Strangeness S=-2 Baryon-Baryon Interaction in Chiral Effective Field Theory

Johann Haidenbauer

IKP, Forschungszentrum GmbH, Jülich, Germany

PANIC 2008, November 9-14, 2008, Eilat

Why are the YN and YY interactions interesting?

- study the role of strangeness in low and medium energy nuclear physics
- test $SU(3)_{flavor}$ symmetry
- input for studies of hypernuclei

YN (S=-1) data

- about 35 data points, all from the 1960s
- 10 new data points, from the KEK-PS E251 collaboration (from $\approx$ 2000)
(cf. $>$ 4000 NN data for $E_{lab} < 350$ MeV!)

YY sector

Strangeness S=-2 channels:

$$
\begin{aligned}
Q &= +2: \ \Sigma^+\Sigma^+ \\
Q &= +1: \ \Xi^0 p,\ \Sigma^+\Lambda,\ \Sigma^0\Sigma^+ \\
Q &= \ \ \ 0: \ \Lambda\Lambda,\ \Xi^0 n,\ \Xi^- p,\ \Sigma^0\Lambda,\ \Sigma^0\Sigma^0,\ \Sigma^-\Sigma^+ \\
Q &= -1: \ \Xi^- n,\ \Sigma^-\Lambda,\ \Sigma^-\Sigma^0 \\
Q &= -2: \ \Sigma^-\Sigma^-
\end{aligned}
$$

There is some experimental information on the $Q = 0$ channel:

- $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He}) - 2B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He}) = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV

  (H. Takahashi et al., Phys. Rev. Lett. 87 (2001) 212502)

- $\Xi^- p$ scattering cross section at $p_{lab} = 500$ MeV/c

  (J.K. Ahn et al., Phys. Lett. B 633 (2006) 214)

YN, YY in chiral effective field theory

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor
practically no information on YY

few investigations so far (for YN only):

- C.K. Korpa et al., PRC 65 (2001) 015208
- S.R. Beane et al., NPA747 (2005) 55

pion-less theory; Kaplan-Savage-Wise resummation scheme

We follow the scheme of E. Epelbaum & Ulf-G. Meißner

(E. Epelbaum, W. Glöckle, Ulf-G. Meißner, NPA747 (2005) 362)

Power counting

$$V_{\rm eff} \equiv V_{\rm eff}(Q, g, \mu) = \sum_{\nu} Q^{\nu}\, \mathcal{V}_{\nu}(Q/\mu, g)$$

- Q ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- g ... pertinent low–energy constants
- μ ... regularization scale
- $\mathcal{V}_{\nu}$ ... function of order one
- $\nu \geq 0$ ... chiral power

Lowest order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

Leading order (LO) contact term for NN

The LO contact term for the NN interaction:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right)$$

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

Considering the large components of the nucleon spinors only, the LO contact term becomes

$$\mathcal{L} = -\frac{1}{2}C_S\left(\varphi_N^\dagger\varphi_N\right)\left(\varphi_N^\dagger\varphi_N\right) - \frac{1}{2}C_T\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)$$

The LO contact term potential resulting from the interaction Lagrangian:

$$V^{NN\rightarrow NN} = C_S + C_T\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

C_S and C_T ... low-energy constants; to be determined in a fit to the experimental data.

Leading order contact terms for YN and YY

$$\mathcal{L}^1 = \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_a \left(\Gamma_i B\right)_b \right\rangle \ , \ldots \ \mathcal{L}^9 = \tilde{C}_i^9 \left\langle \bar{B}_a \bar{B}_b \right\rangle \left\langle \left(\Gamma_i B\right)_a \left(\Gamma_i B\right)_b \right\rangle$$

a, b denote the Dirac indices of the particles

B is the usual irreducible octet representation of $\mathrm{SU(3)}_f$:

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}^2 = \ & \frac{C_i^2}{2} \left\{ \frac{1}{3} \left(\bar{\Lambda}\Gamma_i\Lambda\right) \left(\bar{N}\Gamma_i N\right) + \left[\left(\bar{\boldsymbol{\Sigma}} \cdot \Gamma_i \boldsymbol{\Sigma}\right) \left(\bar{N}\Gamma_i N\right) \right. \right. \\ & \left. \left. + i \left(\bar{\boldsymbol{\Sigma}} \times \Gamma_i \boldsymbol{\Sigma}\right) \cdot \left(\bar{N}\boldsymbol{\tau}\Gamma_i N\right) \right] - \frac{1}{\sqrt{3}} \left[\left(\bar{N}\boldsymbol{\tau}\Gamma_i N\right) \cdot \left(\bar{\Lambda}\Gamma_i \boldsymbol{\Sigma}\right) + H.c. \right] \right\} , \end{aligned}$$

...

Leading order contact terms for YN and YY

The LO contact term potential resulting from the interaction Lagrangian:

$$V^{BB\to BB} = C_S^{BB\to BB} + C_T^{BB\to BB}\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

There are six contact constants (C_S^2, C_T^2, C_S^5, C_T^5, C_S^7 and C_T^7) for the BB interactions!

For fitting to the data it is convenient to consider certain linear combinations of the C_i's so that

$V_{1S0}^{\Lambda N\to\Lambda N} = C_{1S0}^{\Lambda N\to\Lambda N}$, etc.

For an overview it is better to consider other linear combinations, namely the $SU(3)_f$ irreducible representations

YY contact terms

BB contact interactions in terms of $SU(3)_f$ irreducible representations

	Channel	Isospin	$V_{^3S1}$
$S = \;\;0$	$NN \rightarrow NN$	1	V^{10^*}
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{2}\left(V^{8_a} + V^{10^*}\right)$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(-V^{8_a} + V^{10^*}\right)$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(V^{8_a} + V^{10^*}\right)$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	V^{10}
$S = -2$	$\Xi N \rightarrow \Xi N$	0	V^{8_a}
	$\Xi N \rightarrow \Xi N$	1	$\frac{1}{3}\left(V^{10} + V^{10^*} + V^{8_a}\right)$
	$\Xi N \rightarrow \Sigma\Lambda$	1	$\frac{\sqrt{6}}{6}\left(V^{10} - V^{10^*}\right)$
	$\Xi N \rightarrow \Sigma\Sigma$	1	$\frac{\sqrt{2}}{6}\left(V^{10} + V^{10^*} - 2V^{8_a}\right)$
	$\Sigma\Lambda \rightarrow \Sigma\Lambda$	1	$\frac{1}{2}\left(V^{10} + V^{10^*}\right)$
	$\Sigma\Lambda \rightarrow \Sigma\Sigma$	1	$\frac{\sqrt{3}}{6}\left(V^{10} - V^{10^*}\right)$
	$\Sigma\Sigma \rightarrow \Sigma\Sigma$	1	$\frac{1}{6}\left(V^{10} + V^{10^*} + 4V^{8_a}\right)$

YY contact terms

	Channel	Isospin	$V_{^1S0}$
$S = 0$	$NN \to NN$	1	V^{27}
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9V^{27} + V^{8_s}\right)$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-V^{27} + V^{8_s}\right)$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(V^{27} + 9V^{8_s}\right)$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	V^{27}
$S = -2$	$\Lambda\Lambda \to \Lambda\Lambda$	0	$\frac{1}{40}\left(27V^{27} + 8V^{8_s} + 5V^{1}\right)$
	$\Lambda\Lambda \to \Xi N$	0	$\frac{-1}{40}\left(18V^{27} - 8V^{8_s} - 10V^{1}\right)$
	$\Lambda\Lambda \to \Sigma\Sigma$	0	$\frac{\sqrt{3}}{40}\left(-3V^{27} + 8V^{8_s} - 5V^{1}\right)$
	$\Xi N \to \Xi N$	0	$\frac{1}{40}\left(12V^{27} + 8V^{8_s} + 20V^{1}\right)$
	$\Xi N \to \Sigma\Sigma$	0	$\frac{\sqrt{3}}{40}\left(2V^{27} + 8V^{8_s} - 10V^{1}\right)$
	$\Sigma\Sigma \to \Sigma\Sigma$	0	$\frac{1}{40}\left(V^{27} + 24V^{8_s} + 15V^{1}\right)$
	$\Xi N \to \Xi N$	1	$\frac{1}{5}\left(2V^{27} + 3V^{8_s}\right)$
	$\Xi N \to \Sigma\Lambda$	1	$\frac{\sqrt{6}}{5}\left(V^{27} - V^{8_s}\right)$
	$\Sigma\Lambda \to \Sigma\Lambda$	1	$\frac{1}{5}\left(3V^{27} + 2V^{8_s}\right)$
	$\Sigma\Sigma \to \Sigma\Sigma$	2	V^{27}

five contact terms for the $S = -1$ channels

one additional contact term (V^1) for the $I = 0$, $S = -2$ channels

One pseudoscalar-meson exchange

LO $\mathrm{SU(3)}_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right\rangle$$

g_A is the axial-vector strength: $g_A \simeq 1.26$
F_π is the weak pion decay constant: $F_\pi = 92.4$ MeV
$\alpha = F/(F+D)$ and $g_A = F + D$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

One pseudoscalar-meson exchange

$$
\begin{aligned}
\mathcal{L} \quad = \quad & -f_{NN\pi}\bar{N}\gamma^{\mu}\gamma_5\boldsymbol{\tau}N\cdot\partial_{\mu}\boldsymbol{\pi} + if_{\Sigma\Sigma\pi}\bar{\boldsymbol{\Sigma}}\gamma^{\mu}\gamma_5\times\boldsymbol{\Sigma}\cdot\partial_{\mu}\boldsymbol{\pi} \\
& -f_{\Lambda\Sigma\pi}\left[\bar{\Lambda}\gamma^{\mu}\gamma_5\boldsymbol{\Sigma}+\bar{\boldsymbol{\Sigma}}\gamma^{\mu}\gamma_5\Lambda\right]\cdot\partial_{\mu}\boldsymbol{\pi} - f_{\Xi\Xi\pi}\bar{\Xi}\gamma^{\mu}\gamma_5\boldsymbol{\tau}\Xi\cdot\partial_{\mu}\boldsymbol{\pi} \\
& -f_{\Lambda NK}\left[\bar{N}\gamma^{\mu}\gamma_5\Lambda\partial_{\mu}K+\bar{\Lambda}\gamma^{\mu}\gamma_5 N\partial_{\mu}K^{\dagger}\right] \\
& -f_{\Xi\Lambda K}\left[\bar{\Xi}\gamma^{\mu}\gamma_5\Lambda\partial_{\mu}K_c+\bar{\Lambda}\gamma^{\mu}\gamma_5\Xi\partial_{\mu}K_c^{\dagger}\right] \\
& -f_{\Sigma NK}\left[\bar{\boldsymbol{\Sigma}}\cdot\gamma^{\mu}\gamma_5\partial_{\mu}K^{\dagger}\boldsymbol{\tau}N+\bar{N}\gamma^{\mu}\gamma_5\boldsymbol{\tau}\partial_{\mu}K\cdot\boldsymbol{\Sigma}\right] \\
& -f_{\Sigma\Xi K}\left[\bar{\boldsymbol{\Sigma}}\cdot\gamma^{\mu}\gamma_5\partial_{\mu}K_c^{\dagger}\boldsymbol{\tau}\Xi+\bar{\Xi}\gamma^{\mu}\gamma_5\boldsymbol{\tau}\partial_{\mu}K_c\cdot\boldsymbol{\Sigma}\right] - f_{NN\eta_8}\bar{N}\gamma^{\mu}\gamma_5 N\partial_{\mu}\eta \\
& -f_{\Lambda\Lambda\eta_8}\bar{\Lambda}\gamma^{\mu}\gamma_5\Lambda\partial_{\mu}\eta - f_{\Sigma\Sigma\eta_8}\bar{\boldsymbol{\Sigma}}\cdot\gamma^{\mu}\gamma_5\boldsymbol{\Sigma}\partial_{\mu}\eta - f_{\Xi\Xi\eta_8}\bar{\Xi}\gamma^{\mu}\gamma_5\Xi\partial_{\mu}\eta
\end{aligned}
$$

$$
\begin{array}{llllll}
f_{NN\pi} = & f, & f_{NN\eta_8} = & \frac{1}{\sqrt{3}}(4\alpha-1)f, & f_{\Lambda NK} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f, \\
f_{\Xi\Xi\pi} = & -(1-2\alpha)f, & f_{\Xi\Xi\eta_8} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f, & f_{\Xi\Lambda K} = & \frac{1}{\sqrt{3}}(4\alpha-1)f, \\
f_{\Lambda\Sigma\pi} = & \frac{2}{\sqrt{3}}(1-\alpha)f, & f_{\Sigma\Sigma\eta_8} = & \frac{2}{\sqrt{3}}(1-\alpha)f, & f_{\Sigma NK} = & (1-2\alpha)f, \\
f_{\Sigma\Sigma\pi} = & 2\alpha f, & f_{\Lambda\Lambda\eta_8} = & -\frac{2}{\sqrt{3}}(1-\alpha)f, & f_{\Xi\Sigma K} = & -f.
\end{array}
$$

One pseudoscalar-meson exchange

$$V^{B_1 B_2 \to B'_1 B'_2} = -f_{B_1 B'_1 P} f_{B_2 B'_2 P} \frac{(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})}{\mathbf{k}^2 + m_P^2}$$

$f_{B_1 B'_1 P}$ ... coupling constants;
m_P ... mass of the exchanged pseudoscalar meson.

- SU(3) breaking due to the mass splitting of the ps mesons (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV) is taken into account.

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244)
(H. Polinder, J.H., U.-G. Meißner, PLB 653 (2007) 29)

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T_{\rho'\rho}{}^{\nu'\nu,J}(p',p) &= V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \\
&+ \sum_{\rho'',\nu''} \int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V_{\rho'\rho''}{}^{\nu'\nu'',J}(p',p'')\, \frac{2\mu_{\nu''}}{p^2 - p''^2 + i\eta}\, T_{\rho''\rho}{}^{\nu''\nu,J}(p'',p)
\end{aligned}
$$

ρ', ρ = ΛN, ΣN
= $\Lambda\Lambda$, $\Sigma\Sigma$, ΞN, etc.

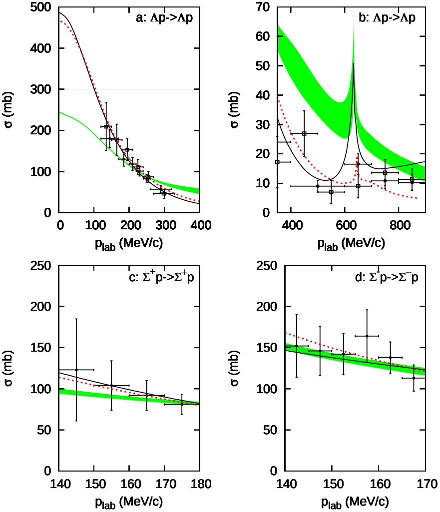
LS equation is solved for particle channels (in momentum space)

The potential in the LS equation is cut off with the regulator function:

$$
f^\Lambda(p',p) = e^{-\left(p'^4+p^4\right)/\Lambda^4}
$$

consider values Λ = 550 - 700 MeV

YN integrated cross sections



EFT '06

(NPA 779 (2006) 244)

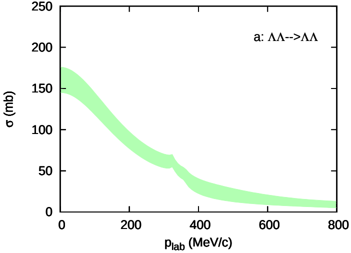
Jülich '04

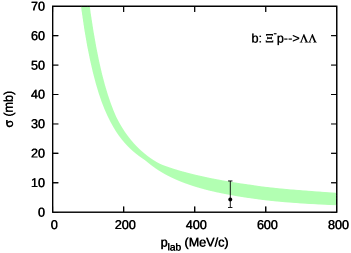
(PRC 72 (2005) 044005)

Nijmegen NSC97f

(PRC 59 (1999) 21)

YY integrated cross sections





cut off Λ= 600 MeV

$a^{\Lambda\Lambda}_{^1S_0}$; $r^{\Lambda\Lambda}_{^1S_0}$ [in fm]

-1.52; 0.59 EFT

(PLB 653 (2007) 29)

-1.32; 4.40 ESC04d (Rijken)

(PRC 73 (2006) 044008)

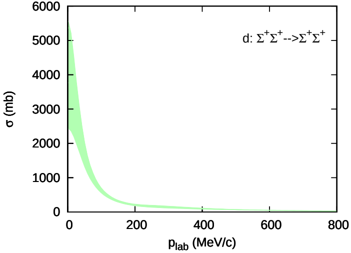
-0.81; 3.80 fss2 (Y. Fujiwara)

(Prog. Part. Nucl. Phys. 58 (2007) 439)

$\sigma_{exp} = 4.3^{+6.3}_{-2.7}$ mb
at p_{lab} = 500 MeV/c

J.K. Ahn et al., 2006

YY integrated cross sections

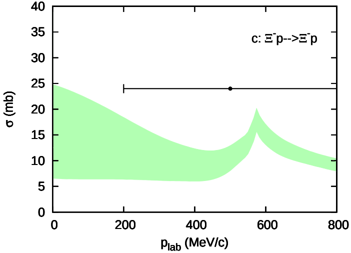


cut off Λ= 600 MeV

$a^{\Sigma^+\Sigma^+}_{^1S_0}$; $r^{\Sigma^+\Sigma^+}_{^1S_0}$ [in fm]

-7.76; 2.00 EFT

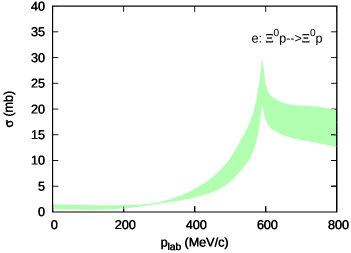
-63.7; 2.37 fss2 (Y. Fujiwara)

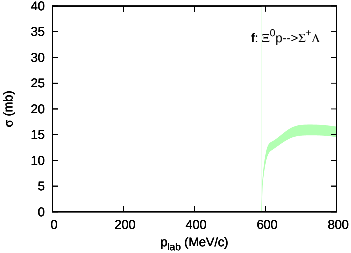


$\sigma^{exp} \leq 24$ mb

J.K. Ahn et al., 2006

YY integrated cross sections





cut off Λ= 600 MeV

$a^{\Xi^0 p}_{^1S_0}$; $r^{\Xi^0 p}_{^1S_0}$ [in fm]

0.19; -37.7 EFT

0.14; 4.67 ESC04d (Rijken)

0.33; -9.19 fss2 (Y. Fujiwara)

$a^{\Xi^0 p}_{^3S_1}$; $r^{\Xi^0 p}_{^3S_1}$ [in fm]

-0.003; – EFT

-0.203; 27.5 fss2 (Y. Fujiwara)

Summary

YN and YY interactions based on *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- LO potential (contact terms, one-pseudoscalar-meson exchange) is derived imposing $SU(3)$ constraints

- Good description of the empirical YN data was achieved (with only 5 free parameters!)
- Results compatible with the sparse empirical information on the YY interaction (1 free parameter)