  $F_p, G_p, H_p$ :

The fit parameters are :

$$F_s, F_p, G_p, H_p \text{ and } \delta = \delta_s - \delta_p$$

# Ke4 decays: event selection and background rejection

## Signal ( $\pi^+\pi^-\ e^\pm\nu$ ) topology :

- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron (LKr info  $E/p \sim 1$ ),
- some missing energy and  $p_T$  ( $\nu$ )
- reconstruct PK (missing  $\nu$  hypothesis)

## Background main sources :

- $K^\pm \rightarrow \pi^+\pi^-\pi^\pm$  (dominant)
  - ↳  $e\nu$  or misidentified as  $e$
- $K^\pm \rightarrow \pi^0(\pi^0)\pi^\pm$ 
  - ↳  $(e+e-\gamma) + 1e$  misidentified as  $\pi$  and  $\gamma$  ( $s$ ) undetected

## Control sample from data ( $\Delta S = \Delta Q$ valid)

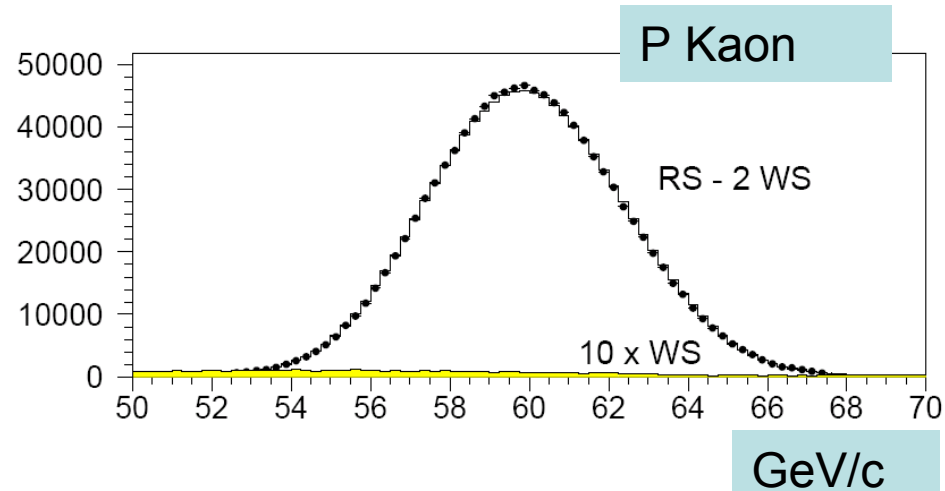
### $K^\pm \rightarrow \pi^\pm\pi^\pm e^\mp\nu$ "Wrong Sign" events

- total charge ( $\pm 1$ ) as Right Sign events
- electron charge opposite to total charge

Rate (RS/WS) events:

**2** if coming from  $K3\pi$

**1** if coming from  $K2\pi(\pi^0)$



Total background level can be kept at  $\sim 0.5\%$  relative level estimated from WS events rate and checked from MC simulation

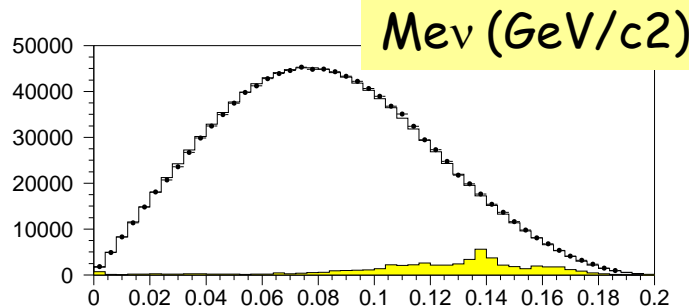
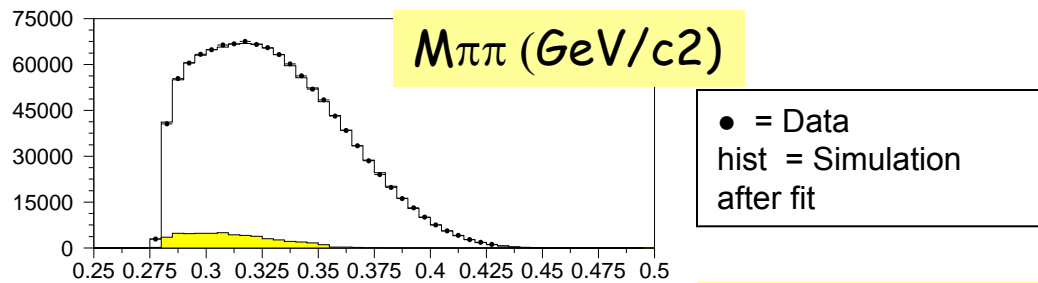


# Ke4 decays: fitting procedure and Form Factors

Total statistics 1.1 Millions Ke4 decays distributed over a grid of **15000 iso populated** boxes =  $10(M_{\pi\pi}) \times 5(\text{MeV}) \times 5(\cos\theta_{\pi}) \times 5(\cos\theta_e) \times 12(\Phi)$

Fits are repeated in each  $M_{\pi\pi}$  bin for  $K^+$  and  $K^-$  separately without any assumption of Form Factor dependence with  $q^2$  ( $S_{\pi}/4m_{\pi}^2 - 1$ ) and  $S_e/4m_{\pi}^2$  using a large (~30 Millions) simulated sample to account for acceptance and experimental conditions.

**Taylor expansion** of Form Factors used to characterize bin to bin variation with  $q^2$ ,  $q^4$  and  $S_e$  (valid in the isospin symmetry limit)



**Preliminary**

	value	stat error	syst error(2003)
$f_s^{\prime\prime}/f_s$	0.158	$\pm 0.007$	$\pm 0.006$
$f_s^{\prime}/f_s$	-0.078	$\pm 0.007$	$\pm 0.007$
$f_e^{\prime}/f_s$	0.067	$\pm 0.006$	$\pm 0.009$
$f_p/f_s$	-0.049	$\pm 0.003$	$\pm 0.004$
constant			
$g_p/f_s$	0.869	$\pm 0.010$	$\pm 0.012$
$g_p^{\prime}/f_s$	0.087	$\pm 0.017$	$\pm 0.015$
$h_p/f_s$	-0.402	$\pm 0.014$	$\pm 0.008$
constant			



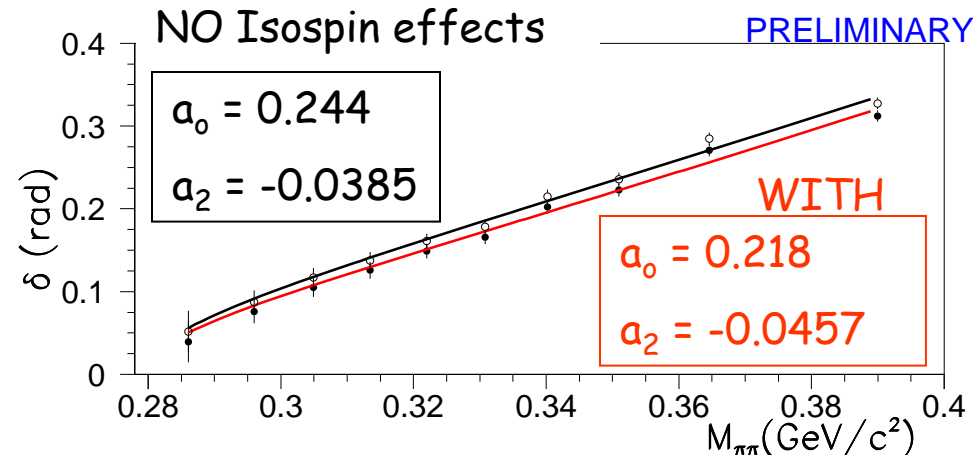
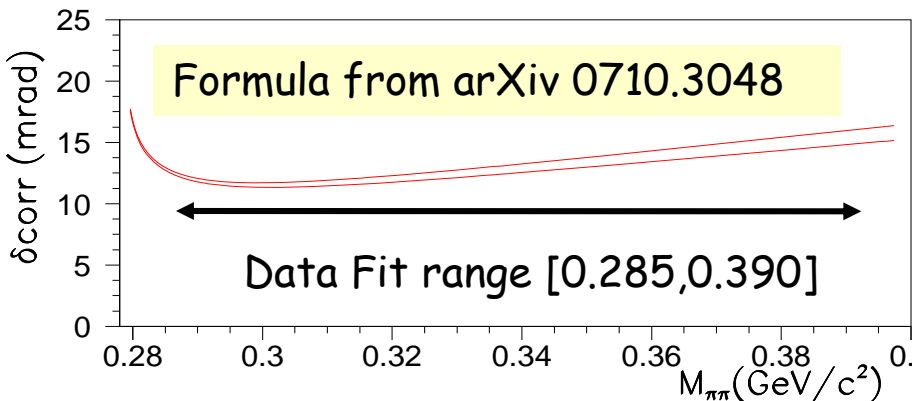
# Ke4 decays: from phase shifts to scattering lengths ( $a_0, a_2$ )

$\pi\pi$  phases at threshold can be predicted from data above 0.8 GeV using **Roy equations** (unitarity, analyticity and crossing symmetries) and 2 subtraction constants  **$a_0$  and  $a_2$**

**Numerical solutions** have been developed (ACGL Phys.Rep.353(2001), DFGS EPJ C24(2002)) valid in the **Isospin symmetry limit**, broken in the experimental world. (**Universal Band**)

**Radiative effects:** included in the simulation, **Mass effects:** recently computed as a correction to the measurements, even larger than current experimental precision! (CGR hep-ph/0811.0775, DK in progress)

Induces a large  **$2 \sigma_{\text{exp}}$  change** on ( $a_0, a_2$ ) values from a 2p fit:  
 $\sigma(a_0): \pm 0.013$  (stat)  $\pm 0.007$  (sys)  
 $\sigma(a_2): \pm 0.0084$  (stat)  $\pm 0.0041$  (sys)



# Ke4 decays: comparison with theoretical predictions

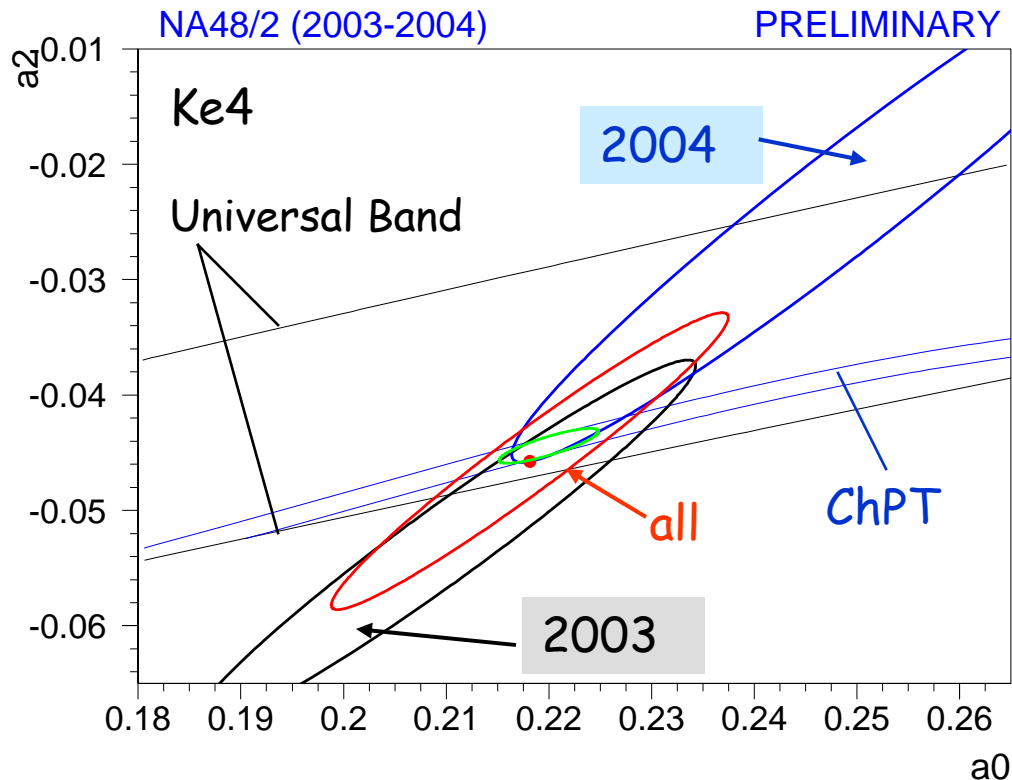
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001)):

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0008$$



$a_0$ ChPT 1p fit	$0.220 \pm 0.005$ stat $\pm 0.002$ syst* $\pm 0.006$ theo**
$a_0$ free	$0.218 \pm 0.013$ stat $\pm 0.007$ syst* $\pm 0.017$ theo**
$a_2$ free 2p fit	$-0.0457 \pm 0.0084$ stat $\pm 0.0041$ syst* $\pm 0.0030$ theo**



\*systematics from 2003 data

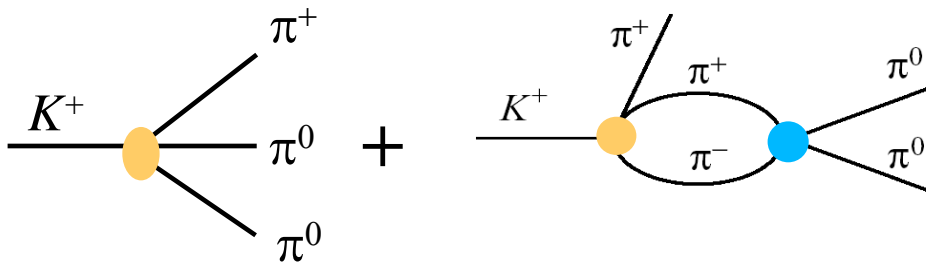
\*\*Theory error evaluated from control of the isospin corrections, inputs to Roy equation numerical solutions ([CGR arXiv:0811.0775](https://arxiv.org/abs/0811.0775))

# Cusp effect : first observation and interpretation (Cabibbo PRL93(2004) )

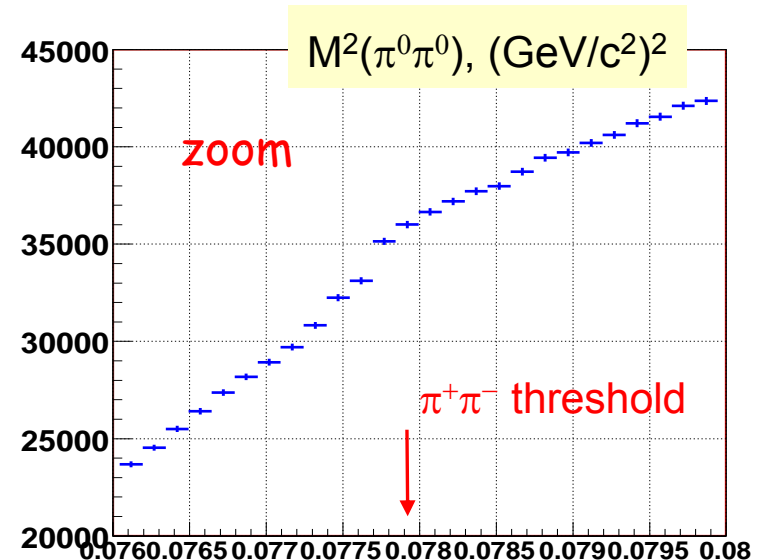
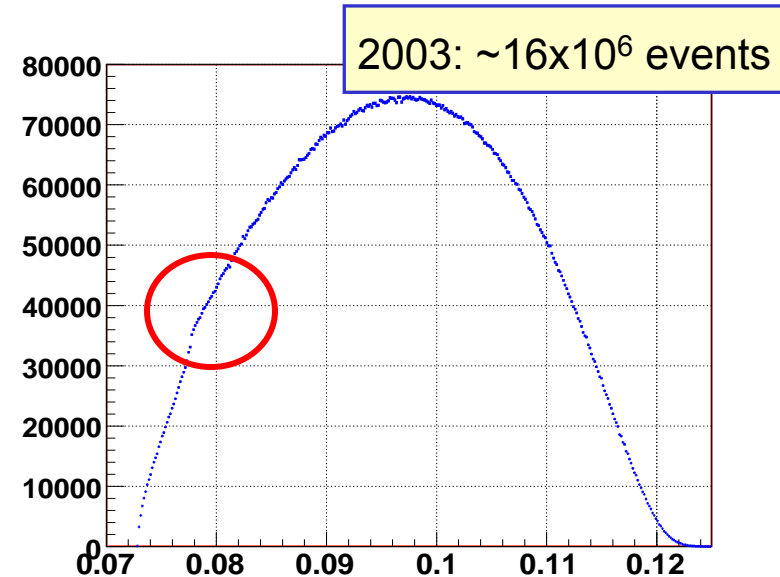
In  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$  decay, the matrix element is usually described as a polynomial expansion using the **Dalitz Plot variables  $u$  and  $v$**

First observation of a cusp structure was made with 16 M events collected in 2003 thanks to the very good mass resolution.

The structure at  $\pi^+ \pi^-$  threshold was interpreted as due to the  $\pi\pi$  rescattering in the  $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$  final state



increased statistics with 44 M more data from 2004



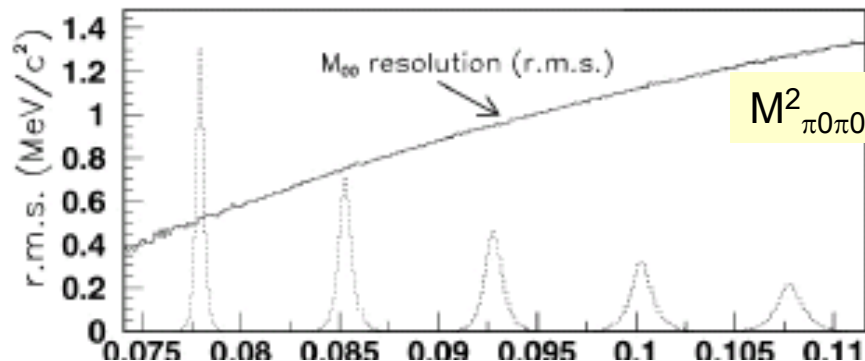
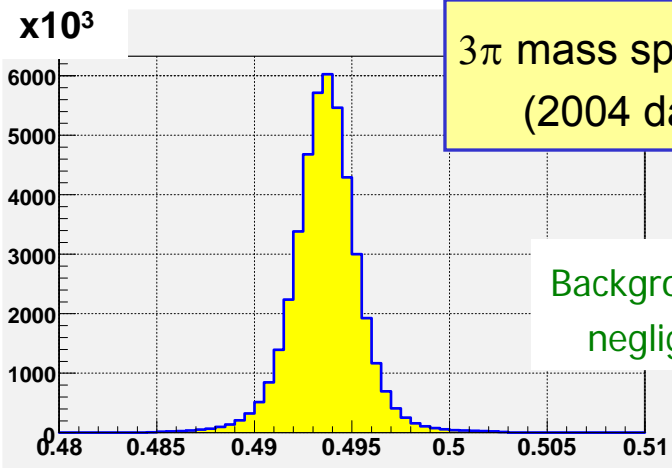
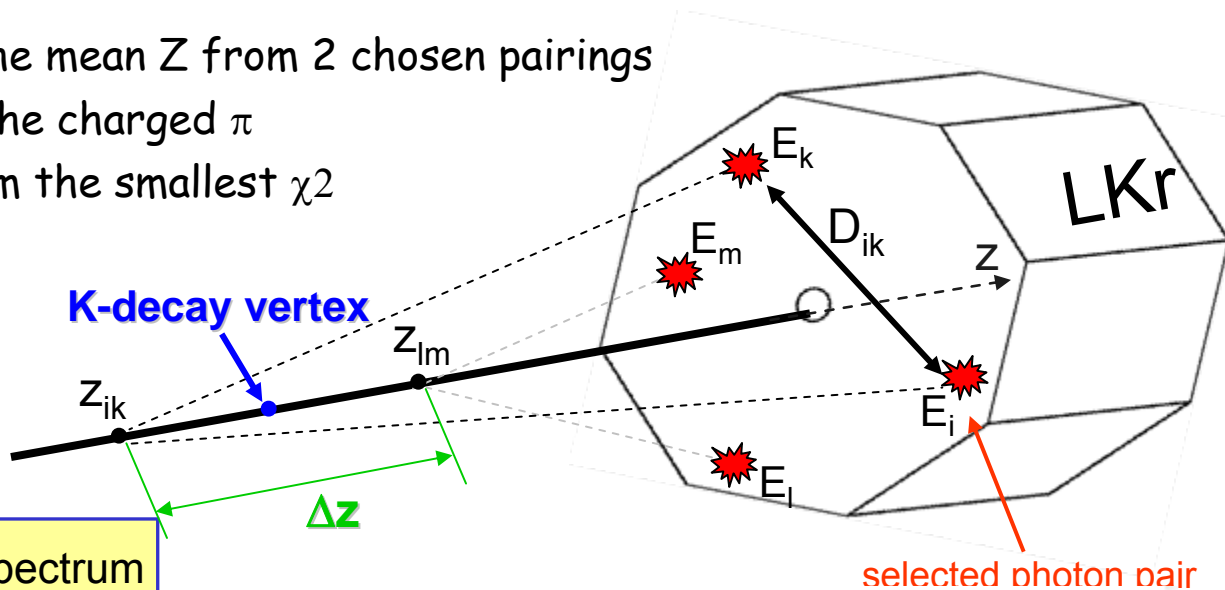
# CUSP effect: $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ event selection

- Each photon pair  $(i,k)$  defines a decay vertex under the assumption of  $\pi^0 \rightarrow \gamma\gamma$  decay

$$Z_{ik}^2 \equiv E_i E_k D_{ik}^2 / m_{\pi^0}^2$$

- Neutral vertex defined as the mean Z from 2 chosen pairings
- Reconstruct K mass adding the charged  $\pi$
- Chose the best 2  $\gamma$  pairs from the smallest  $\chi^2$

$$\chi^2 = \left( \frac{\Delta Z}{\sigma_Z} \right)^2 + \left( \frac{\Delta m_K}{\sigma_{m_K}} \right)^2$$



# CUSP effect: fitting procedure : two approaches

Cabibbo-Isodori [JHEP 0503\(2005\)](#)

Without re-scattering,  $M_0$  parameterization (as PDG)

$$M_0 = A_0(1 + g_0 u/2 + h_0' u^2/2 + k_0' v^2/2)$$

- First order term  $M_1$  describes loop diagrams

$$M_1 = -\frac{2}{3} (a_0 - a_2) m_{\pi^+} M_+ \sqrt{1 - M_{00}^2 / 2m_{\pi^+}^2}$$

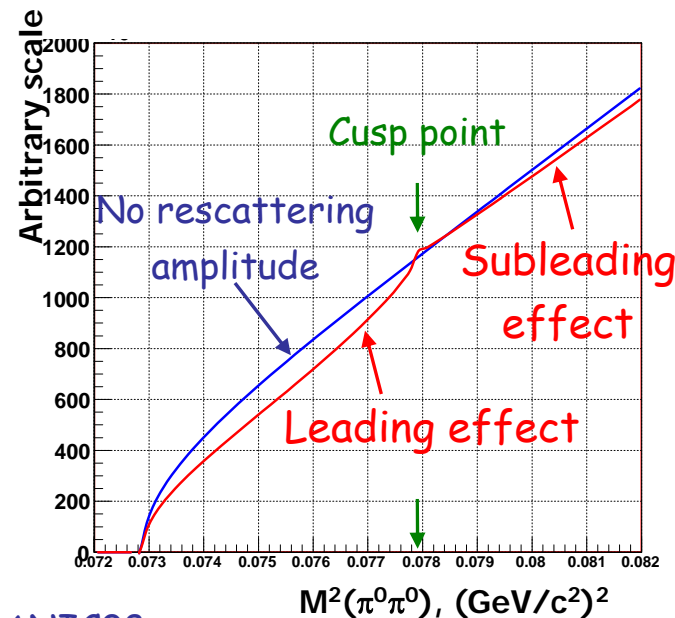
above threshold  $|M|^2 = |M_0|^2 + |M_1|^2$

below threshold  $|M|^2 = |M_0|^2 + |M_1|^2 + 2 M_0 M_1$

- Second order effects included
- Radiative corrections not (yet) included

Bern-Bonn Effective field theory  
[CGKR PLB638 \(2006\)](#), [BFGKR arXiv:0807.0515](#)

- **electromagnetic effects** included in the amplitudes
- **two-loop formulation** different from CI introduces different correlations between parameters



# Cusp : experimental fitting procedure

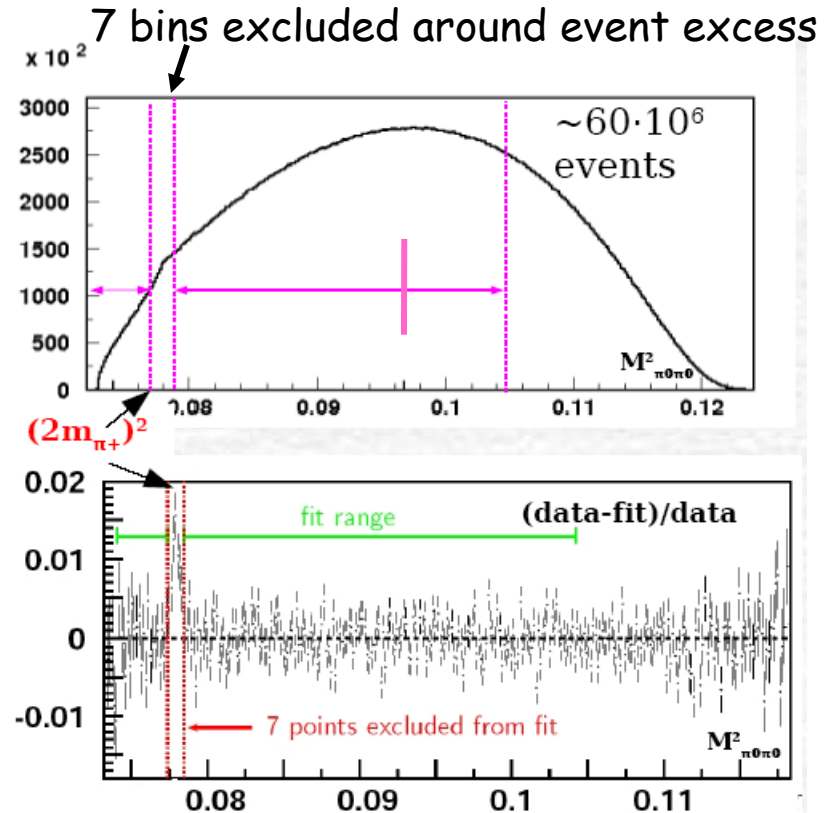
Fit the  $M_{\pi^0\pi^0}^2$  projection using the detector response matrix  $R_{ij}$  obtained from a Monte-Carlo simulation and 4 physics parameters ( $g, h, a_0, a_2$ ) for both approaches

the constant  $k'_0$  (v-dependent term) is fixed to the value recently measured by a 2d fit of the  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  Dalitz plot  
 $k'_0 = 0.0095 \pm 0.0002 \text{ stat} \pm 0.0005 \text{ syst}$

The  $M_+$  amplitude for  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

$$M_+ = A_+ (1 + g_+ u/2 + h'_+ u^2/2 + k'_+ v^2/2)$$

- fixed from NA48/2 measurement PLB649(2007) for CI
- fitted in a simultaneous fit of both Dalitz plots for CGKR



Event excess around the  $M(\pi^+\pi^-)$  threshold can be explained as Pionium decay to  $\pi^0\pi^0$  (Silagadze, JETP Lett.60 (1994))  
 $R = \Gamma(K^\pm \rightarrow \pi^\pm A_{2\pi}) / \Gamma(K^\pm \rightarrow \pi^\pm \pi^+\pi^-)$   
 $= (1.8 \pm 0.3) \times 10^{-5}$   
 while the prediction is  $R = 0.8 \times 10^{-5}$

# Cusp: scattering lengths results

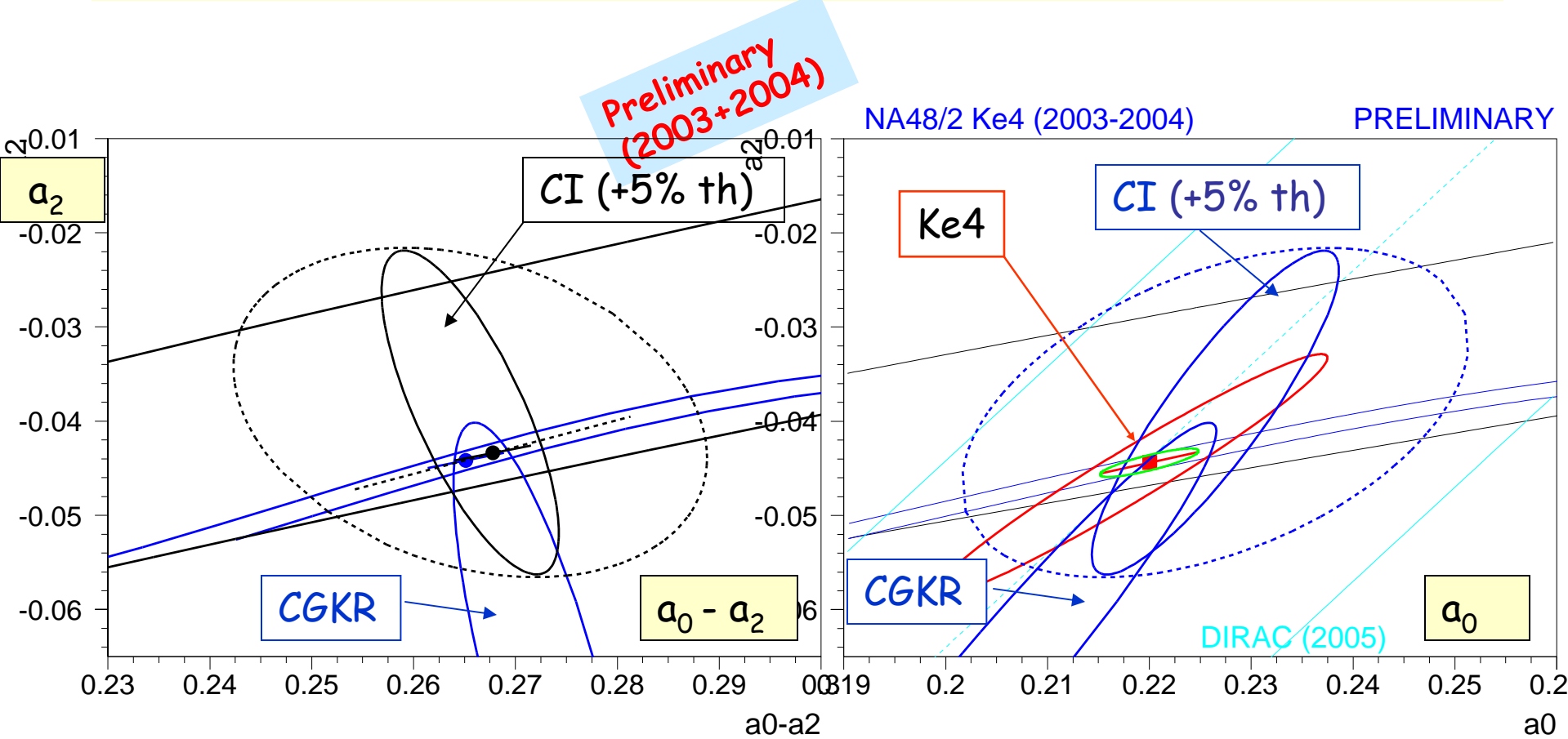
Preliminary  
(2003+2004)

Using ChPT constraint	Note : <b>ext</b> is mainly due to $R=A_+/A_0=1.975\pm 0.015$ th(CI) ~5% probably pessimistic (under evaluation)
CI model	$a_0 - a_2 = 0.268 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{th}}$
CGKR model	$a_0 - a_2 = 0.266 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}}$

$a_2$ free	Note : correlations between $a_2$ and other parameters are larger in CGKR model
CI model	$a_0 - a_2 = 0.266 \pm 0.005_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}} \pm 0.013_{\text{th}}$ $a_2 = -0.039 \pm 0.009_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.002_{\text{ext}}$
CGKR model	$a_0 - a_2 = 0.273 \pm 0.005_{\text{stat}} \pm 0.002_{\text{syst}} \pm 0.001_{\text{ext}}$ $a_2 = -0.065 \pm 0.015_{\text{stat}} \pm 0.010_{\text{syst}} \pm 0.002_{\text{ext}}$



# Cusp and Ke4 : scattering lengths results



Two statistically independent measurements by NA48/2

Large overlap in the  $(a_2, a_0)$  plane

Impressive agreement with ChPT predictions (green ellipse)

# Summary and Comparison with other experimental measurements

**Ke4** : apply **isospin corrections** to published phase points of all experiments and perform  $a_0$  **ChPT fit**

Note : E865 number dominated by highest energy data point, otherwise compatible

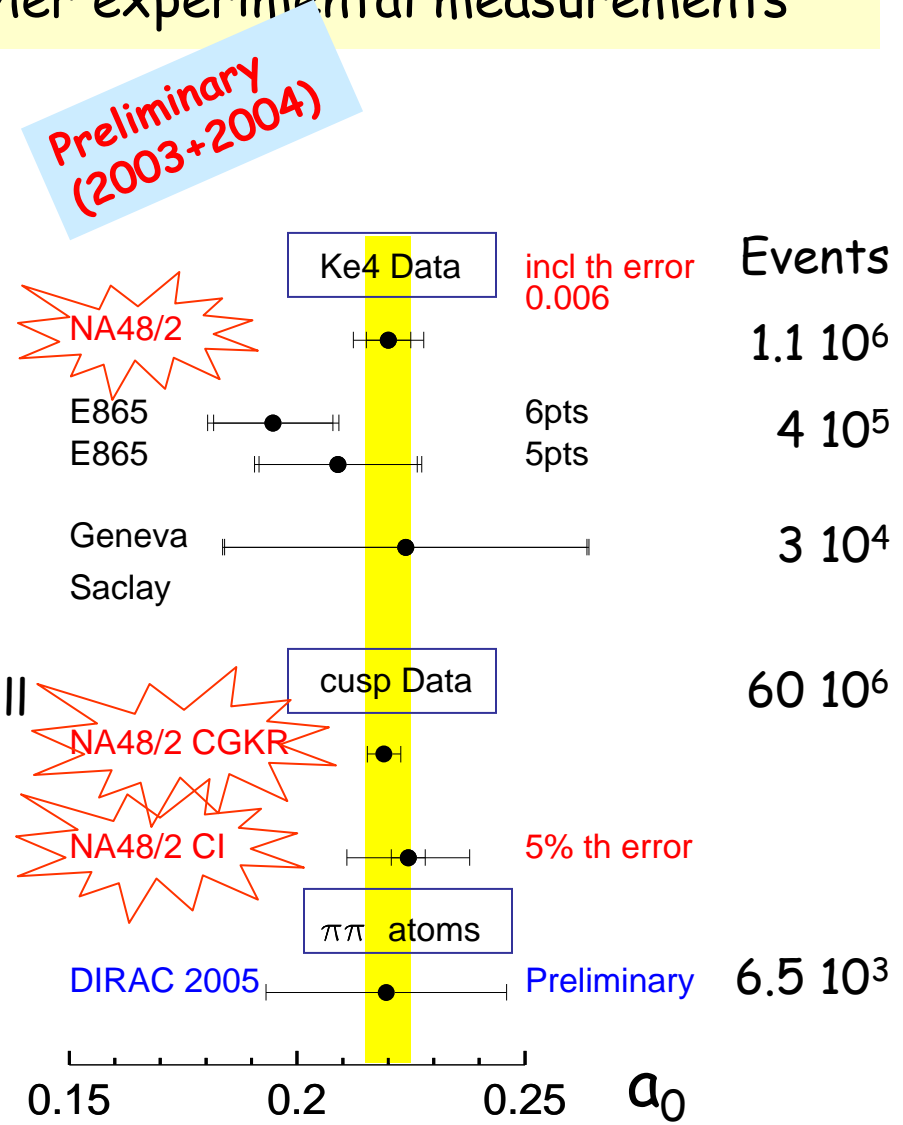
**Cusp** :  $(a_0 - a_2)$  ChPT fit with 2 models

**$\pi\pi$  atoms DIRAC**:  $|a_0 - a_2|$  errors from PLB619 (2005), use ChPT constraint (still being revisited + more Data analyzed)

Yellow band is ChPT prediction

$$a_0 = 0.220 \pm 0.005$$

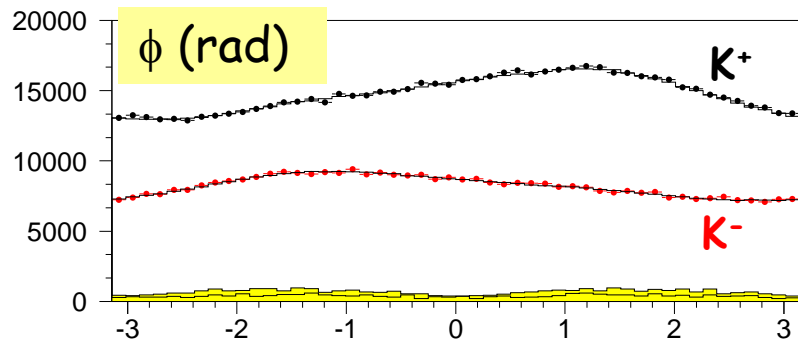
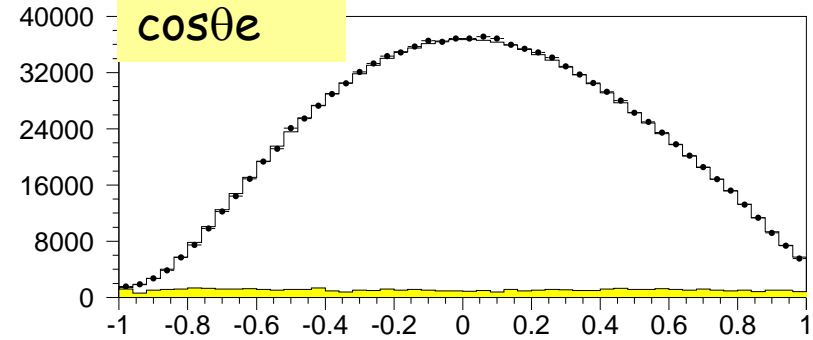
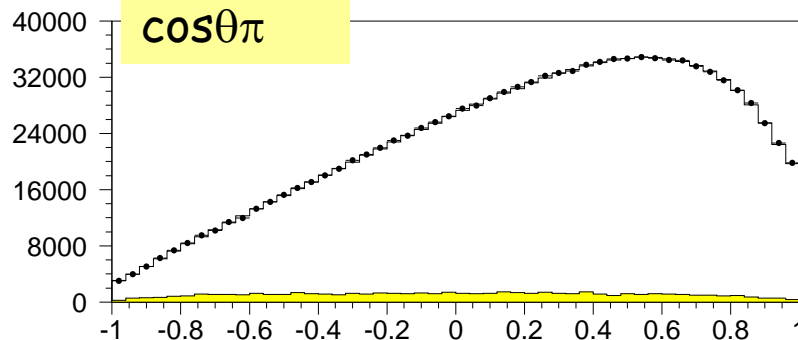
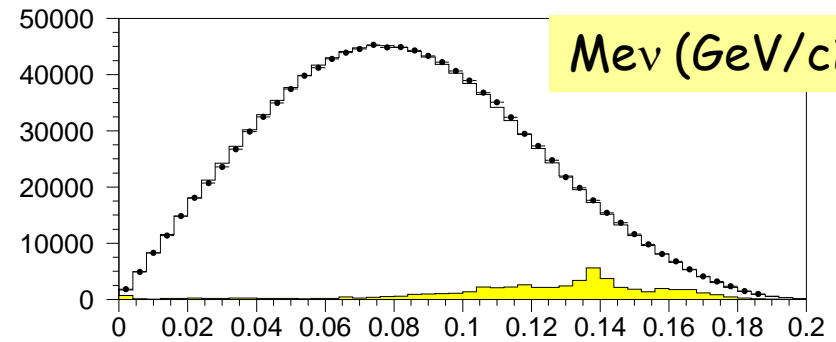
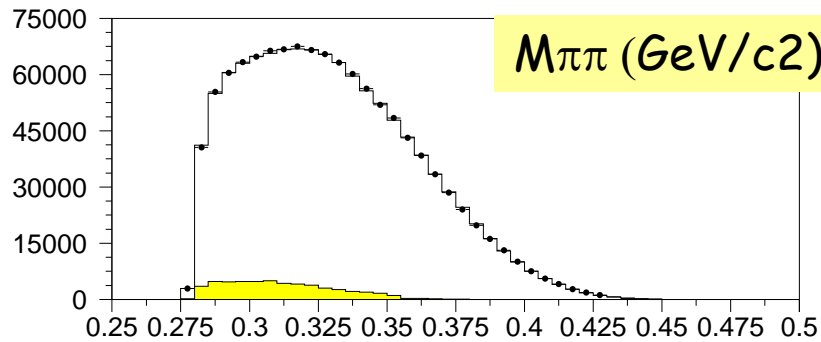
NA48/2 experimental precision now at the same level !



Final publications coming soon, fruitful collaboration with theory groups

# spares

# Ke4 decays : Data/MC comparison after fit

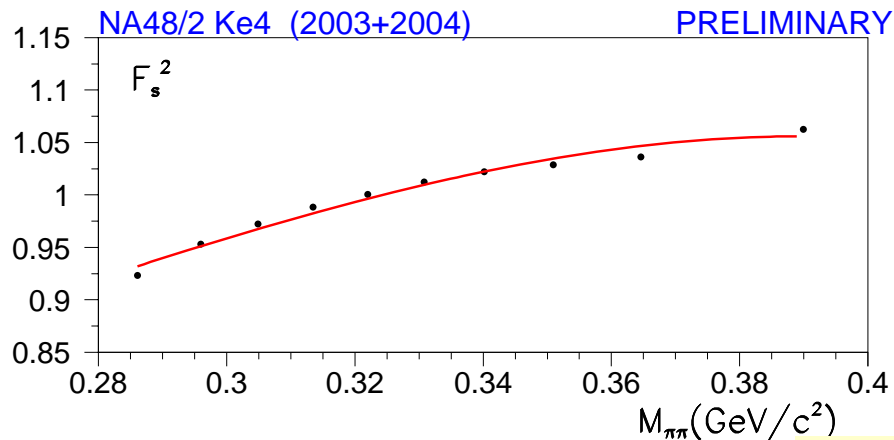


	= Data
	= Simulation after fit
	= WS Background (x 10 to be visible)

CP symmetry :  $(K^+)$   $\phi$  distribution is opposite of  $(K^-)$   $\phi$  distribution

# Ke4 decays : getting F,G,H form factors and phase shift

- **Ten independent fits**, one in each  $M_{\pi\pi}$  bin, assuming  $\sim$ constant form factors over each box. This allows a **model independent** analysis.
- Without the overall normalization (Branching fraction), one can quote **relative form factors** and their **variations with  $q^2, q^4$**  ( $q^2 = (S_{\pi}/4m_{\pi}^2 - 1)$  and  $S_{\pi}/4m_{\pi}^2$ )
- $F_s^2$  is obtained from the relative bin to bin normalization Data/MC after fit
- if projected along Mev, a residual variation is observed.
- a 2-dimension fit of the normalization is performed to get the 3 slopes



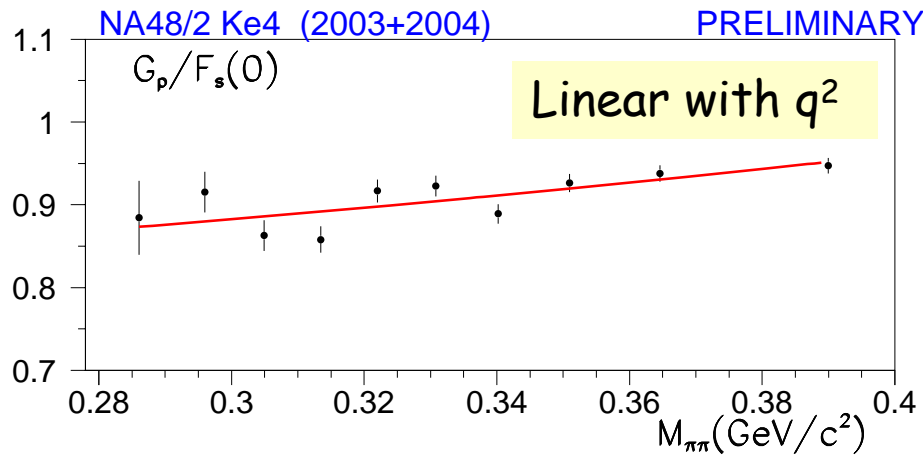
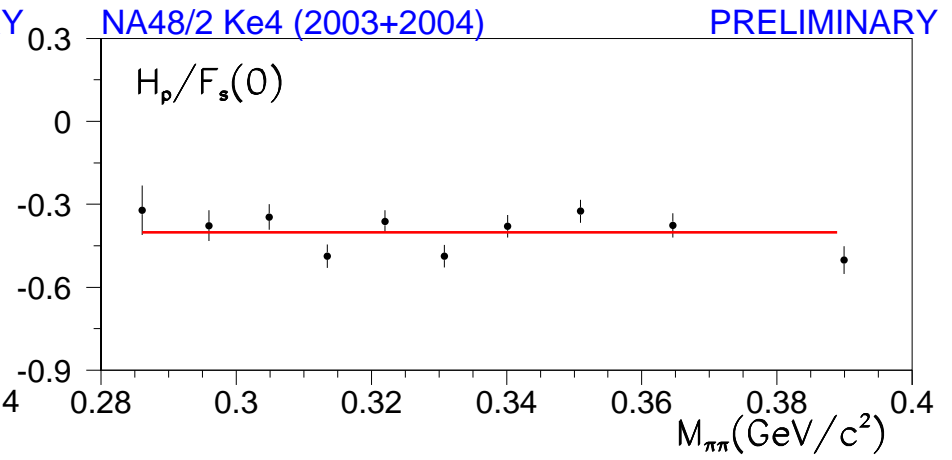
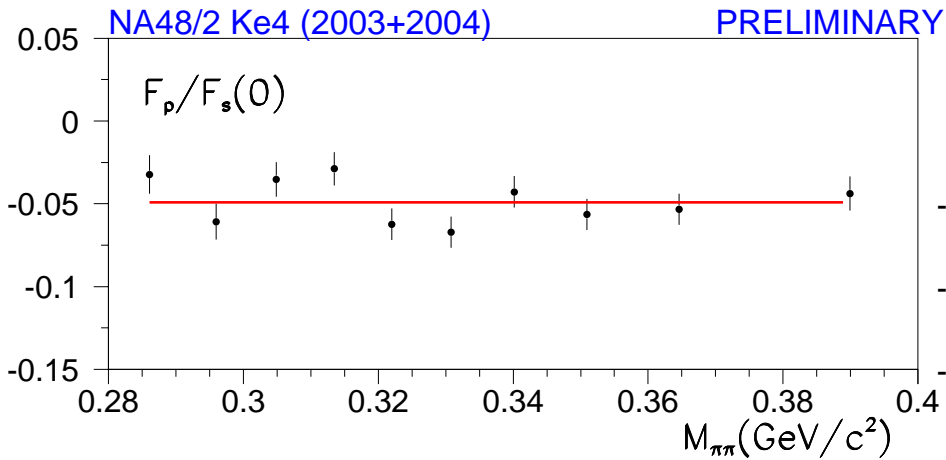
The 3 slopes are correlated

$$F_s^2 \propto (1 + f's q^2 + f''s q^4 + f'e S_{\pi}/4m_{\pi}^2)^2$$

	$f''s$	$f'e$
$f's$	-0.95	0.08
$f''s$		0.02

Other parameterizations could be easily tried if the Taylor expansion does not apply..

# Getting $F_p$ , $G_p$ , $H_p$

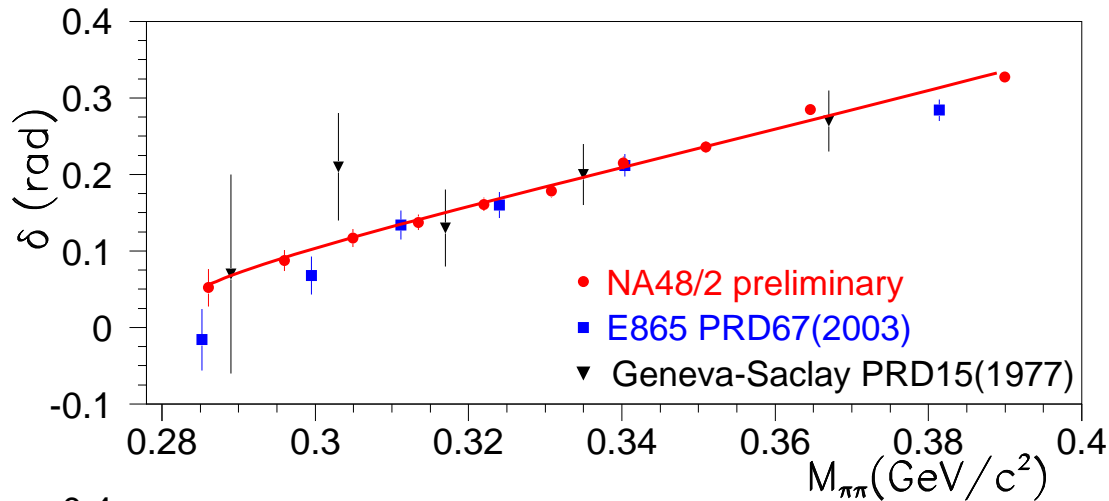


Correlation

$$g'_p$$

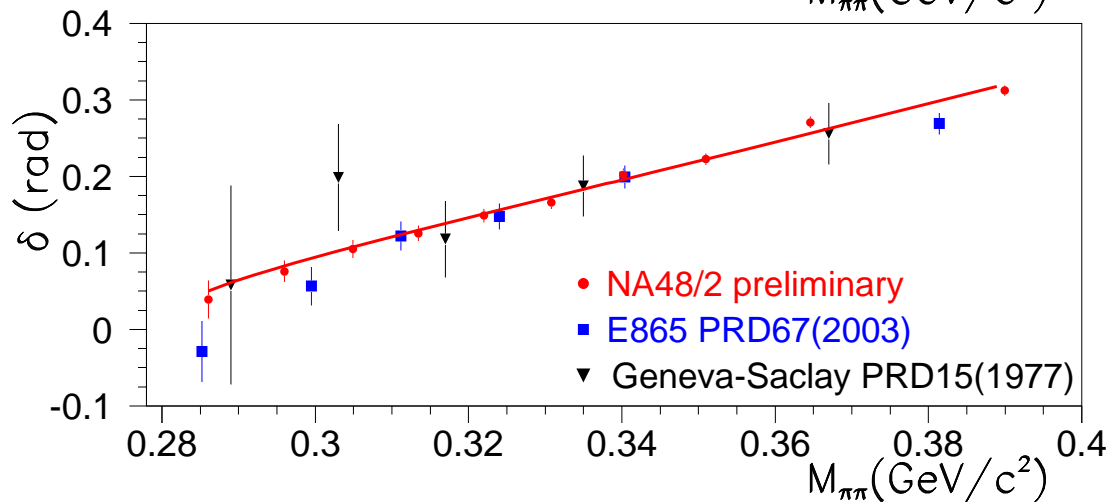
$$g_p(0) \quad -0.914$$

# Comparison of Ke4 phase experimental measurements



Phase points without isospin corrections

Line from a 2p fit to NA48 data alone



All Phase points corrected for isospin mass effects



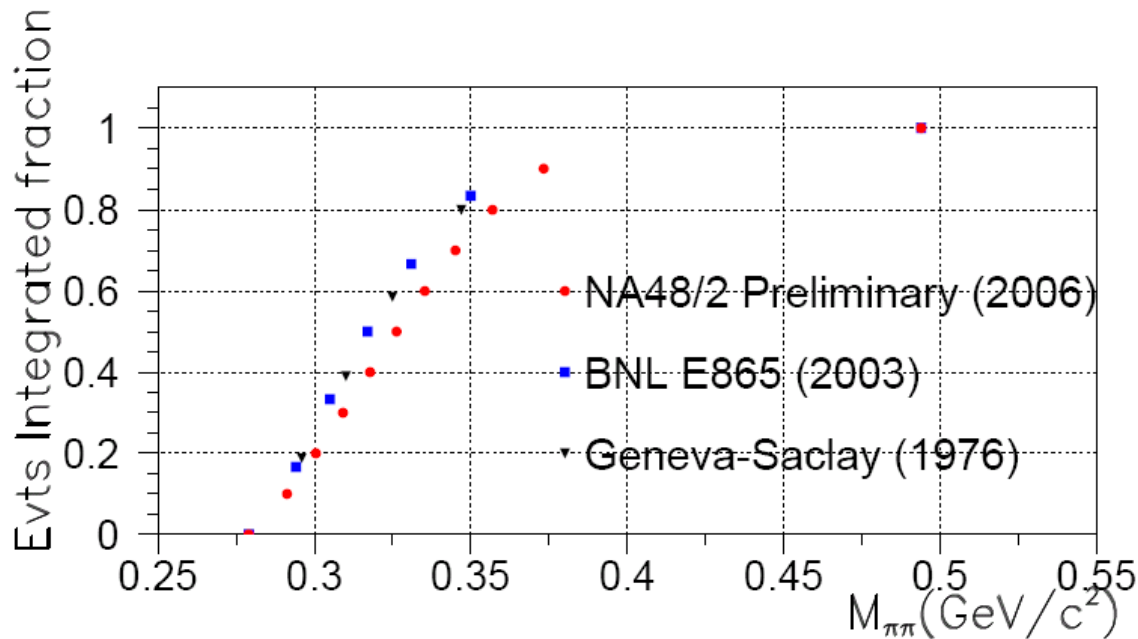
## NA48/2 and E865 sensitivity to $a_0, a_2$

Because of different PK and acceptances, NA48/2 is twice more sensitive to scattering lengths values for the same statistics:

Example : above 0.350 GeV

E865 has 1/6 of the statistics =  $\sim 400\,000 / 6 = \sim 65\,000$  ets

NA48/2 has 1/4 of the statistics =  $\sim 400\,000 / 4 = 100\,000$  ets = 1.5 times more



# K3π : measurement of the k' term

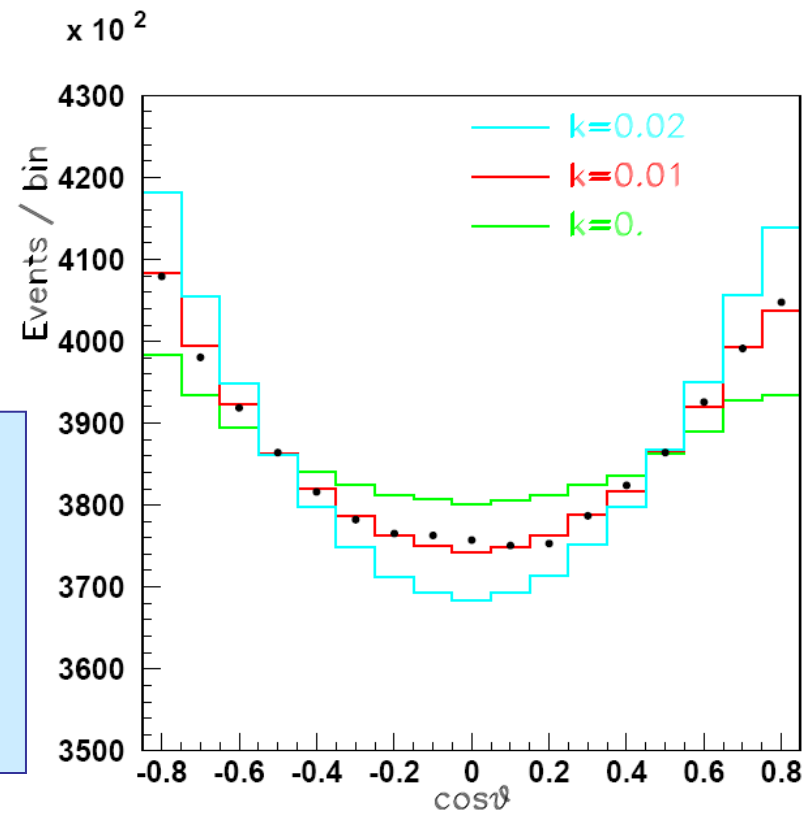
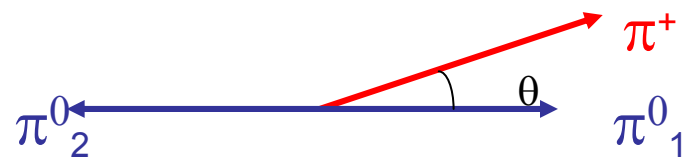
Going to a 2D fit would imply to use  $(M_{00}^2, M_{+0}^2)$  variables. An alternate choice is  $(M_{00}^2, \cos\theta)$  where  $\theta$  is the angle between the charged  $\pi$  and the direction of the  $\pi^0$ 's in their rest frame.

Use a modified matrix element :

$$M_0 = A_0 \left( 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k' v^2 \right)$$

re-fit in  $M_{00}^2$  range [0.082, 0.097]  $(\text{GeV}/c^2)^2$

no incidence on previous  $(\mathbf{a}_0 - \mathbf{a}_2)$  result.



Preliminary result (2003+2004 data, K<sup>+</sup> and K<sup>-</sup>)

$k' = 0.0095 \pm 0.0002_{\text{stat}} \pm 0.0005_{\text{syst}}$

Note: the different meaning  $(g_0, h', k')$  wrt PDG  $(g_0, h, k)$

## Cusp: Dalitz plot slopes result

Reminder:  $M(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$

$M_0 \sim (1 + g_0 u/2 + h'_0 u^2/2 + k'_0 v^2/2)$  but ...

$|M_0|^2$  (PDG)  $\sim (1 + gu + hu^2 + kv^2)$  so  $g_0 \approx g$ ,  $h'_0 \approx h - g^2/4$ ,  $k'_0 \approx k$

-  $k'_0$  is extracted from a 2-dimensional fit

- Other parameters are fitted including a fixed  $k'_0$  value

$$k'_0 = 0.0095 \pm 0.0002_{\text{stat.}} \pm 0.0005_{\text{syst}}$$

$g_0$

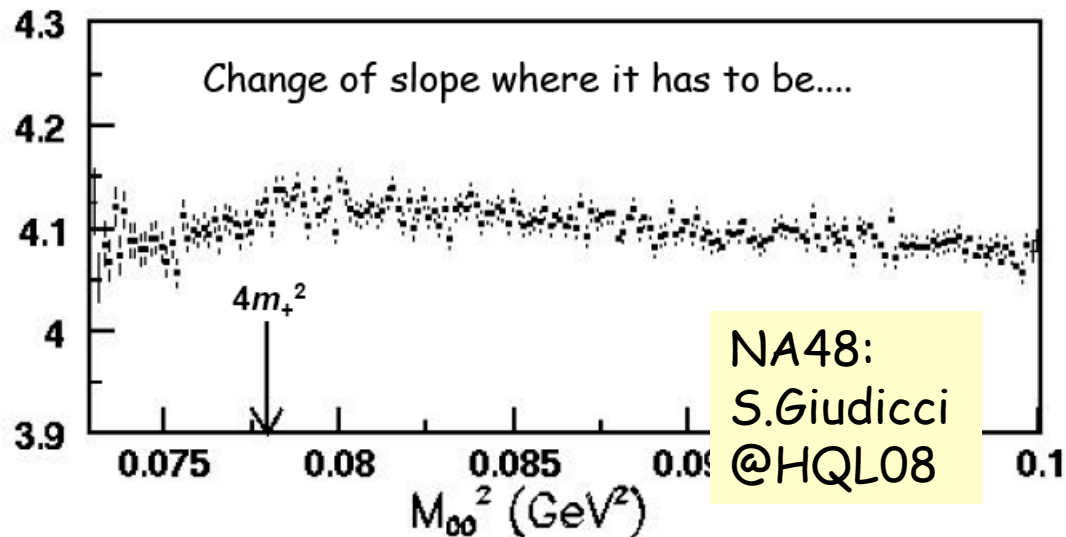
$h'_0$

Preliminary  
(2003+2004)

	$g_0$	$h'_0$
CI model	$0.652 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$	$-0.039 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$
CGKR model	$0.621 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$	$-0.049 \pm 0.001_{\text{stat}} \pm 0.003_{\text{syst}}$
ChPT fit	CI $a_0 - a_2 = 0.268$ $a_0 = 0.2244$ $a_2 = -0.0434$	GCKR $a_0 - a_2 = 0.266$ $a_0 = 0.2191$ $a_2 = -0.0470$

# The cusp in $K_L \rightarrow 3\pi^0$ decays: work in progress

Ratio data / prediction



$$(a_0 - a_2)m_\pi$$

ChPTH

DIRAC  $\pi^+\pi^-$  atom (2005)

NA48  $K^+ \rightarrow \pi^+\pi^0\pi^0$  (2006)

KTeV  $K_L \rightarrow 3\pi^0$  (2008)

0.16 0.18 0.2 0.22 0.24 0.26 0.28

KTeV:  
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