LATTICE GAUGE THEORY IN TECHNICOLOR

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(YS, BS, & TD, arXiv:0803.1707 [hep-lat])

1. Beyond the Standard Model — more gauge groups, more reps
   - β function scenarios
2. Method: Schrödinger Functional (background field method) and the lattice phase diagram
3. Results for:

   The β function of the SU(3) gauge theory with $N_f = 2$ fermions in the **6 rep**

4. and:

   $m, T \neq 0$: Phase diagram on a finite lattice

BEYOND THE STANDARD MODEL on a lattice:

Strong coupling gauge theories — specifically

- Technicolor; walking
- vs. Unparticles?
- Supersymmetry

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Current studies: (Lattice 2008)

- SU(3) with fund rep quarks: $N_f = 8, 12$
- SU(2) with adjoint rep quarks
- SU(3) with sextet quarks ***
- SU(N) with 2-index symm rep quarks (*quenched*)
- SU(2) SUSY

Why this model?

- Banks–Zaks fixed point (Caswell 1974; Banks & Zaks 1981) — Is it really there?
- Scale separation: $C_2(R) = \frac{10}{3}$ vs. $\frac{4}{3}$ for fund rep

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Banks–Zaks: Perturbation theory

$$\beta(g^2) = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6 + \cdots$$

Here $b_1 > 0$, $b_2 < 0$ [as in QCD with $8.05 < N_f < 16\frac{1}{2}$]

$\Longrightarrow$ IR-attractive fixed point at $g_*^2 \simeq 10.4$ — a strong coupling

What can happen NONPERTURBATIVELY?

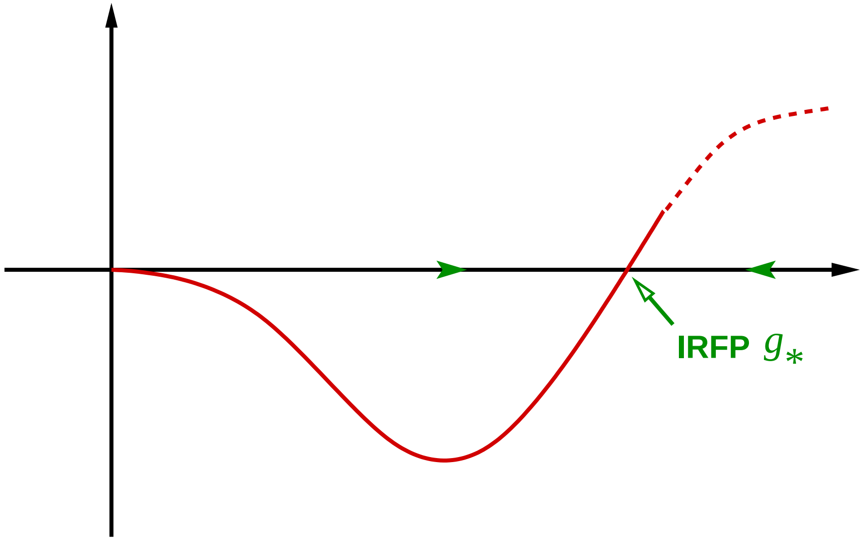
g_*^2 weak

IRFP $\Rightarrow$ conformal dynamics at large distances

$\Rightarrow$ no confinement, no χSB,

no particles!

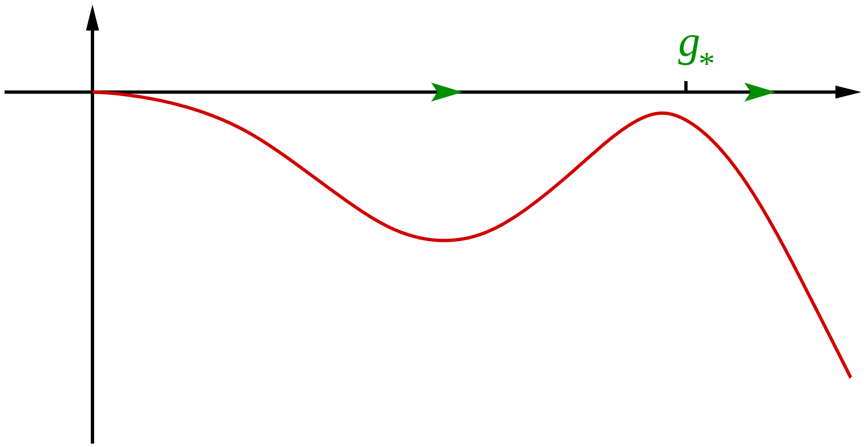
[*unparticles?*]



g_*^2 strong

χSB $\Rightarrow$ fermions decouple, back to β fn of pure gauge theory

[Technicolor . . . maybe walking]



CALCULATING THE β FUNCTION: the Schrödinger Functional

- Wilson fermions — because
  1. boundary values (background field) can be set on a single time slice
  2. control over N_f
- SF: fix spatial links U_i on time boundaries $t = 0, L$
- Calculate the free energy $\Gamma \equiv -\log Z$ since $\Gamma \equiv \frac{1}{g^2(L)} S_{YM}^{cl}$ gives the running coupling $g^2(L)$.

But we can't calculate Γ directly, so:

Choose boundary values U_i to depend on a parameter η. Then

$$\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S_{YM}}{\partial \eta} - \mathrm{tr} \left(\frac{1}{D_F^\dagger} \frac{\partial (D_F^\dagger D_F)}{\partial \eta} \frac{1}{D_F} \right) \right\rangle = \frac{K}{g^2(L)}, \qquad K \equiv \frac{\partial S_{YM}^{cl}}{\partial \eta} = 37.7\ldots$$

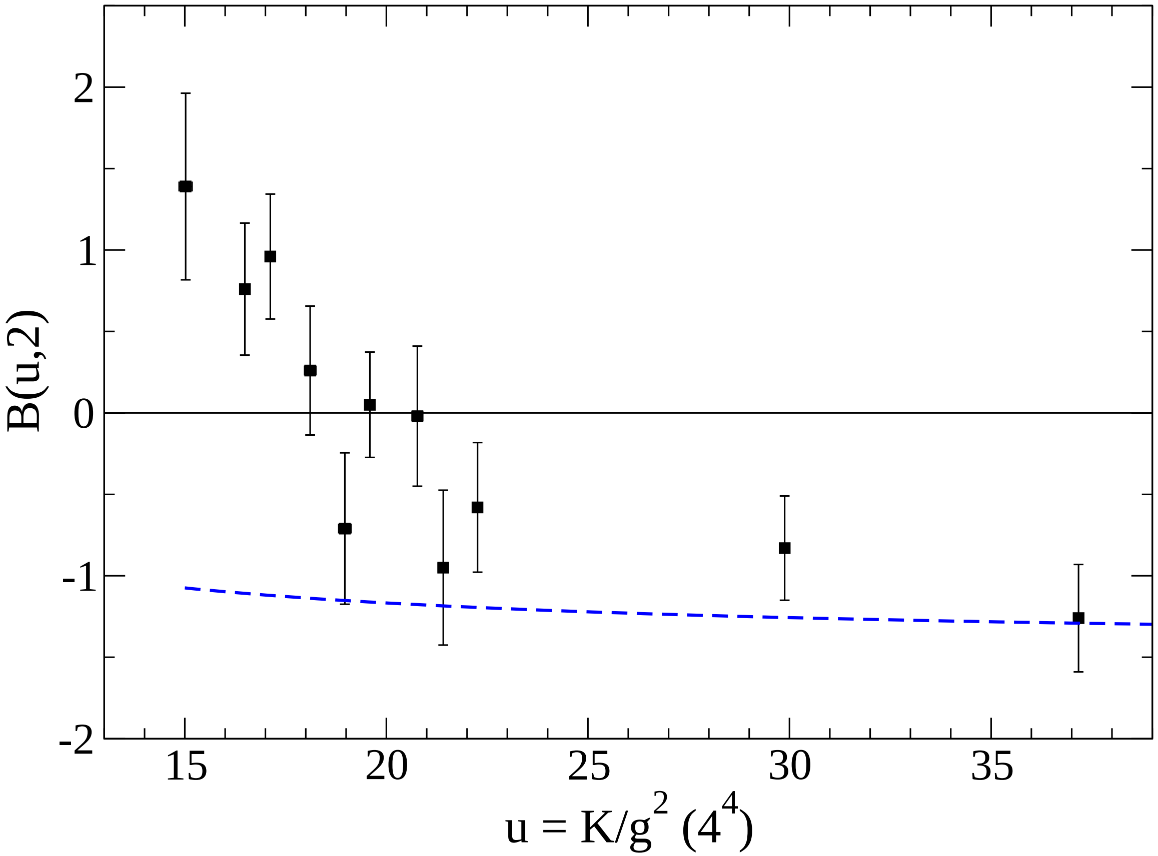
EXTRACTING PHYSICS

1. Fix lattice size L, couplings $\beta \equiv 6/g_0^2$, $\kappa = \kappa_c(\beta)$
2. Calculate $K/g^2(L)$ and $K/g^2(2L)$. Use common lattice spacing (= UV cutoff) $a = L/4$.
3. Result: Discrete Beta Function

$$B(u, 2) = \frac{K}{g^2(2L)} - \frac{K}{g^2(L)}\,,$$

a function of $u \equiv K/g^2(L)$.

The DISCRETE BETA FUNCTION



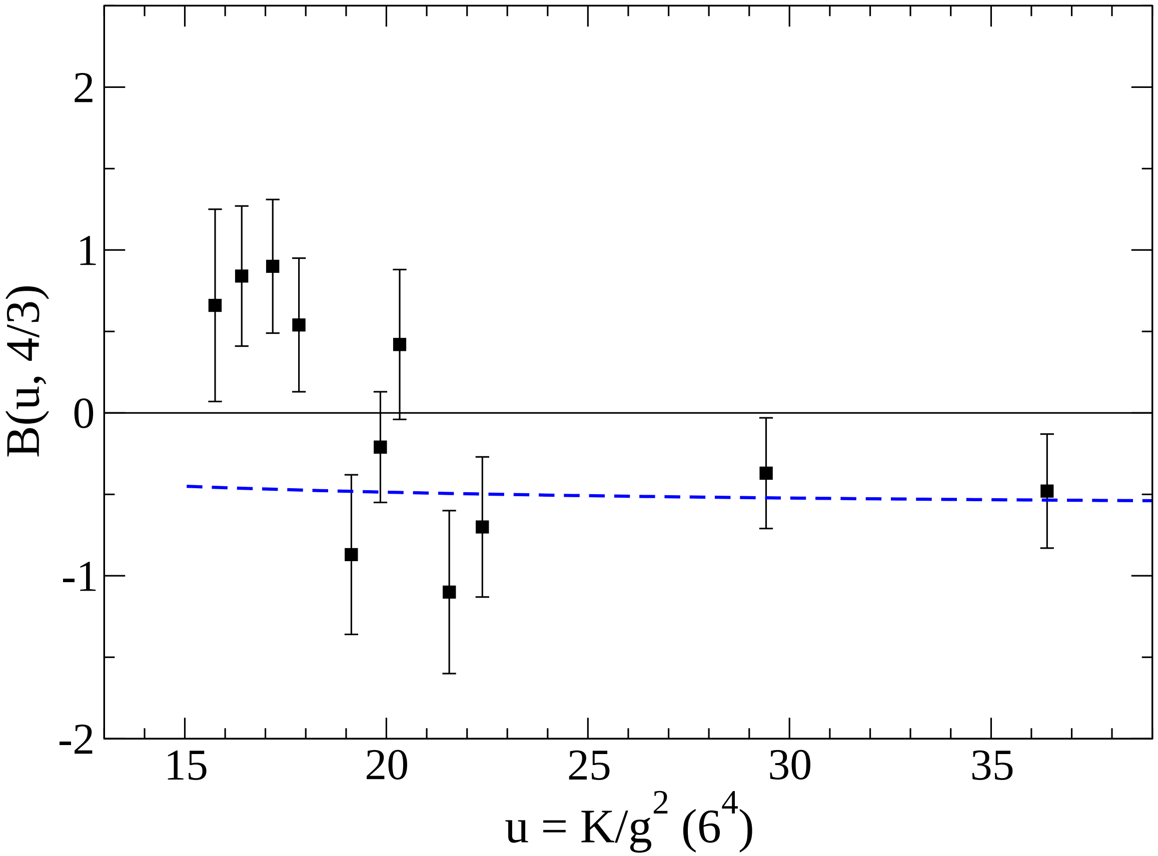
$4^4 \longrightarrow 8^4$

$B(u,2)$ crosses zero at $g^2 \simeq 2.0$

not at $g^2 \simeq 10$!

$\Longrightarrow$ IR theory is CONFORMAL

The DISCRETE BETA FUNCTION



Cf. $6^4 \longrightarrow 8^4$

Caveat cursor

- Is there only one, unique running coupling?
  - Perturbatively, yes.
  - If the $q\bar{q}$ potential is *almost* Coulombic: $V(r) \simeq g^2(r)/r$
- Is it really an IRFP?
- Can we extend the picture off the $\kappa_c(\beta)$ curve?

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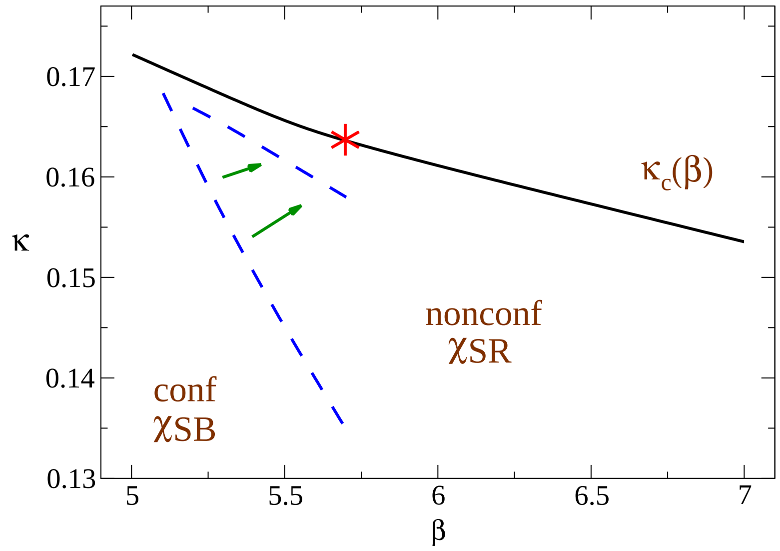
"PHASE DIAGRAM" in finite volume

$N_t = 8, 12$: "finite temperature," confinement and chiral phase transition!

$\Rightarrow$ No evidence of scale separation

Note *weak coupling* at IRFP



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ANSWERS will come from:

- More knowledge of phase diagram
- Checking beta function with more volumes
- Eventually: scaling towards the continuum limit

MORE QUESTIONS

- Properties of (near-) conformal theory