Off-shell helicity amplitudes in high-energy factorization

Piotr Kotko

Institute of Nuclear Physics (Cracow)

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A. van Hameren, P.K., K. Kutak
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Motivation & Plan

Motivation

- scattering amplitudes in QCD
  - collinear factorization $\Rightarrow$ on-shell amplitudes
  - high-energy factorization $\Rightarrow$ off-shell amplitudes
- there are plenty of tools for automated calculation of tree-level amplitudes in collinear factorization; most of them use helicity method
- there are no such tools for high-energy factorization
- main problem $\Rightarrow$ gauge invariance

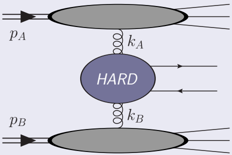
Plan

- definition of amplitudes in high-energy factorization
- forward jets and one-leg off-shell amplitudes (they are rather special)
- two-leg off-shell amplitudes
- two independent MC codes

Introduction

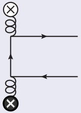
High-energy CCH (Catani, Ciafaloni, Hautmann) factorization[1]

The CCH was originally stated for heavy quarks production in photo-, lepto- and hadro-production.



$$d\sigma_{AB\to Q\bar{Q}} \simeq \int d^2k_{TA} \int \frac{dx_A}{x_A} \int d^2k_{TB} \int \frac{dx_B}{x_B}$$
$$\mathcal{F}(x_A, k_{TA})\, d\sigma_{g^*g^*\to Q\bar{Q}}(x_A, x_B, k_{TA}, k_{TB})\, \mathcal{F}(x_B, k_{TB}),$$

where $\mathcal{F}$ are unintegrated gluon densities with BFKL evolution.

$d\sigma_{g^*g^*\to Q\bar{Q}} \rightarrow$  where $\otimes \text{(gluon)} = |\vec{k}_{TA}|\, p_A^\mu$, $\otimes \text{(gluon)} = |\vec{k}_{TB}|\, p_B^\mu$

At high energies the single longitudinal components of momentum transfers dominate

$$k_A^\mu \simeq x_A p_A^\mu + k_{TA}^\mu, \qquad k_B^\mu \simeq x_B p_B^\mu + k_{TB}^\mu.$$

It can be shown that $d\sigma_{g^*g^*\to Q\bar{Q}}$ is gauge invariant.

- this is not a strict theorem of PQCD (same as for e.g. SIDIS)
- extremaly useful in phenomenology studies, even with *HARD* amplitude at tree level

The approaches to *HARD* part

- in terms of the Lipatov's effective action it corresponds to Quasi-Multi-Regge kinematics; one can use resulting Feynman rules[2]
- two new methods for multiple final states (using helicity method and implemented in MC codes)

[1] S. Catani, M. Ciafaloni, F. Hautmann (1990, 1991, 1994) [2] E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov (2005)

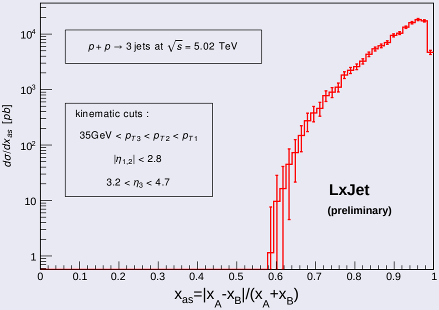
Forward processes and one-leg off-shell amplitudes

Low x and asymmetric kineatics

$$x_A = \sum_i \frac{|\vec{p}_{T\,i}|}{\sqrt{S}} \exp\left(\eta_i\right), \quad x_B = \sum_i \frac{|\vec{p}_{T\,i}|}{\sqrt{S}} \exp\left(-\eta_i\right)$$

$\Rightarrow$ small longitudinal fractions are probed in highly asymmetric configuration.
This accounts for a simplification:

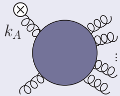
- large fractions $x_B \to$ collinear approach (with on-shell partons)
- small fractions $x_A \to k_T$-factorization (with off-shell partons)



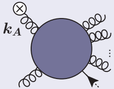
$$d\sigma_{AB\to X} \simeq \sum_b \int d^2k_{T\,A} \int \frac{dx_A}{x_A} \int dx_B\, \mathcal{F}\left(x_A, k_{T\,A}\right) f_{b/B}\left(x_B\right)\, d\sigma_{g^* a\to X}\left(x_A, x_B, k_{T\,A}\right)$$ [1]

One-leg off-shell amplitudes

An example contribution to N-jet process: $g^* g \to gg \ldots g$



- not gauge-invariant (some pieces are missing)



$$\neq 0 \quad \Longleftrightarrow \quad \mathcal{M}\left(\varepsilon_1 \ldots, k_i, \ldots, \varepsilon_N\right) \neq 0$$

- one cannot use helicity method, e.g. $\varepsilon_k^\mu\left(q\right) = \varepsilon_k^\mu\left(q'\right) + k^\mu \beta_k\left(q, q'\right)$

[1] S. Catani, M. Ciafaloni, F. Hautmann (1991); M. Deak, F. Hautmann, H. Jung, K. Kutak (2010); K. Kutak, S. Sapeta (2012)

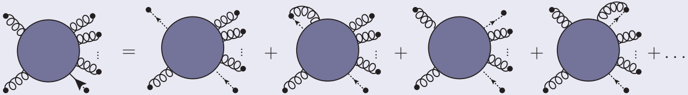
Gauge-restoring amplitude

There exists an "amplitude" $\mathcal{W}$ such that $\widetilde{\mathcal{M}} = \mathcal{M} + \mathcal{W}$ satisfies $\widetilde{\mathcal{M}}(\varepsilon_1, \ldots, k_i, \ldots, \varepsilon_N) = 0$.
The "gauge-restoring" amplitude $\mathcal{W}$ can be obtained by using the ordinary QCD Slavnov-Taylor identities.

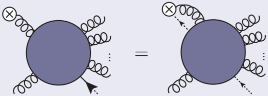
- introduce a reduction formula for the off-shell amplitude ($\tilde{G}$ – the Green function)

$$\mathcal{M}(\varepsilon_1, \ldots, \varepsilon_N) = \lim_{k_A \cdot p_A \to 0} \lim_{k_1^2 \to 0} \ldots \lim_{k_N^2 \to 0} \left(\left| \vec{k}_{TA} \right| p_A^{\mu_A} \right) \left(k_1^2 \varepsilon_1^{\mu_1} \right) \ldots \left(k_N^2 \varepsilon_N^{\mu_N} \right) \tilde{G}_{\mu_A \mu_1 \ldots \mu_N} (k_A, k_1, \ldots, k_N),$$

- apply Slavnov-Taylor identities to $\tilde{G}$ to determine gauge contributions



- after applying the reduction formula (and using axial gauge for internal propagators) the single term survives



The r.h.s term is precisely the amount of gauge-invariance violation and can be calculated (note however, this is not the "gauge-restoring" amplitude yet, as it contains the external ghost line).

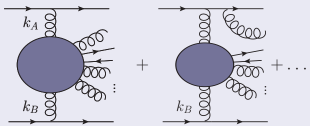
- trading the external ghosts for the longitudinal projections of the gluons and summing the gauge contributions we get (color ordered result)

$$\mathcal{W}_{\mathrm{ord}} (\varepsilon_1, \ldots, \varepsilon_N) = -\left(\frac{-g}{\sqrt{2}} \right)^N \frac{\left| \vec{k}_{TA} \right| \varepsilon_1 \cdot p_A \ldots \varepsilon_N \cdot p_A}{k_1 \cdot p_A \; (k_1 - k_2) \cdot p_A \ldots (k_1 - \ldots - k_{N-1}) \cdot p_A}$$

- works for any number of gluons at the amplitude level
- one can use helicity method and Berends-Giele recursion
- corresponds to Lipatov's $R \to G \ldots G$ effective vertex

Two-leg off-shell amplitudes

When there are two off-shell gluons the previous method does not work without extending QCD action.



An amplitude $g^*(k_A)\,g^*(k_B) \to X$ can be disentangled from $q_A q_B \to q'_A q'_B X$. However, if we want to have

$$k_A^\mu = x_A p_A^\mu + k_{T\,A}^\mu, \quad k_B^\mu = x_B p_B^\mu + k_{T\,B}^\mu$$

the quarks q'_A, q'_B cannot be on-shell $\Rightarrow$ amplitude for $q_A q_B \to q'_A q'_B X$ is not gauge invariant

We want to have both on-shellness and high-energy kinematics:

- the amplitude $q_A\,(p_A)\,q_B\,(p_B) \to q'_A\,(p'_A)\,q'_B\,(p'_B)\,X$ need not to be physical
- introduce on-shell complex momenta for the quarks (the gauge invariance is still there)

For $l_1^2 = l_2^2 = 0$ we define

$$l_3^\mu = \frac{1}{2}\langle l_2;-|\,\gamma^\mu\,|l_1;-\rangle$$

$$l_4^\mu = \frac{1}{2}\langle l_1;-|\,\gamma^\mu\,|l_2;-\rangle$$

They have properties: $l_3^2 = l_4^2 = 0$, $l_{1,2}\cdot l_{3,4} = 0$, $l_1 \cdot l_2 = -l_3 \cdot l_4$

We can decompose $k_{T\,A,B}$ into complex vectors l_3, l_4, in such a way that

$$p_A^\mu = (\Lambda + x_A)\,l_1^\mu - \frac{l_4 \cdot k_{T\,A}}{l_1 \cdot l_2}\,l_3^\mu, \qquad p_B^\mu = (\Lambda + x_B)\,l_2^\mu - \frac{l_3 \cdot k_{T\,B}}{l_1 \cdot l_2}\,l_4^\mu$$

$$p'^{\mu}_A = \Lambda l_1^\mu + \frac{l_3 \cdot k_{T\,A}}{l_1 \cdot l_2}\,l_4^\mu, \qquad p'^{\mu}_B = \Lambda l_2^\mu + \frac{l_4 \cdot k_{T\,B}}{l_1 \cdot l_2}\,l_3^\mu$$

We get both the on-shellness $p_{A,B}^2 = p'^{\,2}_{A,B}$ and high-energy limit for any Λ. Moreover the external spinors for quarks $|p_A;-\rangle \propto |l_1;-\rangle$, $|p_B;-\rangle \propto |l_2;-\rangle$ etc.

- in order to extract the physical amplitude take the limit $\Lambda \to \infty$, either numerically or analytically (then eikonal couplings and propagators appear)
- corresponds to Lipatov's $RR \to X$ effective vertex

Example applications

We have implemented the two methods in the two independent MC codes

- C++ code using FOAM and ROOT (currently QCD with single off-shell leg) – LxJet
- FORTRAN code similar to HELAC (QCD, QED, Weak) – OSCARS (Off-Shell Currents And Related Subjects)

Nuclear modification factors with OSCARS

$$gg^* \to b\bar{b}\,\mu^+\mu^-$$

$p_{T\,q(\bar{q})} > 20\,\mathrm{GeV}$ $\quad y_{q(\bar{q})} < 2.5$

$p_{T\,\mu^\pm} > 20\,\mathrm{GeV}$ $\quad y_{\mu^\pm} < 2.1$

$\Delta R_{q,\bar{q}} > 0.4$ $\quad \Delta R_{q(\bar{q}),\mu^\pm} > 0.4$

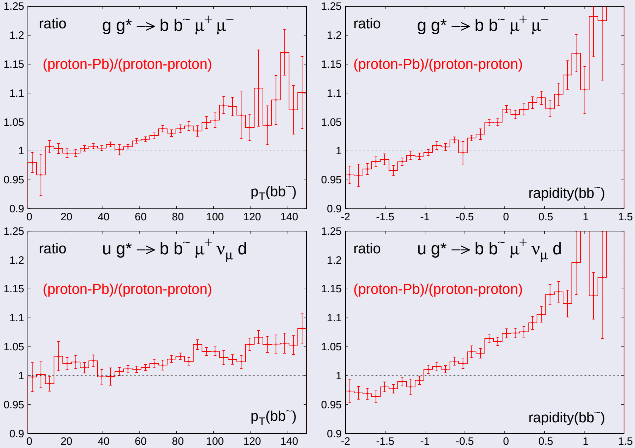
$$ug^* \to b\bar{b}\,\mu^+\nu_\mu\, d$$

$p_{T\,q(\bar{q})} > 20\,\mathrm{GeV}$ $\quad y_{q(\bar{q})} < 2.5$

$20\,\mathrm{GeV} < p_{T\,\mu^+} < 50\,\mathrm{GeV}$

$y_{\mu^+} < 2.1$ $\quad \not{E}_T > 20\,\mathrm{GeV}$

$\Delta R_{q,\bar{q}} > 0.4$ $\quad \Delta R_{q(\bar{q}),\mu^\pm} > 0.4$



The plots are for 5.02 TeV. We use unintegrated PDFs of K. Kutak and S. Sapeta.

- high-energy factorization makes use of unintegrated gluon densities and off-shell amplitudes
- the last can be obtained eg. from Lipatov's effective action, but there are no tools for automated numerical calculation (in the contrary to on-shell tree-level amplitudes with multiple final states)
- we have constructed two methods: one dedicated for forward scattering with single off-shell gluon, and the second that can be used also for two off-shell gluons
- we have implemented the methods in the two independent MC programs
- phenomenological studies are in progress...

BACKUP

Gauge-restoring amplitude (comments)

Remarks concerning gauges and ghosts

- It is allowed to use two different gauges for on-shell lines and internal off-shell lines.
- Ghosts do exist in the axial gauge (but usually decouple)[1]

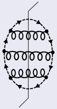
  A ghost-gluon coupling in the axial gauge is  $= ig\, f_{abc}\, n^{\mu}$, where n is a gauge vector.

  The inverse ghost propagator is proportional to $n \cdot k$.
- Usually, when squaring an amplitude one uses sum over *physical* gluon polarization

$$\sum_{\lambda} \varepsilon_k^{(\lambda)\,\mu}(q)\, \varepsilon_k^{(\lambda)\,\nu\,*}(q) = -g^{\mu\nu} + \frac{q^{\mu} k^{\nu} + q^{\nu} k^{\mu}}{q \cdot k},$$

  with some light-like momentum q.

  Alternatively, one can use external gluons in the Feynman gauge and *cut ghost loops*. $\longrightarrow$



- The last remark allows us to trade an external ghost with momentum k to a gluon projected onto some light-like momentum q

$$\text{external ghost} \to \frac{\varepsilon_k \cdot q}{k \cdot q}$$

[1] e.g. G. Leibrandt, *Rev. Mod. Phys.* (1987)