

Phase Structure in Hadronic/Quark Models and its
Implementation in Heavy-Ion Simulations

OUTLINE

- basics of hadronic model
- nuclear matter properties
- generating an EOS with a critical endpoint
- molecular / hydro simulation

- including quark degrees of freedom
- first results on phase diagram
- simulations in progress

J. Steinheimer, V. Dexheimer, G. Zeeb, D. Zschiesche, SWS
Goethe University, Frankfurt

hadronic model based on non-linear realization of chiral symmetry

Degrees of Freedom **SU(3) multiplets:**

	n (d du)	p (u ud)	
Baryons	Σ^- (s dd)	Σ^0 Λ (s du)	Σ^+ (s uu)
	Ξ^- (ss d)	Ξ^0 (ss u)	}
			hyperons

	κ^0 (\bar{s} d)	κ^+ (\bar{s} u)
Scalar Mesons	δ^- (\bar{u} d)	δ^0, σ, ζ δ^+ (\bar{d} u)
	κ^- (\bar{u} s)	$\bar{\kappa}^0$ (\bar{d} s)

$$\sigma \sim \langle \bar{u} u + \bar{d} d \rangle \quad \zeta \sim \langle \bar{s} s \rangle \quad \delta^0 \sim \langle \bar{u} u - \bar{d} d \rangle$$

	K^{*0} (\bar{s} d)	K^{*+} (\bar{s} u)
Vector Mesons	ρ^- (\bar{u} d)	ρ^0, ω, ϕ ρ^+ (\bar{d} u)
	K^{*-} (\bar{u} s)	\bar{K}^{*0} (\bar{d} s)

plus pseudoscalars, axial vectors and gluonic field χ

construction of the model

A) **SU(3)** interaction

$$\sim \text{Tr} [\bar{B}, M]_{\pm} B \quad , \quad (\text{Tr} \bar{B} B) \text{Tr} M$$

B) meson interactions

$$\sim V(M) \quad \langle \sigma \rangle = \sigma_0 \neq 0 \quad \langle \zeta \rangle = \zeta_0 \neq 0$$

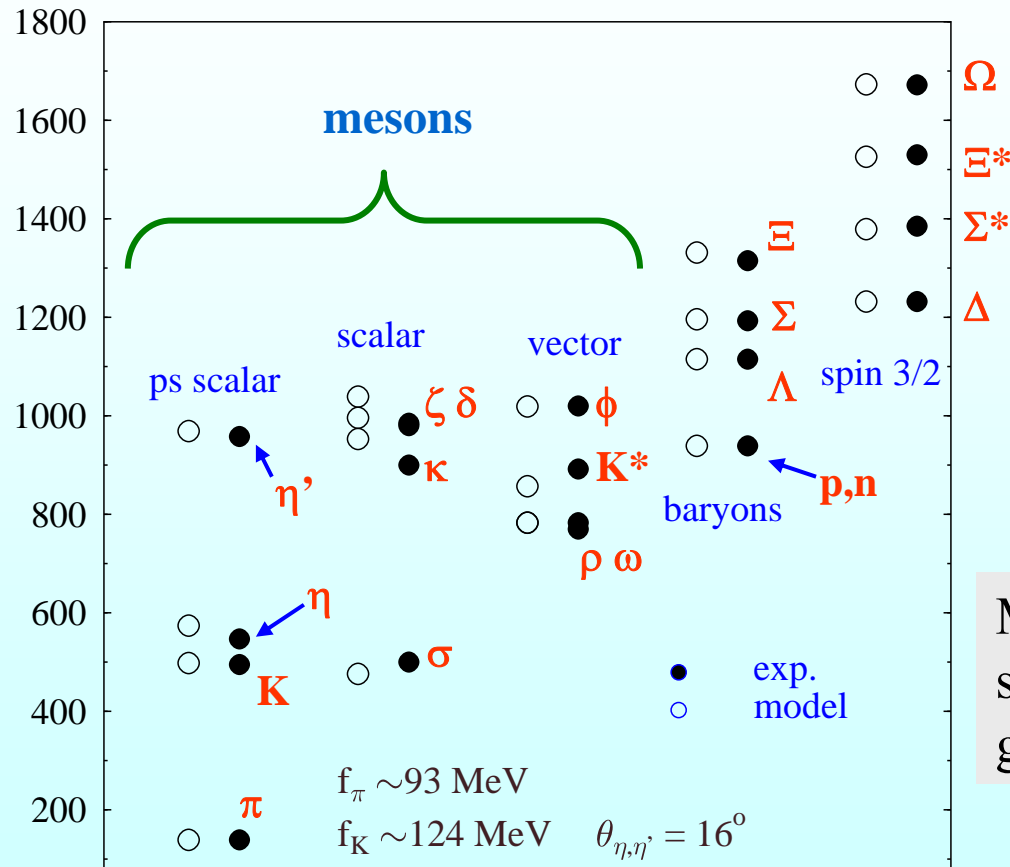
C) chiral symmetry $\Rightarrow m_{\pi} = m_K = 0$

$$\text{explicit breaking} \sim \text{Tr} [c \Sigma] \quad (\cong m_q \bar{q} q)$$

\Rightarrow light pseudoscalars, breaking of SU(3)

fit parameters to hadron masses

Hadron Masses [MeV]



Model can reproduce hadron spectra via dynamical mass generation

Lagrangian (in mean-field approximation)

$$L = L_{BS} + L_{BV} + L_V + L_S + L_{SB}$$

baryon-scalars:

$$L_{BS} = - \sum \bar{B}_i (g_i^\sigma \sigma + g_i^\zeta \zeta + g_i^\delta \delta) B_i$$

baryon-vectors:

$$L_{BV} = - \sum \bar{B}_i (g_i^\omega \phi + g_i^\rho \phi + g_i^\phi \phi) B_i$$

meson interactions:

$$\begin{aligned} L_{BS} = & -k_0/2 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 \\ & + k_2/2 (\sigma^4 + 2\zeta^4 + \delta^4 + 6\sigma^2\delta^2) + k_3 \chi \sigma^2 \zeta \\ & - k_4 \chi^4 - \chi^4 \ln \chi/\chi_0 + \varepsilon \chi^4 \ln [(\sigma^2 - \delta^2) \zeta / (\sigma_0^2 \zeta_0)] \end{aligned}$$

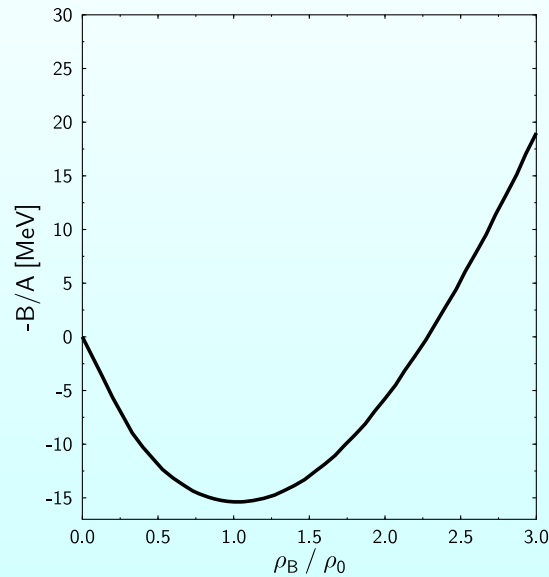
$$L_V = -k'_0/2 \chi^2 (\omega^2 + \rho^2 + \phi^2) + g_4 (\omega^4 + \rho^4 + \phi^4 + 6\omega^2\rho^2)$$

explicit symmetry breaking: $L_{SB} = -(\chi/\chi_0)^2 (c_1 \sigma + c_2 \zeta)$

important reality check

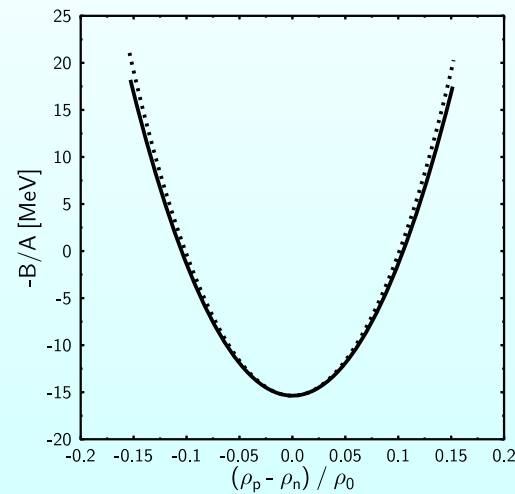
nuclear matter properties at saturation density

equation of state $E/A(\rho)$



asymmetry energy

$E/A(\rho_p - \rho_n)$



binding energy $E/A \sim -15.2$ MeV

saturation $(\rho_B)_0 \sim .16/\text{fm}^3$

compressibility ~ 223 MeV

asymmetry energy ~ 31.9 MeV

phenomenology: 200 - 250 MeV

30 - 35 MeV

**Task: self-consistent relativistic mean-field calculation
coupled 7 meson/photon fields + equations for nucleons in 1 to 3 dimensions**

parameter fit to known nuclear binding energies and hadron masses

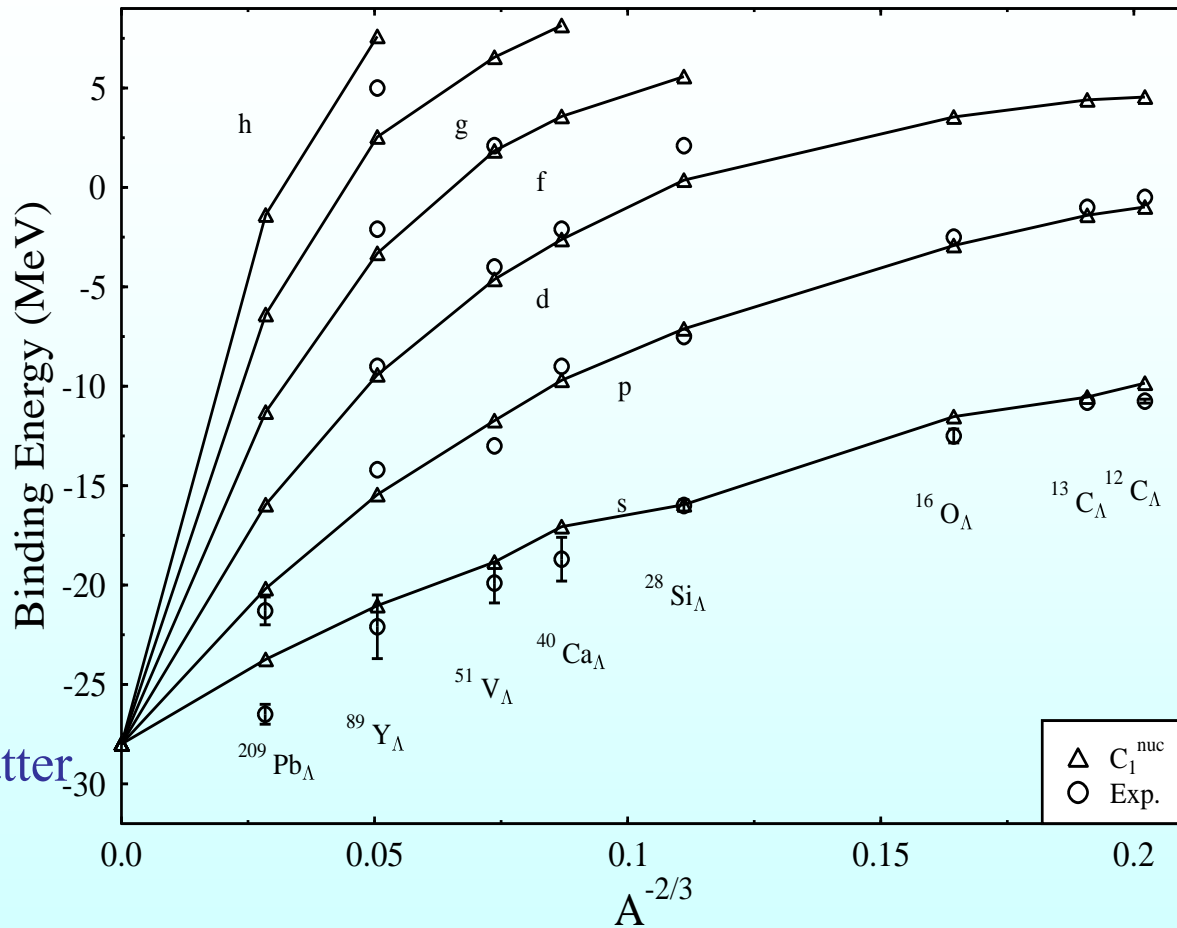
2d calculation of all measured (~ 800) even-even nuclei

error in energy $\varepsilon (A \geq 50) \sim 0.21 \%$ (NL3: 0.25 %)
 $\varepsilon (A \geq 100) \sim 0.14 \%$ (NL3: 0.16 %)

relativistic nuclear
structure models

good charge radii $\delta r_{\text{ch}} \sim 0.5 \%$ (+ LS splittings)

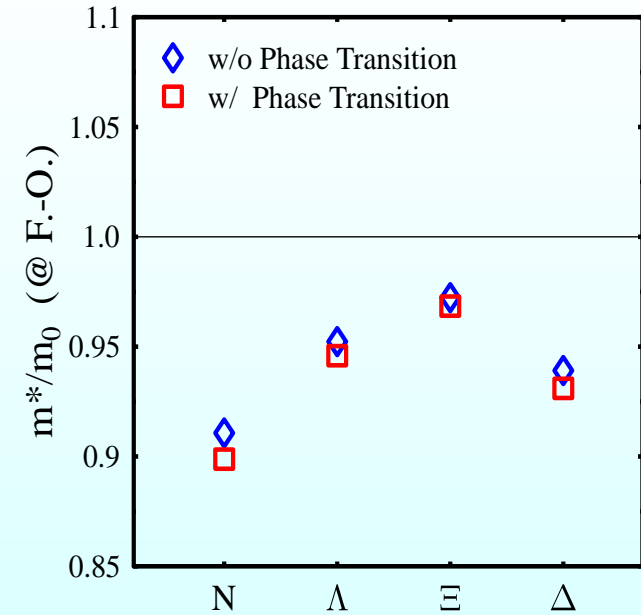
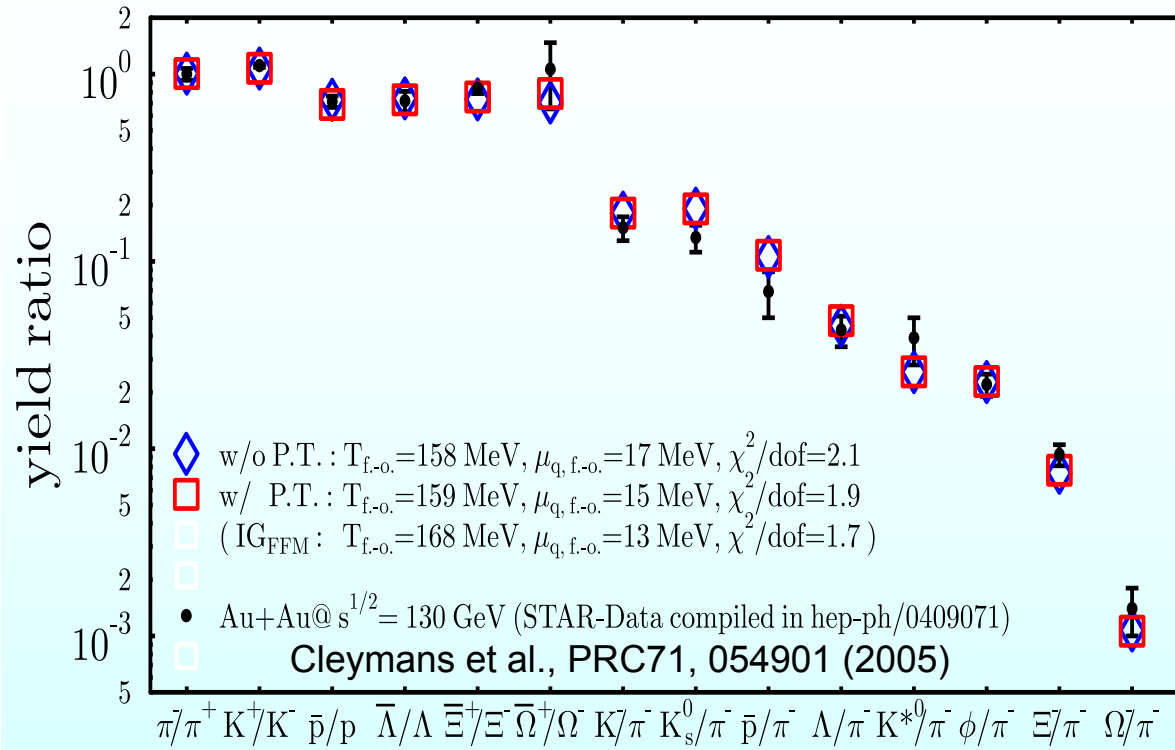
Hypernuclei - Λ single-particle energies



Nuclear matter

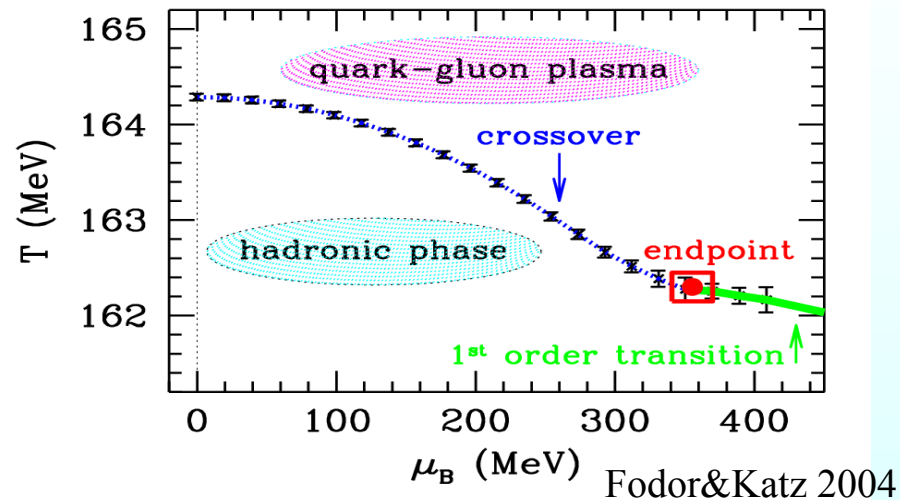
Model and experiment agree well

fitting particle ratios measured at RHIC



	T [MeV]	μ_B [MeV]
	158.0	51.0
ideal gas	168.0	39.0

QCD Phase diagram



Chiral SU(3) model:
only baryon octet included, then the
chiral transition is crossover

But: by including additional d.o.f.,
the transition can become first order

→ models relying exclusively on order
parameter dynamics predict significantly
lower T_c in dense matter (Stephanov 2004)

→ contribution of heavy hadronic states
substantial at high T (Gerber and Leutwyler
1989)

→ Hadron resonance gas gives reasonable
description of lattice thermodynamics
(Karsch, Redlich, Tawfik 2003)

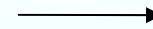
→ m_π dependence of T_c far too strong in chiral
order parameter σ - models (Dumitru, Röder,
Ruppert 2004). Reduced by heavy states
(Gerber and Leutwyler 1989).

Investigate role of heavy hadronic
states on the location of the
chiral critical point in the chiral
SU(3) model

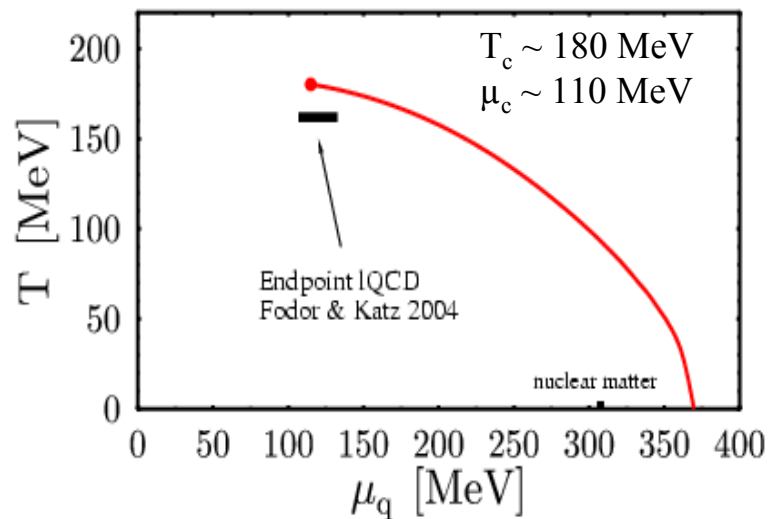
phase transition compared to lattice simulations

heavy states/resonance spectrum is effectively described by single resonance with adjustable scalar and vector couplings

$$m_R = m_0 + g_R \sigma$$



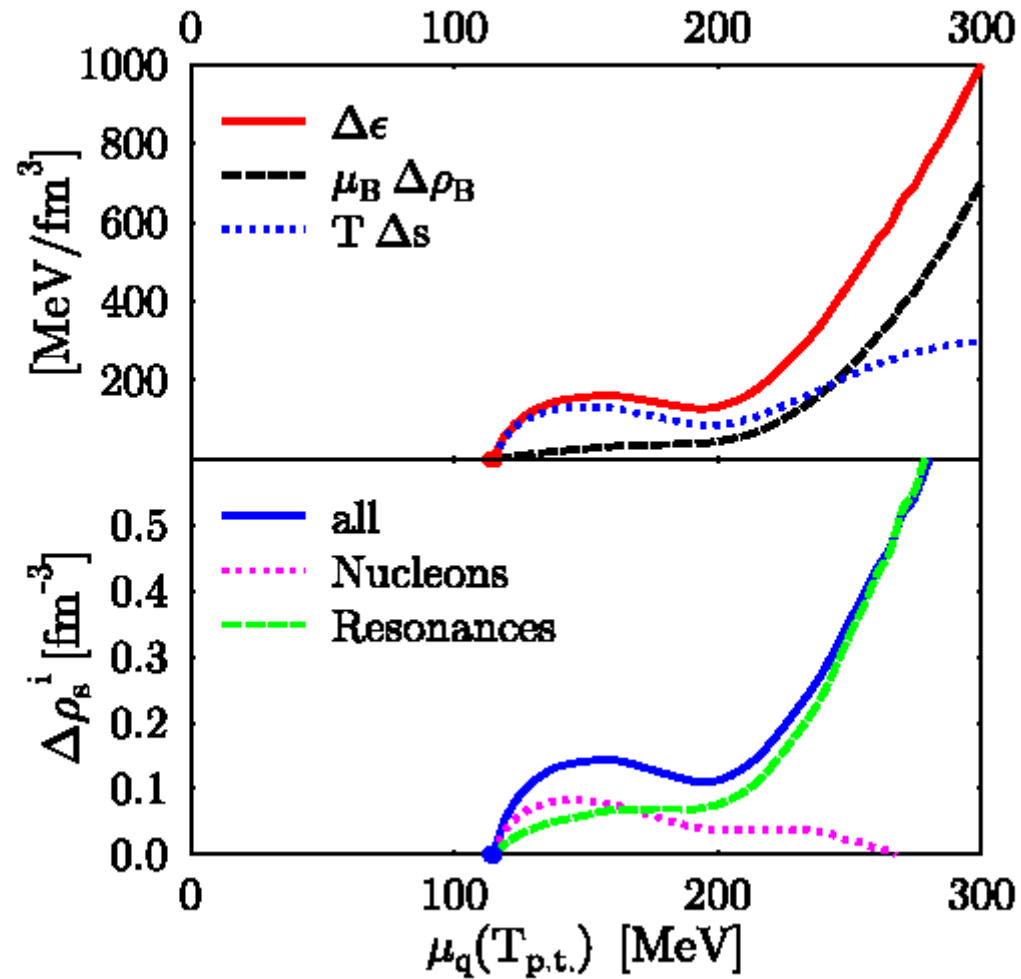
$$g_{R\omega} = r_v g_{N\omega}$$



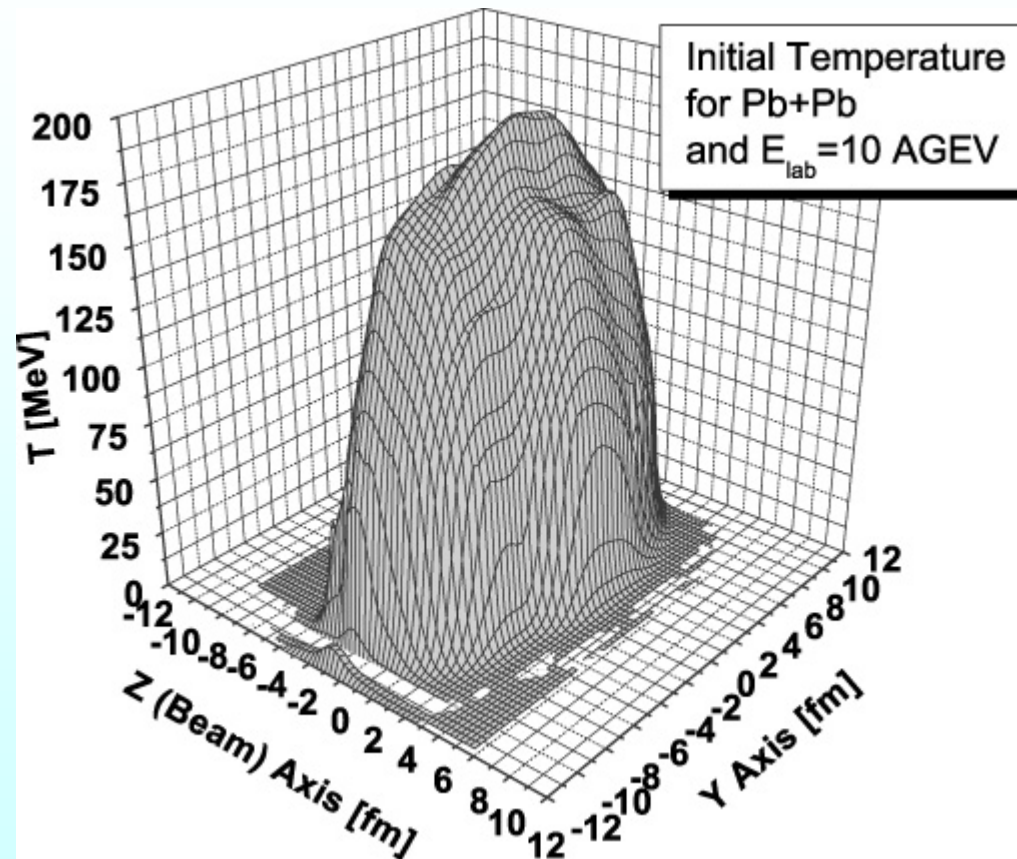
reproduction of LQCD phase diagram, especially T_c, μ_c
+
successful description of nuclear matter saturation

phase transition becomes first-order for degenerate baryon octet $\sim N_f = 3$ with $T_c \sim 185 \text{ MeV}$

latent energy as function of chemical potential

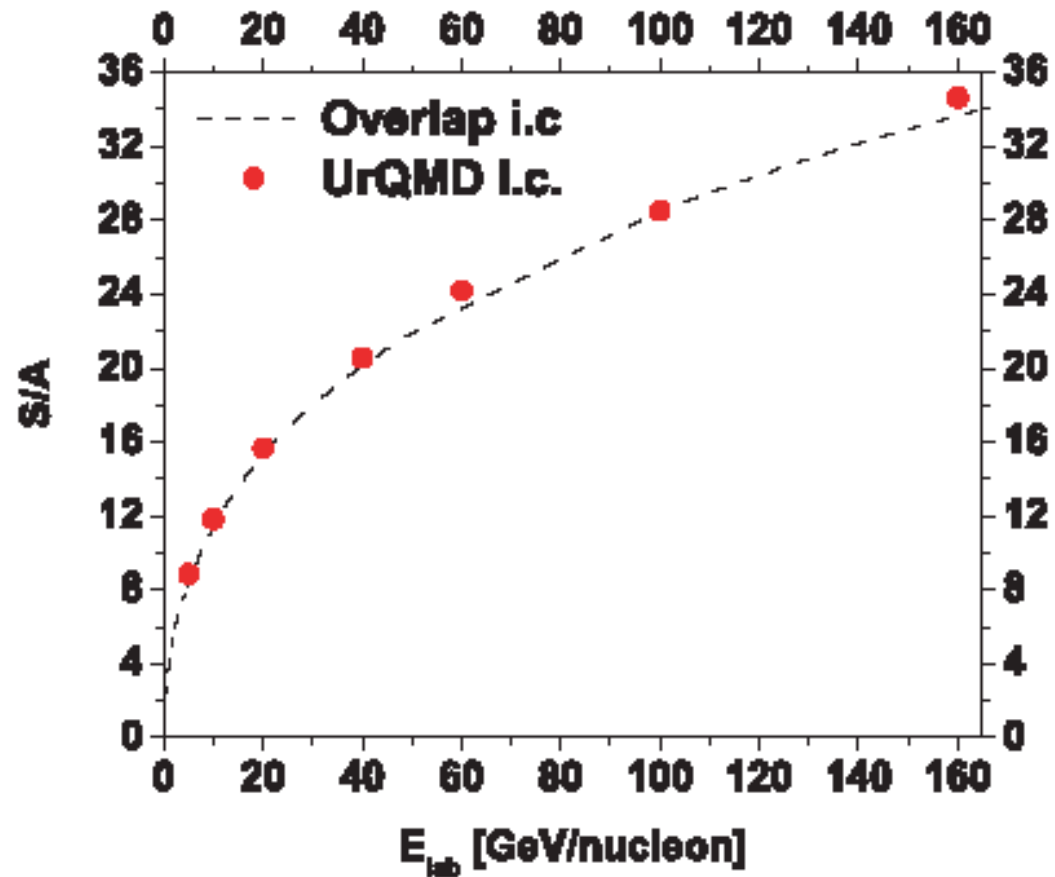


Temperature distribution from UrQMD simulation as initial state for (3d+1) hydro calculation



Use smeared-out particle distributions as starting point for hydro simulation

Comparison of gross properties of initial conditions

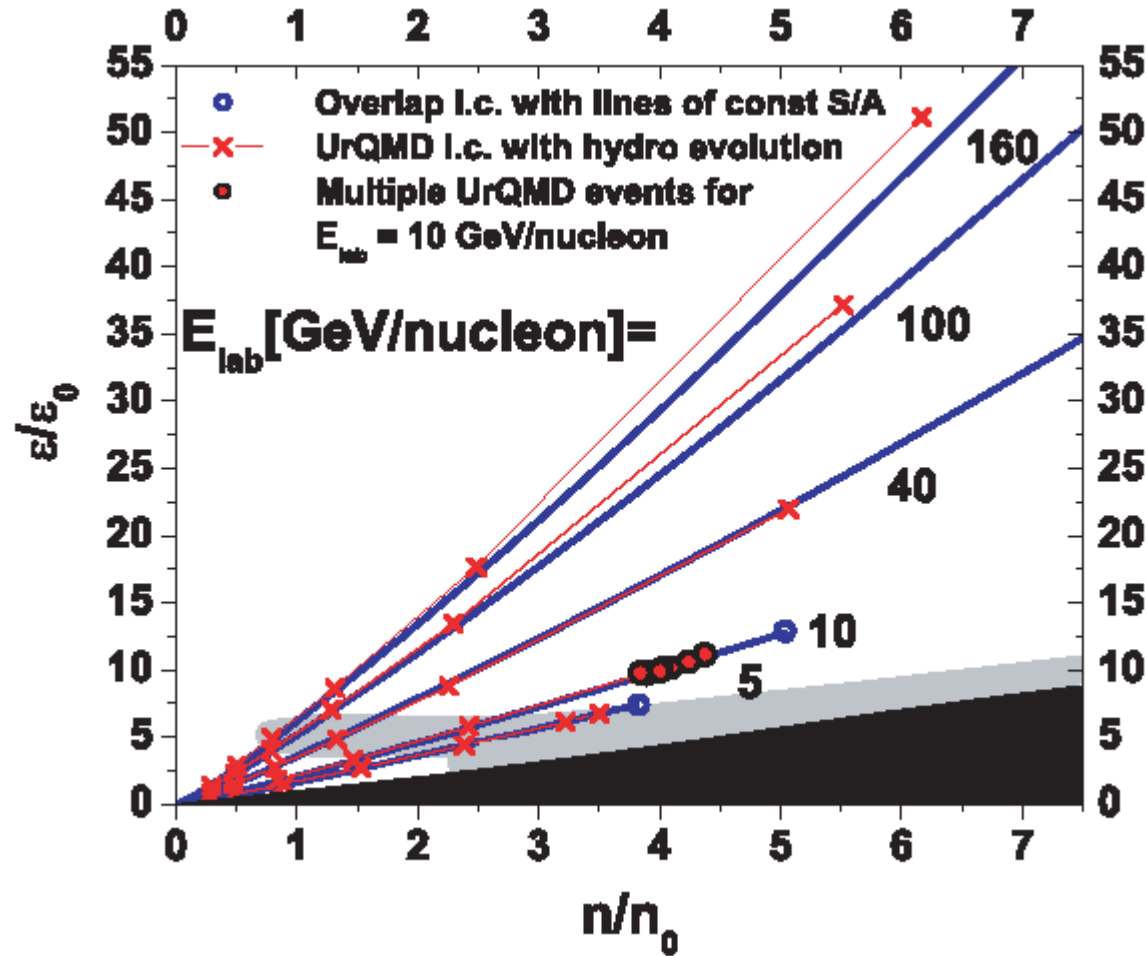


Overlap of projectile and target

$$\rho_B^{\text{initial}} = 2\gamma_{\text{c.m.}}\rho_0,$$

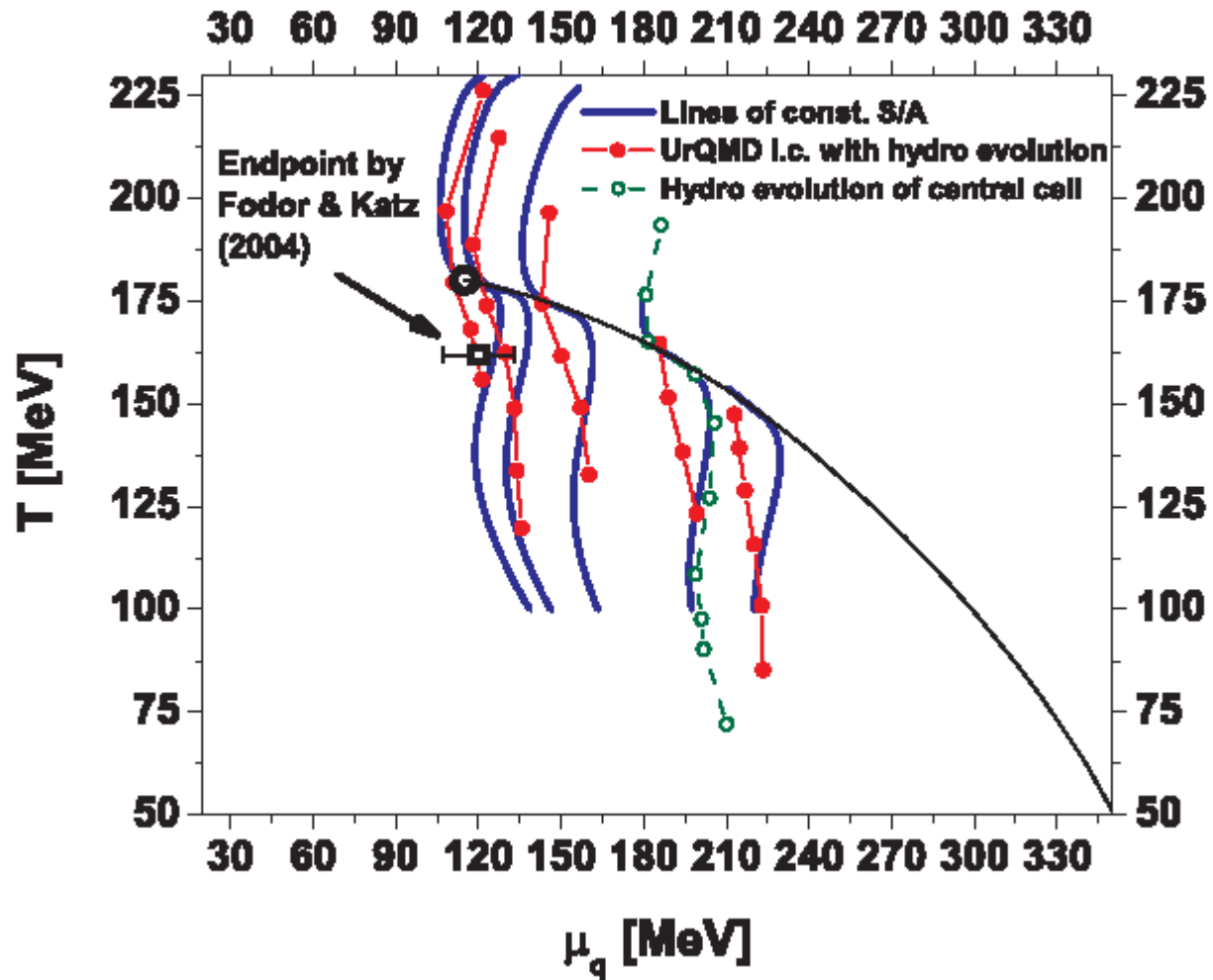
$$\epsilon^{\text{initial}} = \sqrt{s}\rho_0\gamma_{\text{c.m.}}$$

Evolution of the collision system



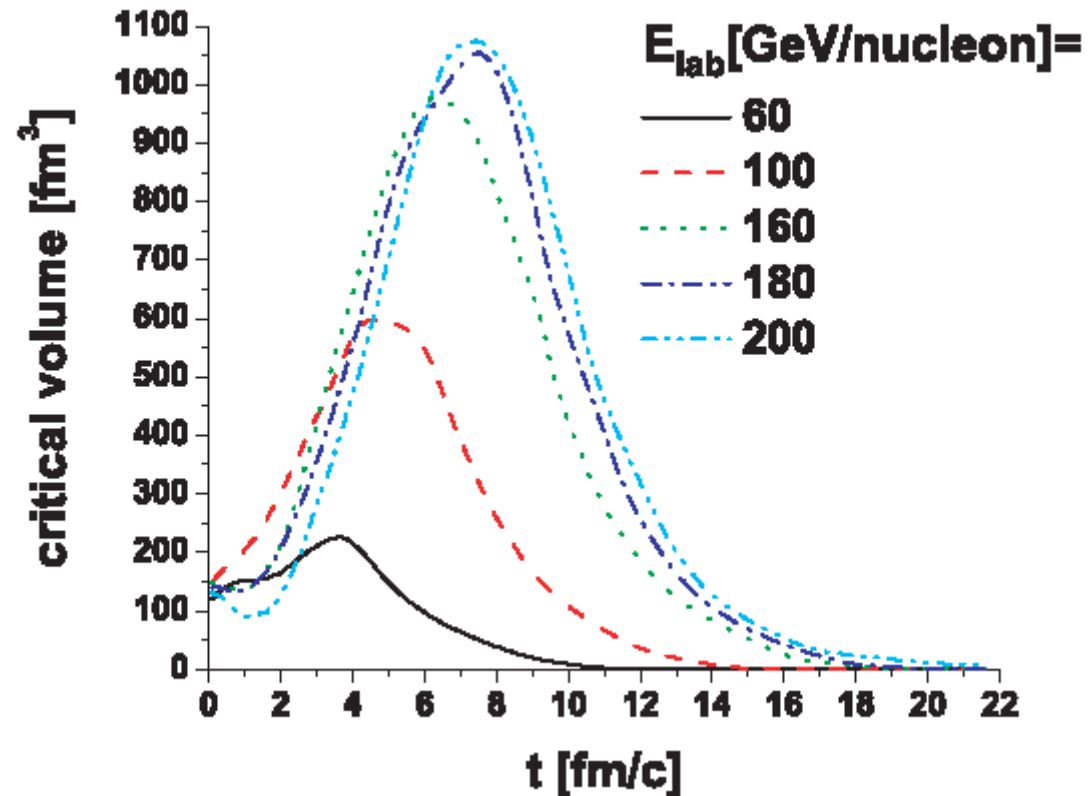
$E_{\text{lab}} \approx 5\text{-}10 \text{ AGeV}$ sufficient to overshoot phase border, $100\text{-}160 \text{ AGeV}$ around endpoint

Isentropes, UrQMD and hydro evolution



lines of constant entropy per baryon, i.e. perfect fluid expansion
 $S/A \approx 34$ goes through endpoint

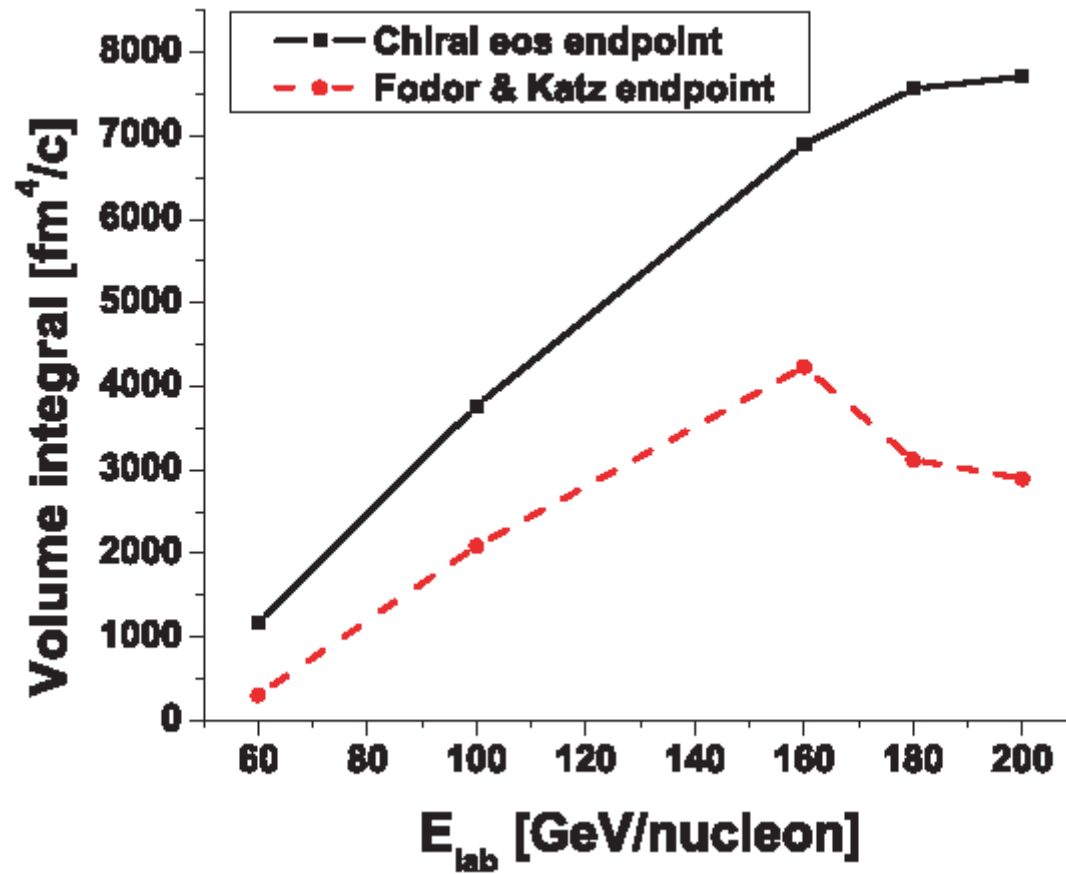
effective volume sampling the critical end point



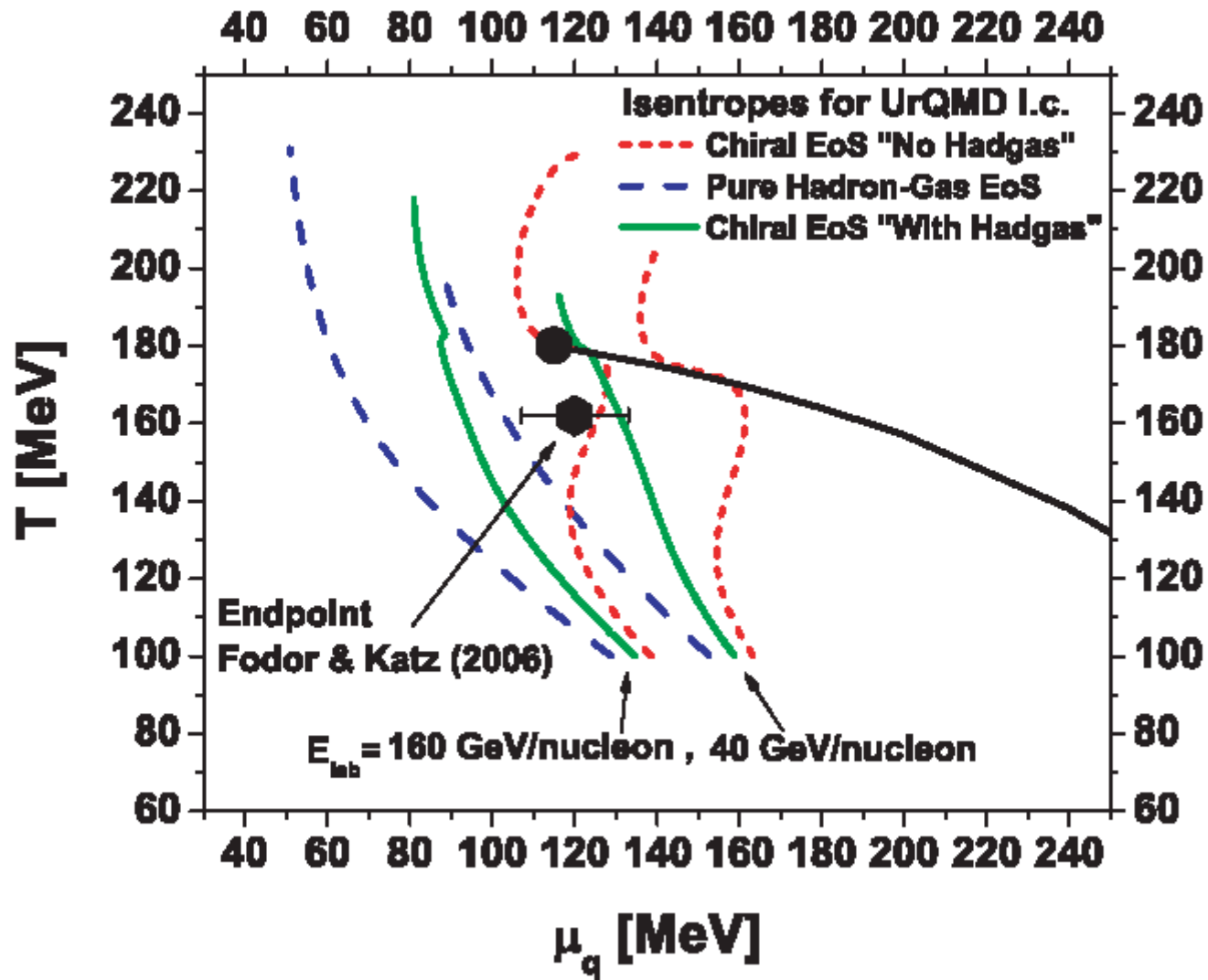
window $T = T_c \pm 10 \text{ MeV}$ $\mu = \mu_c \pm 10 \text{ MeV}$

maximum shifts in time

time integrated volume around the critical end point



Isentropes for different beam energies in the $\mu - T$ diagram



modified PNJL model

$$\Phi = \frac{1}{N_c} \text{Tr}_c L \quad L(\vec{x}) = \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

order parameter of the phase transition

$$\langle \Phi \rangle = 0 \quad \text{confined phase}$$

$$\langle \Phi \rangle \neq 0 \quad \text{deconfined phase}$$

$V(\Phi, \bar{\Phi}; T, \mu)$ effective potential for Polyakov loop

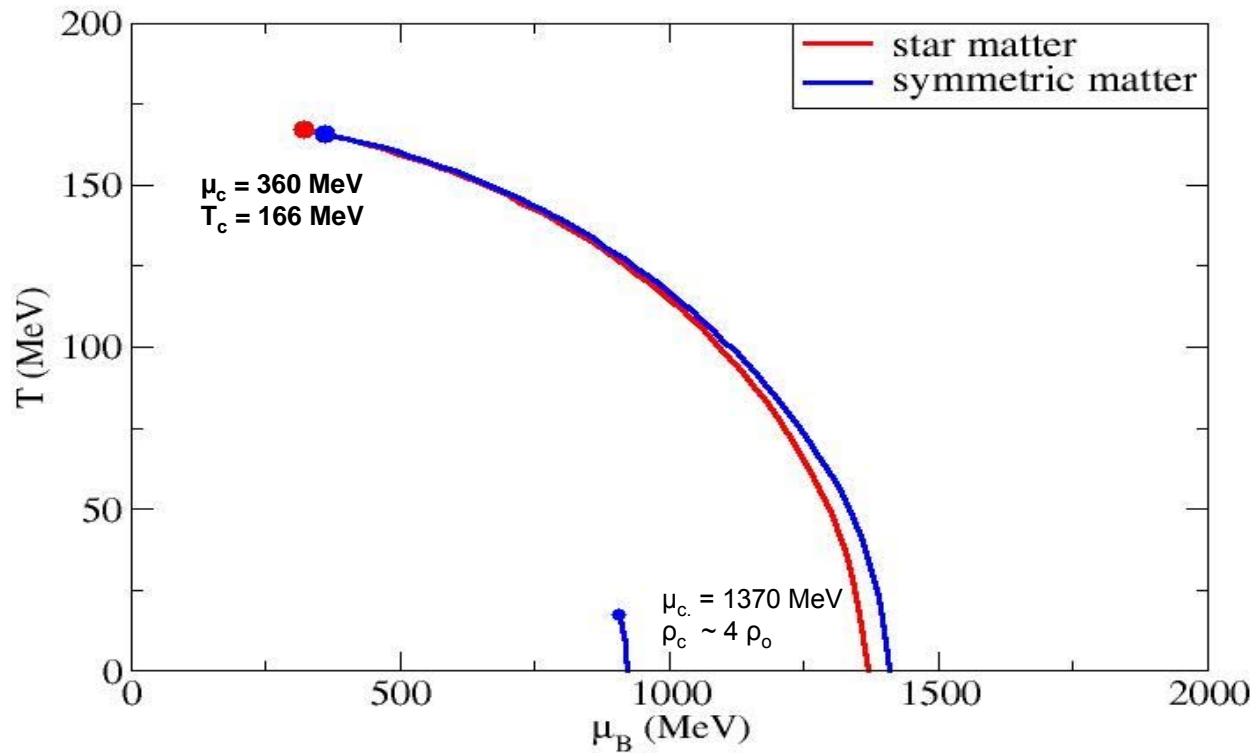
Baryonic and quark mass shift $\delta m_B \sim f(\Phi)$ $\delta m_q \sim f(1-\Phi)$

quarks couple to mean fields via g_σ, g_ω

minimize grand canonical potential

Ratti et al. PRD 73 014019 (2006)
Fukushima, PLB 591, 277 (2004)

Phase Diagram for HQM model

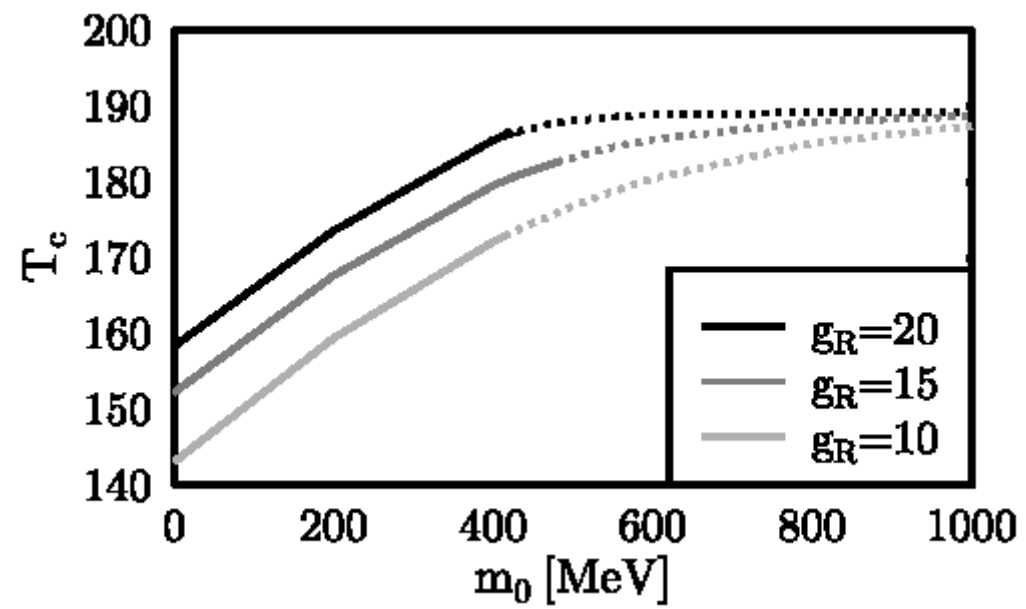


hybrid hadron-quark model - so far SU(2)
critical endpoint tuned to lattice results

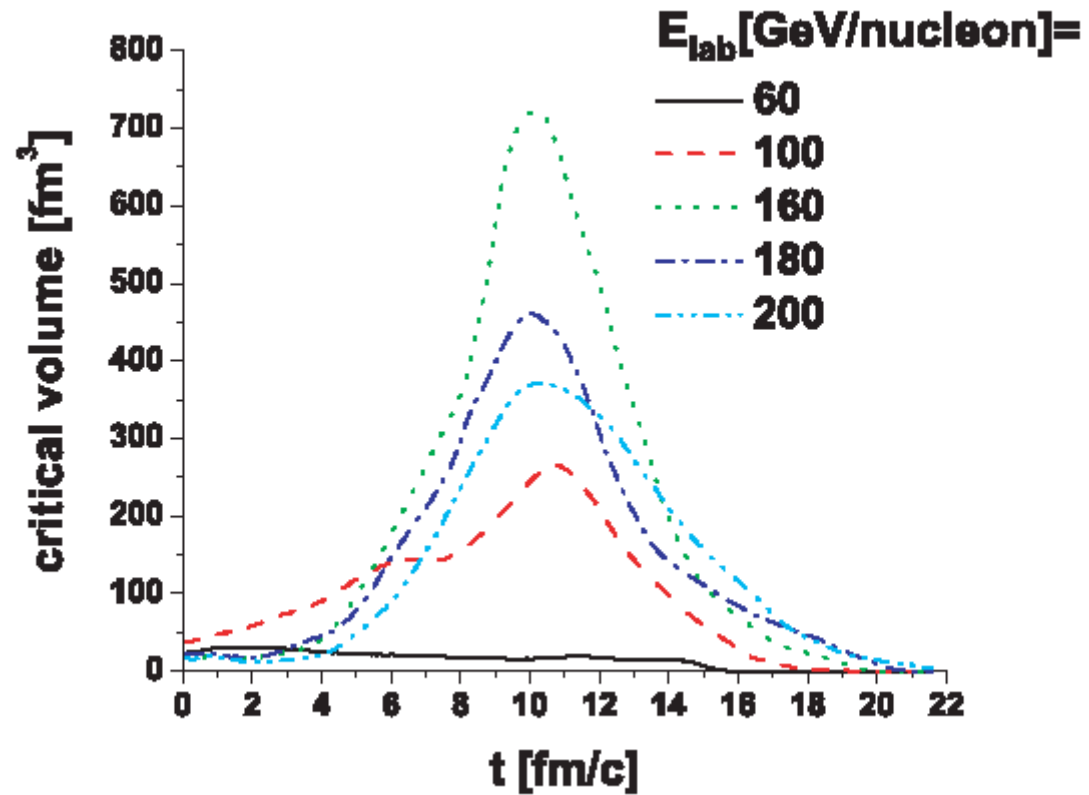
see talk by V. Dexheimer

SUMMARY / OUTLOOK

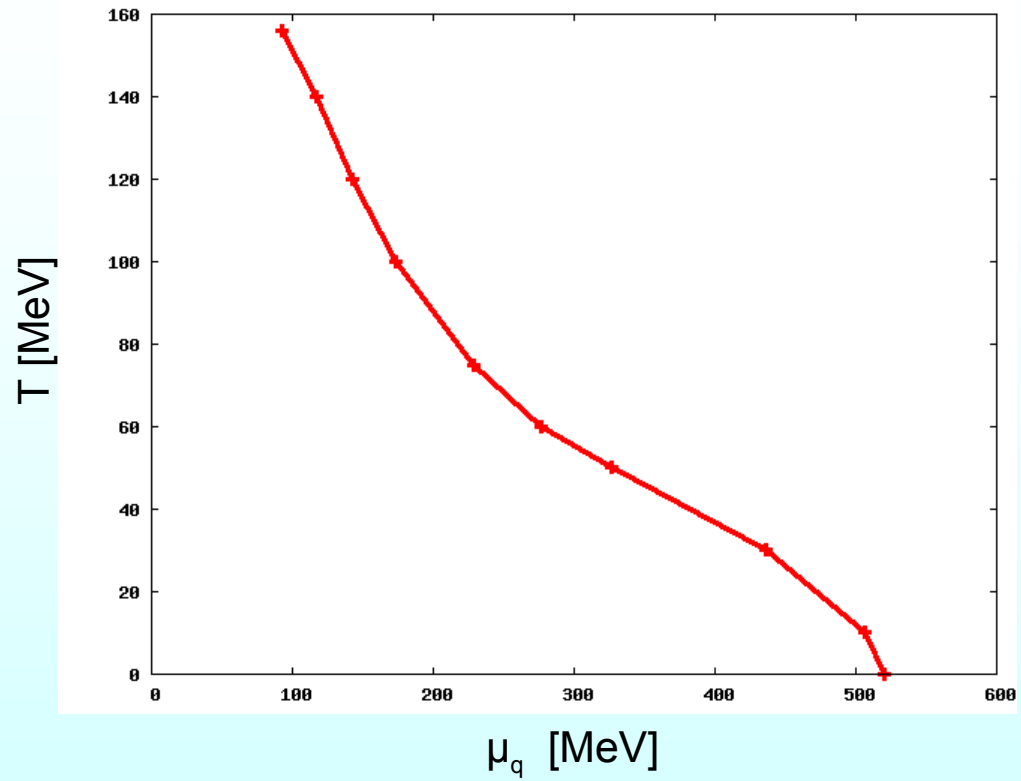
- general hadronic model
 - works well with basic vacuum properties, nuclear matter, nuclei, ...
 - phase diagram with critical end point via resonances
 - implement EOS in combined molecular dynamics/ hydro simulations
 - quarks included essentially following the PNJL spirit
 - realistic phase transition line
-
- more exhaustive study of the model space (Dexheimer)
 - resonances in both phases
 - transport study including freeze-out (Petersen, Steinheimer, in progress)



amount of volume scanning the critical endpoint (lattice)



Line of 1st order phase transition



hybrid hadron-quark model

critical endpoint at $T = 156 \text{ MeV}$, $\mu_q = 92 \text{ MeV}$