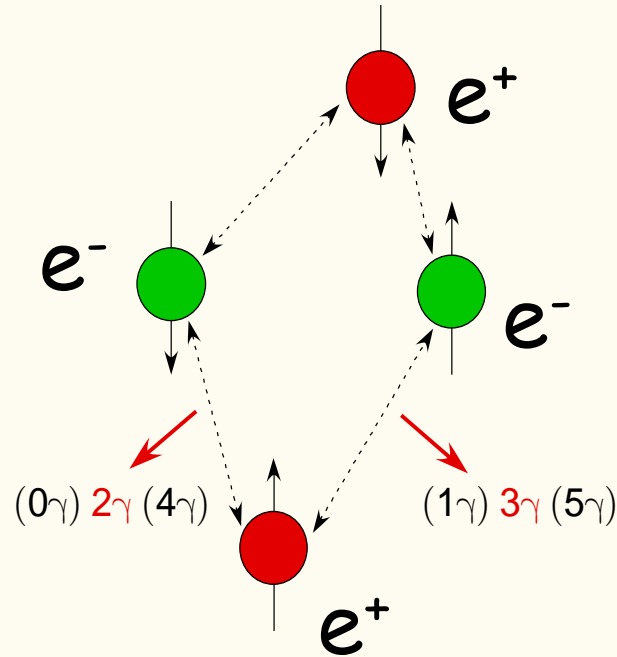


# Positronium and Polyelectrons



PANIC 2008  
Eilat, November 2008

Andrzej Czarnecki  University of Alberta

# Outline

## Positronium, its ion, and the molecule

1951

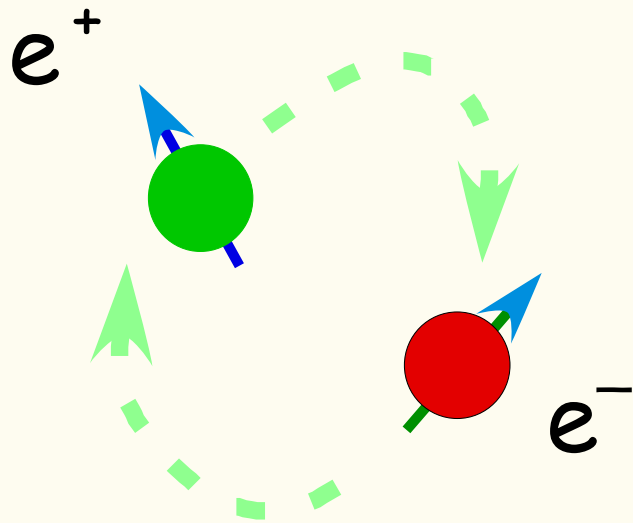
1981

2007

**Ion: theoretical prediction for the decay rate. Comparison with experiment.**

**Molecule: spectrum. Dipole transition vs. annihilation.**

# Positronium and its ion



Very similar to hydrogen, except

- no hadronic nucleus
- annihilation
- reduced mass reduced

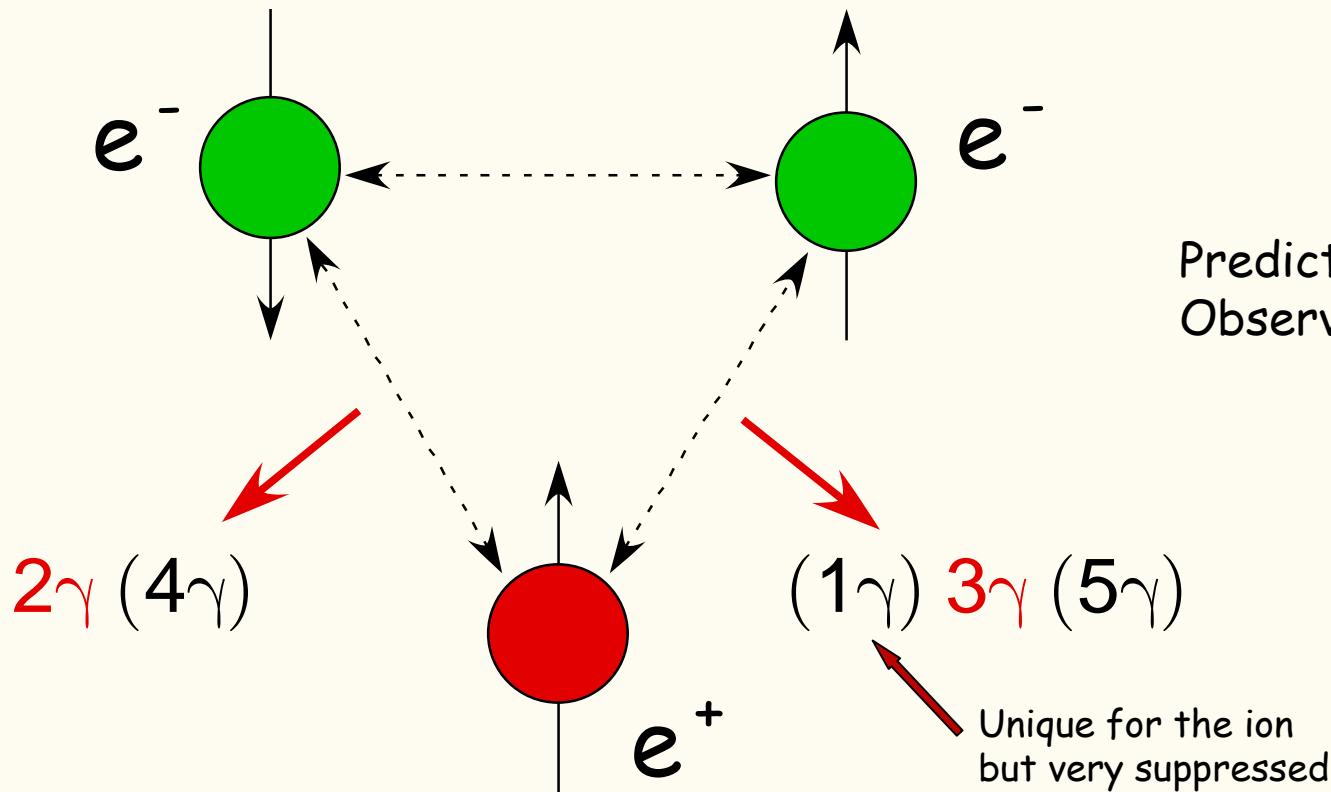
$$m_e \rightarrow \frac{m_e}{2}$$

Two spin states:

singlet (para-Ps; short-lived, 0.1 ns)

triplet (ortho-Ps; long-lived, ~150 ns)

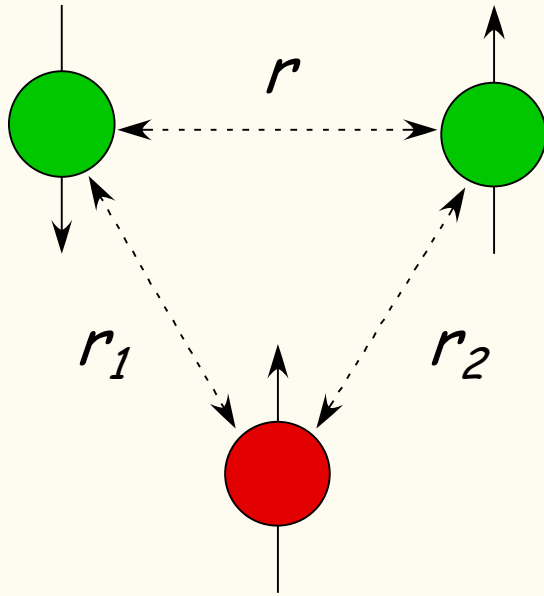
# Positronium ion: a new test of bound-state QED



New positronium-ion source  
at the FRM II reactor in Munich:  
measurements of branching ratios.



# Theory of the Ps ion: the wavefunction



$$\psi(\vec{r}_1, \vec{r}_2) = \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}} \phi(r_1, r_2, r)$$

The wave function is not known analytically,  
but can be found using the variational method.

The ion resembles a positronium "shell"  
and a loosely-bound electron.

Now we can easily estimate the dominant decay channels.

# Recent measurement of the ion decay rate

63401 (2006)

PHYSICAL REVIEW LETTERS

week  
17 FEBRU

## Measurement of the Decay Rate of the Negative Ion of Positronium ( $\text{Ps}^-$ )

Frank Fleischer,<sup>1,\*</sup> Kai Degreif,<sup>1</sup> Gerald Gwinner,<sup>1,†</sup> Michael Lestinsky,<sup>1</sup>  
Vitaly Liechtenstein,<sup>2</sup> Florian Plenge,<sup>1</sup> and Dirk Schwalm<sup>1</sup>

<sup>1</sup>*Max-Planck-Institut für Kernphysik, Heidelberg, Germany*

<sup>2</sup>*Kurchatov Institute, Moscow, Russia*

(Received 28 November 2005; published 13 February 2006)

A new determination of the decay rate of the negative ion of positronium ( $\text{Ps}^-$ ), using a beam-foil method and a stripping-based detection technique, is reported. The measured result of  $\Gamma = 2.089(15) \text{ ns}^{-1}$  is a factor of 6 more precise than the previous experimental value of  $\Gamma = 2.09(9) \text{ ns}^{-1}$ , and is in excellent agreement with the theoretical value of  $\Gamma = 2.086(6) \text{ ns}^{-1}$ .

$$\Gamma_{\text{exp}}(\text{Ps}^-) = 2089(15) \mu\text{s}^{-1}$$

$$7 \cdot 10^{-3}$$

(factor 4-5  
improvement  
expected)

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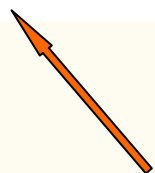
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$$\Gamma_{\text{exp}}(\text{Ps}^-) = 2089(15) \mu\text{s}^{-1}$$

$$7 \cdot 10^{-3}$$

(factor 4-5  
improvement  
expected)



We wanted to  
improve this

# Positronium ion decay: refinements

Corrections  $O(\alpha)$

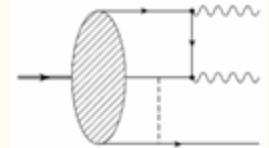
single hard photon loops

Corrections  $O(\alpha^2)$ ; challenge: divergences  $\rightarrow \ln \alpha$

soft

$O(k^2)$  corrections to the amplitude  $M$

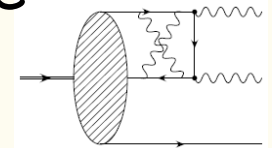
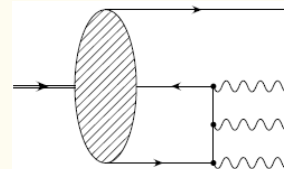
Breit hamiltonian  $\rightarrow$  correction to  $\phi(r=0)$



hard

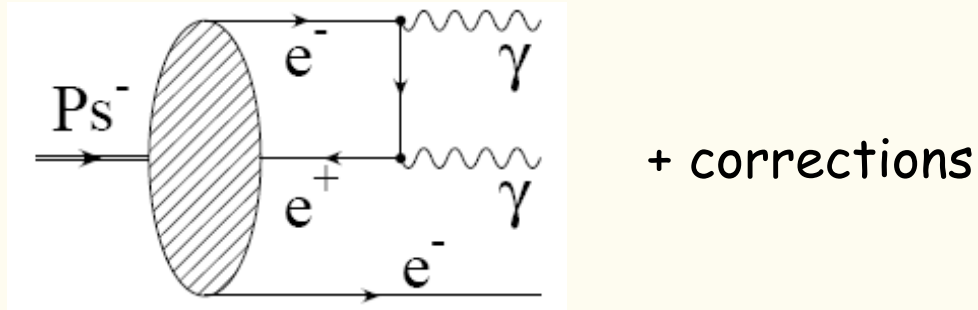
Short-distance two-loop photon exchange

Real photon radiation





# Decay rate prediction

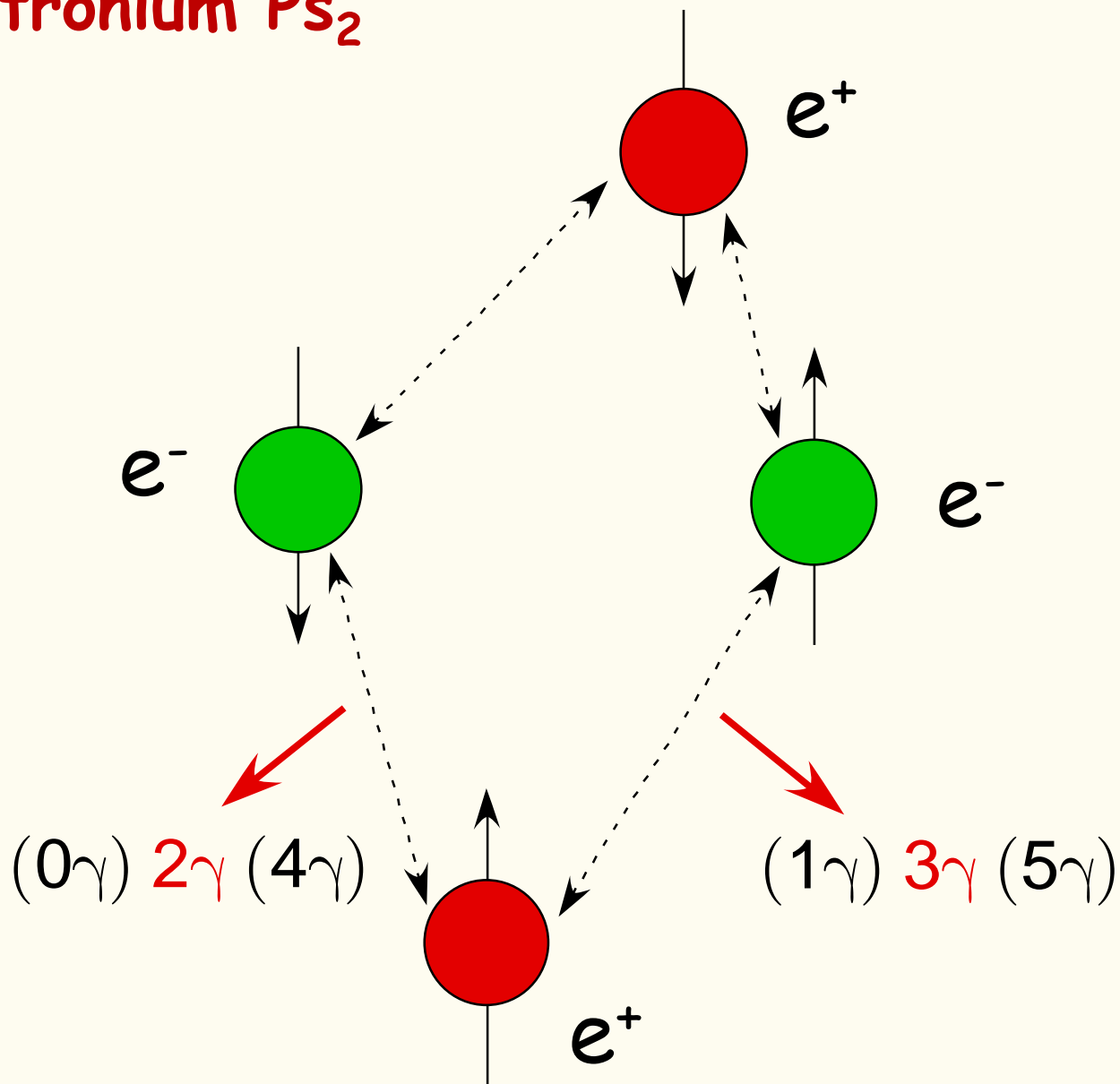


$$\Gamma(\text{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar} (1 + C) \langle \delta^3(r_{12}) \rangle$$

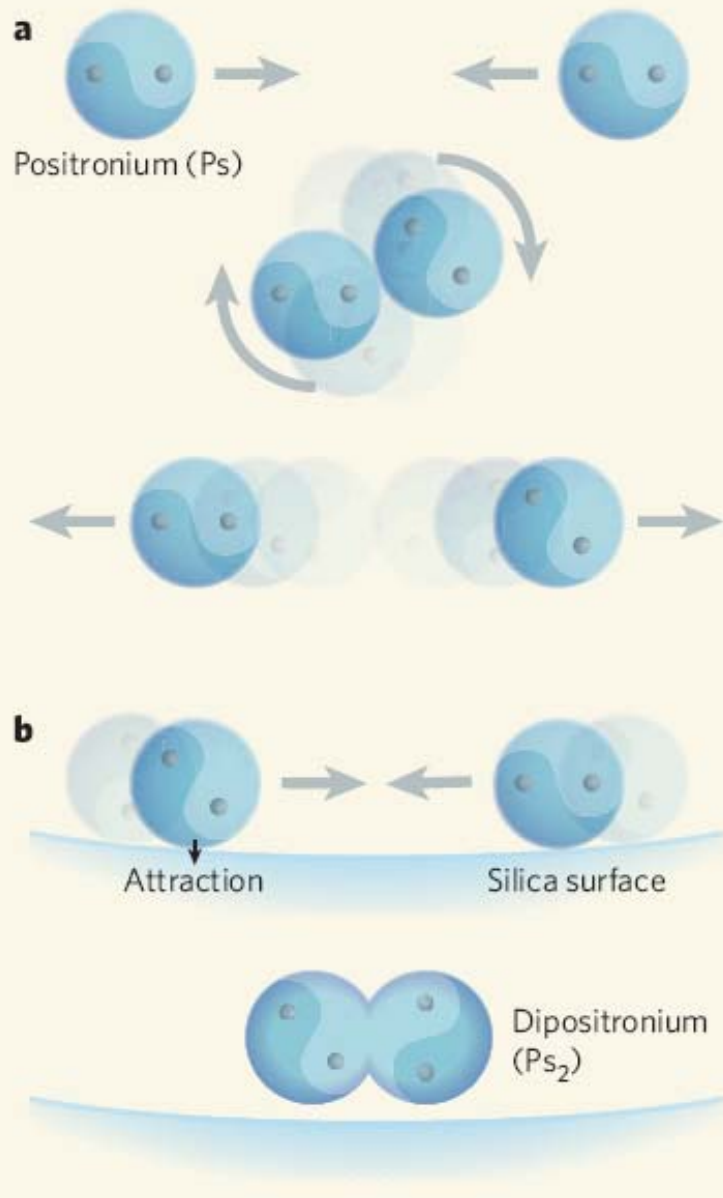
$$\Gamma(\text{Ps}^-) = 2.087963(12) \text{ ns}^{-1}$$

with M. Puchalski and S. Karshenboim  
PRL **99**, 203401 (2007)

# Dipositronium $Ps_2$



# Discovery of dipositronium 2007



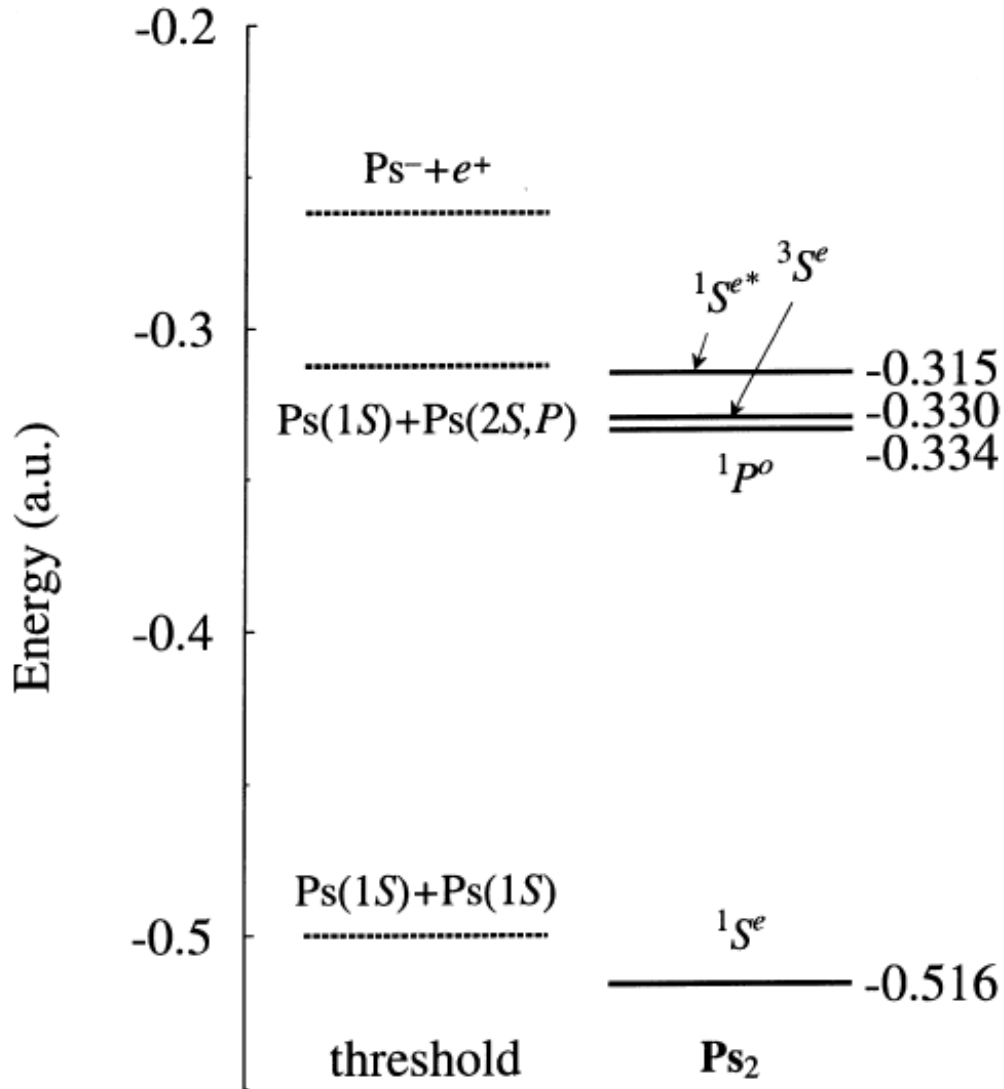
Molecule formation kills long-lived positronia.

At higher temperature, fewer atoms on the surface, fewer molecules formed.

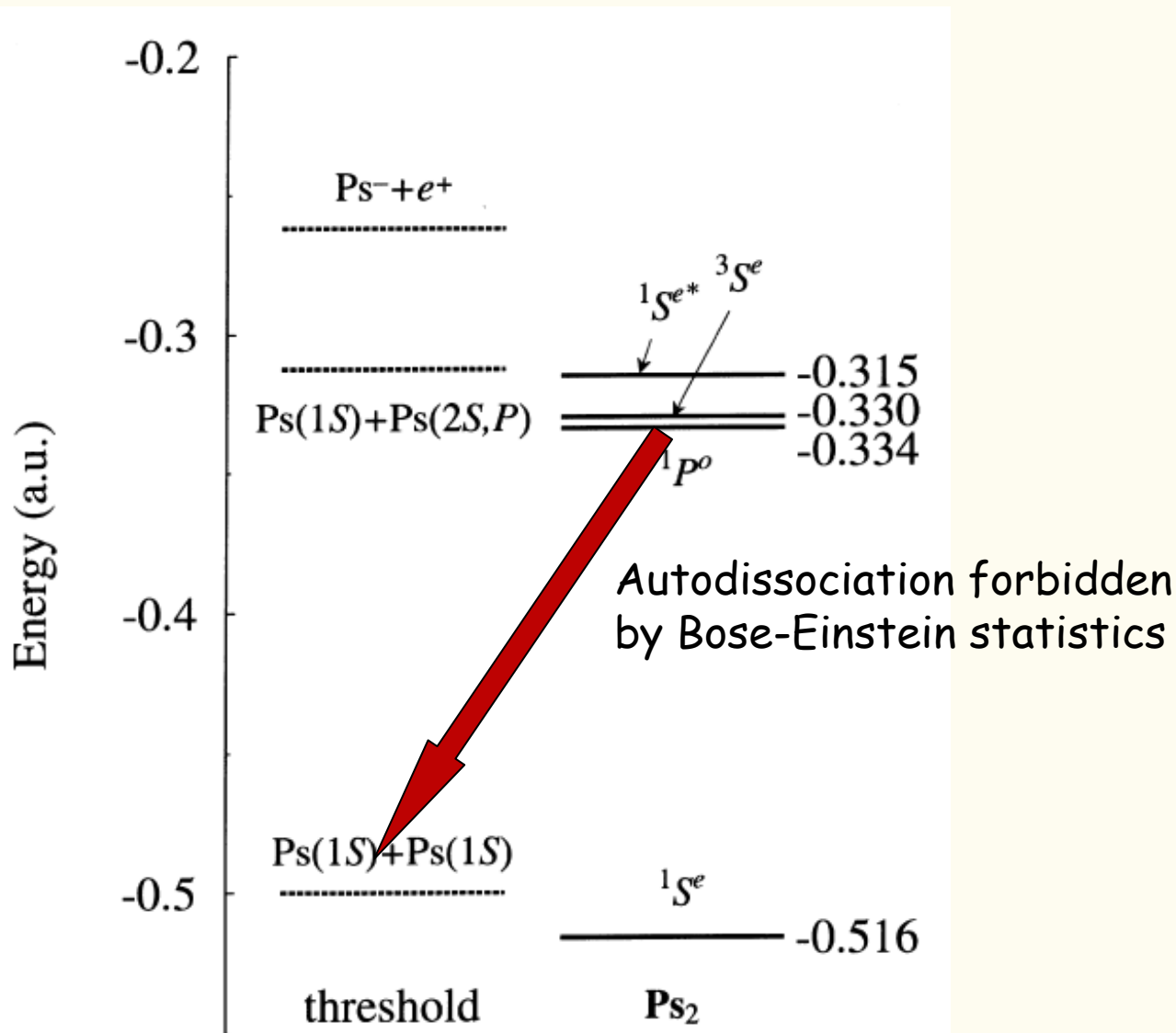
Indeed: at high-T, more long-lived positronia observed.

Cassidy & Mills, Nature 2007

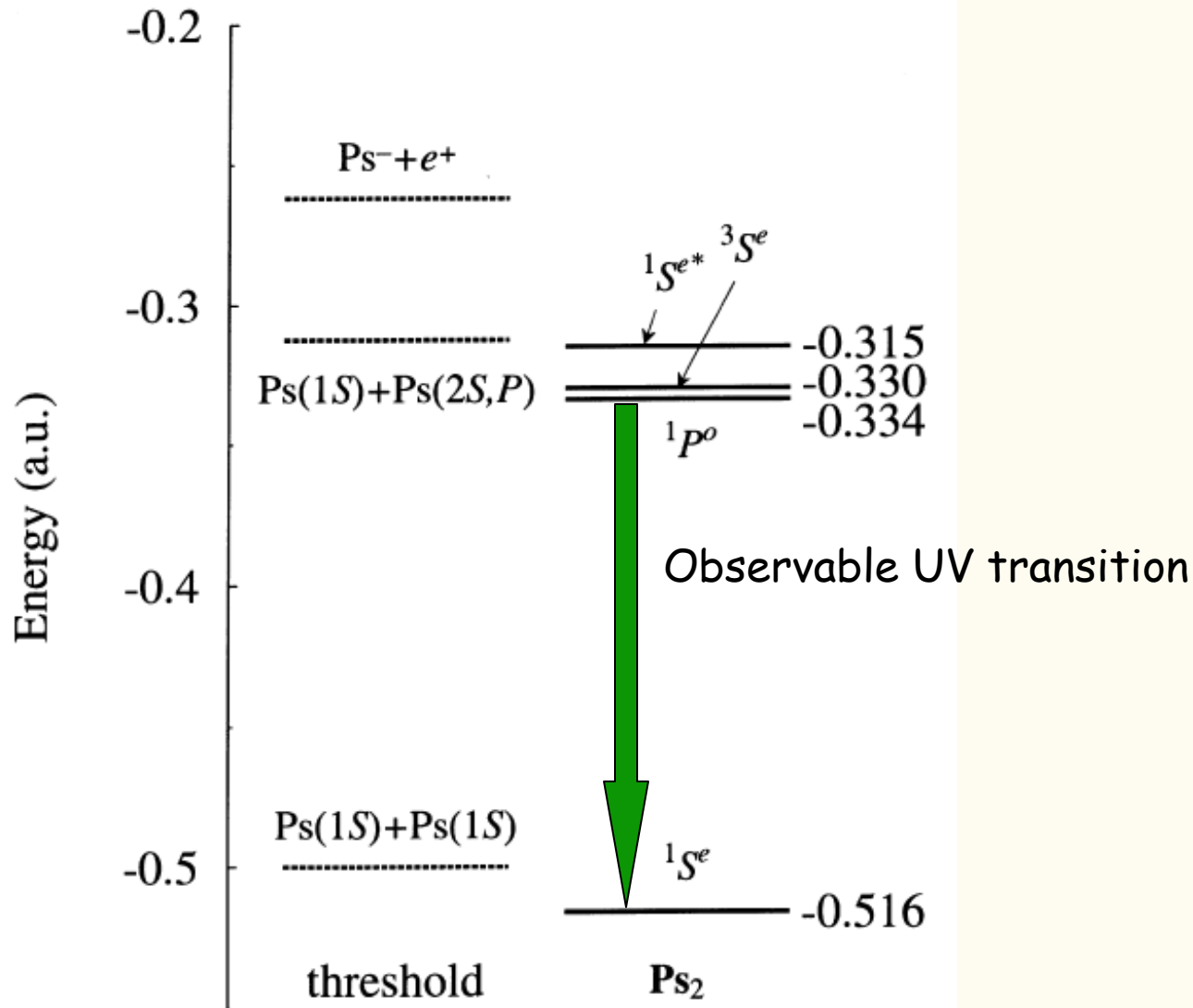
# Spectrum of the molecule $\text{Ps}_2$



# A direct signal of the molecule: transition line.



# A direct signal of the molecule: transition line.



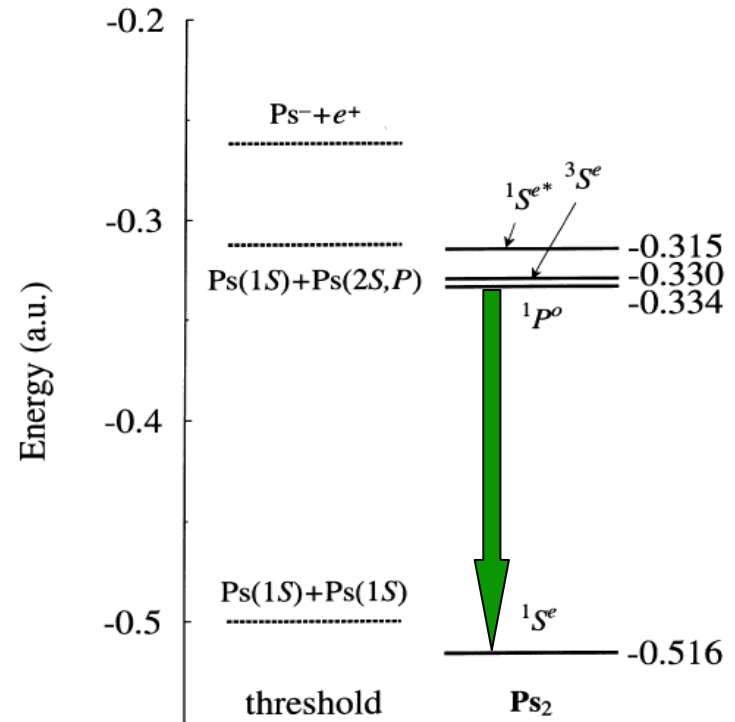
## Two questions about this transition:

What is its accurate energy?

$$\Delta E = E_P - E_S = 0.1815867(8) \text{ a.u.} \\ \simeq 4.9 \text{ eV}$$

How often does it appear  
(before annihilation)?

$$\text{BR}(P \rightarrow S) = \frac{\Gamma_{\text{dip}}(P \rightarrow S)}{\Gamma_{\text{annih}}(P) + \Gamma_{\text{dip}}(P \rightarrow S)} = 0.191(2)$$



## Summary

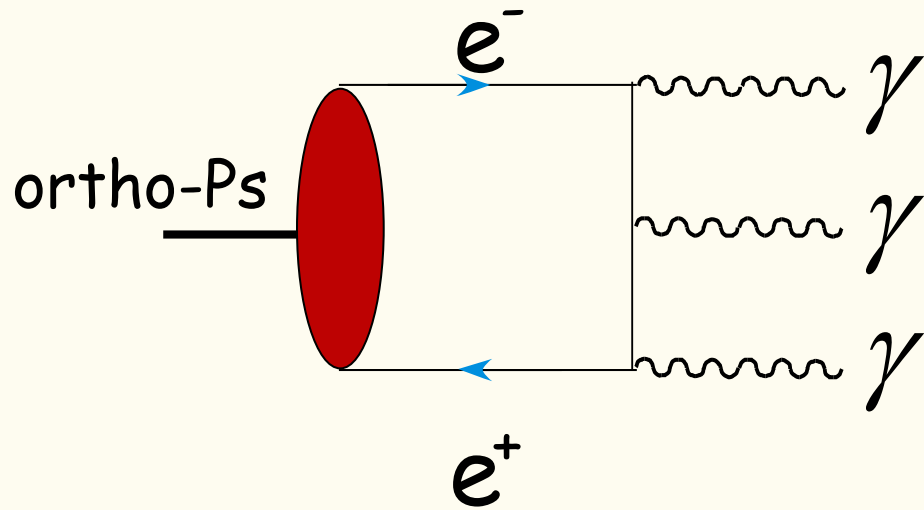
New states of positronium-like systems, ions and molecules: opportunity to test three- and four-body QED.

The Coulomb wave functions not known analytically for  $e^+ e^- e^-$  and  $e^+ e^- e^+ e^-$ .

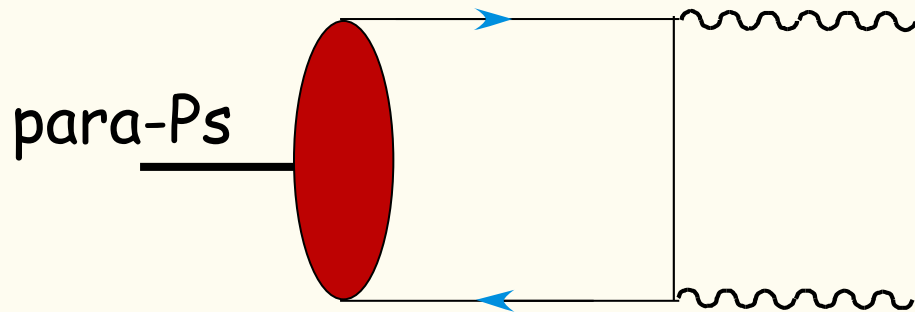
Variational methods very accurate.  $O(\alpha^2)$  corrections computed for the ion decay rate and the  $Ps_2$  P-S interval. Both are being measured.



# Decays of positronium (atoms)



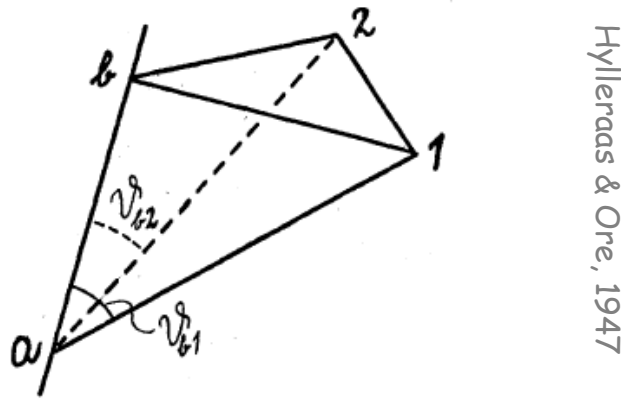
$$\Gamma_{\text{exp}}(\text{o-Ps}) = 7.0404(14) \mu\text{s}^{-1}$$
$$2 \cdot 10^{-4}$$



$$\Gamma_{\text{exp}}(\text{p-Ps}) = 7.9909(17) \text{ns}^{-1}$$
$$2 \cdot 10^{-4}$$

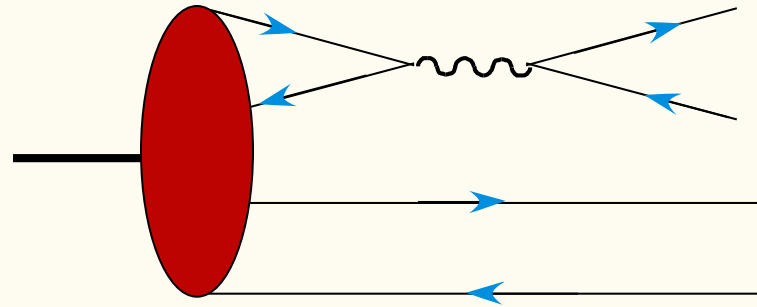
# Energy levels: ground state and P-excitation

Wave function determined variationally, using Coulomb potential; M. Puchalski



Coordinate system for the positronium molecule

Relativistic corrections: perturbations.  
Annihilation dominates.

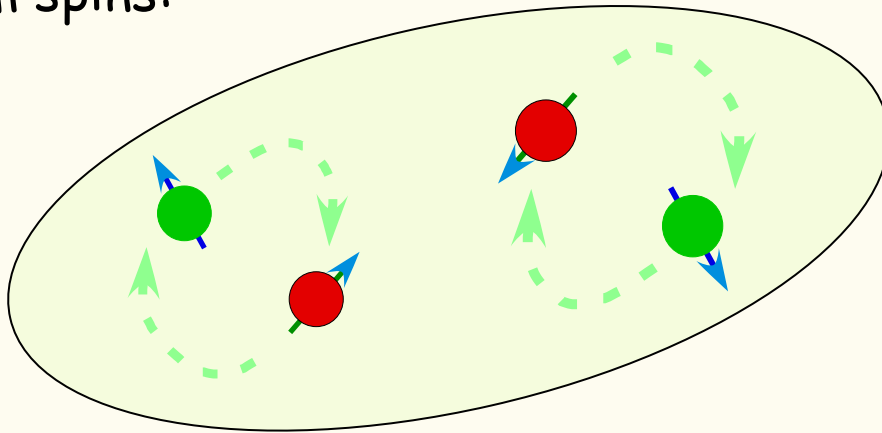


Interval P-S determined with  $5 \times 10^{-6}$  accuracy (slightly smaller than in Ps, "dielectric effect").

# Competition: dipole transition vs. annihilation

S-state annihilates quite rapidly.

Assume it consists of two weakly-interacting Ps atoms, with random spins:



There is a para-positronium pair with probability  $2 \cdot \frac{1}{4} = \frac{1}{2}$

The decay rate:

$$\Gamma_s \approx \frac{1}{2} \cdot \frac{1}{\tau_{pPs}} = \frac{1}{0.25 \text{ ns}}$$

P-state: half of this rate.

# Competition: dipole transition vs. annihilation

In an isolated P-excited Ps atom lives  $\sim 3.2$  ns,  
In in  $Ps_2$  the decay is about twice faster.

$$d_{\text{atom}} = \langle S | \vec{d} | P \rangle$$

$$d_{\text{molecule}} = \left\langle SS \left| \vec{d} \right| \frac{PS+PS}{\sqrt{2}} \right\rangle = \sqrt{2} d_{\text{atom}}$$

Branching ratio

$$BR(P \rightarrow S) = 19\% \sim \frac{1}{\frac{1}{1.6} + \frac{1}{0.5}}$$

# Breakdown of corrections to the Ps ion width

Correction	Value
$\alpha A^{3\gamma}$	0.002 693 245
$\alpha A^{2\gamma}$	-0.005 882 770
$-2\alpha^2 \ln\alpha$	0.000 524 019
$\alpha^2 B^{4\gamma}$	0.000 001 480
$\alpha^2 B^{3\gamma}$	-0.000 064 352
$\alpha^2 B_{\text{squared}}$	0.000 008 652
$\alpha^2 B_{\text{hard}}^{\text{fin}}$	-0.000 218 3(34)
$\alpha^2 B_{\text{aa}}$	0.000 017 750
$\alpha^2 B_{\text{H1}}$	0.000 078 366
$\alpha^2 B_{\text{H2}}$	0.000 541 484
$3\alpha^3 \ln^2\alpha / (2\pi)$	-0.000 004 491
$2.5(2.5)\alpha^3 \ln\alpha$	-0.000 004 8(48)
Total $C$	-0.002 309 7(59)

$$\Gamma(\text{Ps}^-) = 2\pi \frac{\alpha^5 m_e c^2}{\hbar} (1 + C) \langle \delta^3(r_{12}) \rangle$$

## Ratio of three- to two-photon annihilation

$$\text{BR}(\text{Ps}^- \rightarrow \gamma\gamma\gamma) \equiv \frac{\Gamma(\text{Ps}^- \rightarrow \gamma\gamma\gamma)}{\Gamma(\text{Ps}^-)}$$

$$= \alpha \left[ A^{3\gamma} + \alpha(B^{3\gamma} - AA^{3\gamma}) - \frac{7}{3} A^{3\gamma} \alpha^2 \ln \frac{1}{\alpha} + \dots \right]$$

$$= 0.002\,635\,8(8).$$

## Positronium and Polyelectrons

**Id:** 196

**Place:** *Dan Panorama Hotel*

Room: Sea Star

**Starting date:** 10-Nov- 17:55

**Duration:** 15' 2008 (Asia/Jerusalem)

**Presenters:** CZARNECKI, Andrzej

**Included in session:** [Parallel II: Standard Model and physics beyond it](#)