High energy QCD: moving from leading to next-to-leading orded

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Perturbation Theory, DIS, Partons, DGLAP, Hard phenomena: jets

Perturbative QGP, Quark Gluon Plasma

$\nwarrow \; \mathbf{Q^2 \to \infty}$

$\nearrow \; \mathbf{T \to \infty}$

QCD

$\swarrow \; \mathbf{m \to 0}$

$\searrow \; \mathbf{s \to \infty}$

χ **– Lagrangians**

RFT: All Collider experiments, Cosmic rays, ultra relativistic neutrinos

Confinement, String Theory

Open questions/Goals

- **What is the high energy limit of QCD?**
  many candidates: BFKL Pomeron Calculus, Lipatov's effective action, elements of Field Theory of Bartels, JIMWLK+KLWMIJ Hamiltonians for Wilson line operators....

- **How does the unitarity of QCD get manifested in high energy scattering amplitudes? Is it possible to rigorously derive an effective theory of QCD in terms of color singlet exchange amplitudes?**

- **How do gluon densities grow with energy? Do they saturate? Scales?**

- **What are applicability limits of factorization theorems?**

- **What are final states in collisions of dense objects (jets, multiplicities, correlations)?**

- **How to get thermalization in high energy collisions of very dense objects (nuclei)?**

High Energy Scattering

Target (ρ^t) **Projectile (ρ^p)**

$$\langle \mathbf{T} | \quad \rightarrow \qquad\qquad \leftarrow \quad | \mathbf{P} \rangle$$

S-matrix:

$$\mathbf{S}(\mathbf{Y}) \;=\; \langle \mathbf{T} \langle \mathbf{P} | \; \hat{\mathbf{S}}(\rho^{\mathrm{t}}, \, \rho^{\mathrm{p}}) \; | \mathbf{P} \rangle \, \mathbf{T} \rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}}, \, \rho^{\mathrm{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T} \langle \mathbf{P} | \; \hat{\mathcal{O}}(\rho^{\mathrm{t}}, \, \rho^{\mathrm{p}}) \; | \mathbf{P} \rangle \, \mathbf{T} \rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged $\mathbf{S}$-matrix:

$$\mathbf{\Sigma(Y) \equiv \langle P| \hat{S}(\rho^t, \rho^p) |P\rangle}$$

evolve with rapidity as $\mathbf{H} \rightarrow$ the HE effective Hamiltonian

$$\mathbf{\Sigma(Y + \delta Y) = e^{-\delta Y\, H}\, \Sigma(Y)}$$

$\mathbf{H}$ defines the high energy limit of QCD and is universal

Spectrum of $\mathbf{H}$ defines energy dependence of All observables.

$$\mathbf{H = H^{LO}(\alpha_s) + H^{NLO}(\alpha_s^2) + \ldots;} \qquad \mathbf{H = H[\rho^t, \delta/\delta\rho^t]}$$

Charge densities ρ are important parameters (define dilute or dense limits)

QCD

QCD Lagrangian

$$\mathcal{L} \; = \frac{1}{4}\,\mathbf{G}_{\mu\nu}\,\mathbf{G}^{\mu\nu} \; + \; \overline{\psi}\,(\mathbf{i}\;\not{\partial}\; - \;\mathbf{g}\,\not{\mathbf{A}} - \mathbf{m})\;\psi$$

The field strength

$$\mathbf{G}_{\mathbf{a}}^{\mu\nu} \; = \; \partial^{\mu}\,\mathbf{A}_{\mathbf{a}}^{\nu} \; - \; \partial^{\nu}\,\mathbf{A}_{\mathbf{a}}^{\mu} \; - \; \mathbf{g}\,\mathbf{f}^{\mathbf{abc}}\,\mathbf{A}_{\mathbf{b}}^{\mu}\,\mathbf{A}_{\mathbf{c}}^{\nu}$$

Equations of motion:

Maxwell equation:

$$\partial_{\mu}\,\mathbf{G}^{\mu\nu} \; = \; \mathbf{g}\,\mathbf{J}^{\nu}\,; \qquad \mathbf{J}_{\mathbf{a}}^{\nu} \; = \; \bar{\psi}\,\gamma^{\nu}\,\tau^{\mathbf{a}}\,\psi \; - \; \mathbf{f}^{\mathbf{abc}}\,\mathbf{G}_{\mathbf{b}}^{\nu\mu}\,\mathbf{A}_{\mathbf{c}}^{\mu}$$

Dirac equation

$$(\mathbf{i}\,\gamma^{\mu}\,\mathbf{D}_{\mu} \; - \; \mathbf{m})\,\psi \; = \; \mathbf{0}$$

Light Cone

LC time $x^+ = (t + z)/\sqrt{2}$ $\qquad\qquad \mathbf{x^- = (t - z)/\sqrt{2}}$

LC gauge

$$\mathbf{A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3) = 0}$$

The Gauss law constraint

$$\partial_\mu \mathbf{G}^{\mu +} = \mathbf{g\,J^+}$$

is solved for the A^- **field**

$$-(\partial^+)^{\mathbf{2}}\,\mathbf{A_a^-} + \partial^+\partial_{\mathbf{i}}\,\mathbf{A_a^i} = \mathbf{g\,J_a^+}$$

$$\mathbf{A_a^-} = -\frac{\partial^{\mathbf{i}}}{\partial^+}\mathbf{A_a^i} + \frac{\mathbf{g}}{(\partial^+)^{\mathbf{2}}}\mathbf{J_a^+}$$

Same story with quarks

Light Cone Hamiltonian

Canonical variables: $\mathbf{A^i}, \quad \mathbf{\Pi^i} = \frac{\delta L}{\delta(\partial^- A^i)} = \mathbf{G^{+i}} = \partial^+ \mathbf{A^i}$

Light Cone Hamiltonian:

$$\mathbf{H^{LC}_{QCD}} = \int \mathbf{dx^- d^2x_\perp} \left[\mathbf{\Pi^i} \, \partial^- \mathbf{A^i} - \mathbf{L} \right] = \mathbf{H_E} + \mathbf{H_M}$$

The electric and magnetic parts

$$\mathbf{H_E} = \frac{1}{2} \int \frac{\mathbf{dk^+}}{(2\,\pi)} \mathbf{d^2x} \;\; \mathbf{\Pi^-_a}(\mathbf{k^+}, \mathbf{x}) \, \mathbf{\Pi^-_a}(-\mathbf{k^+}, \mathbf{x})$$

$$\mathbf{H_M} = \frac{1}{4} \int \frac{\mathbf{dk^+}}{(2\,\pi)} \mathbf{d^2x} \;\; \mathbf{G^a_{ij}}(\mathbf{k^+}, \mathbf{x}) \, \mathbf{G^a_{ij}}(-\mathbf{k^+}, \mathbf{x})$$

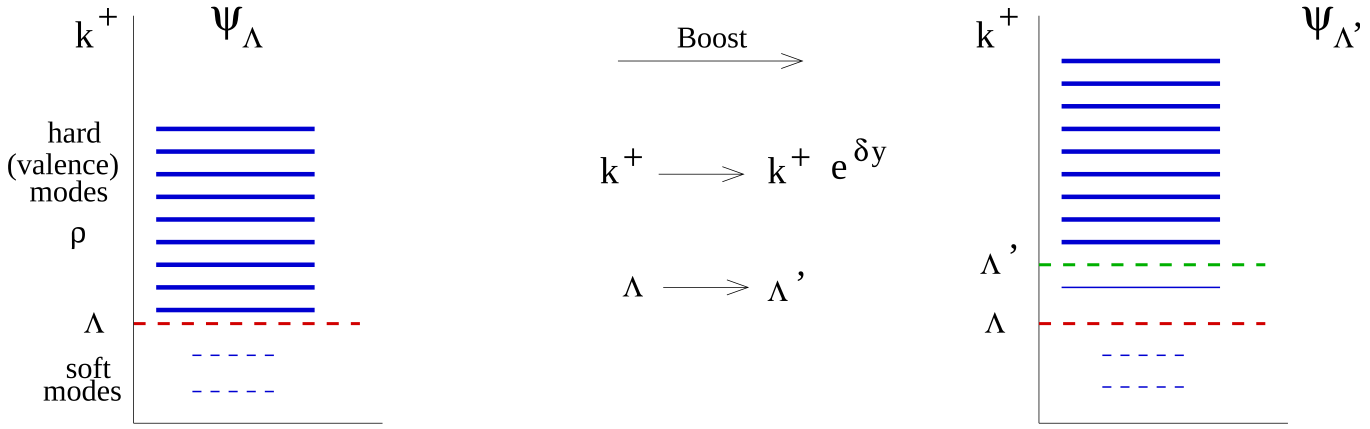
The chromoelectric field

$$\mathbf{\Pi^-_a}(\mathbf{k^+}, \mathbf{x}) = \partial^+ \mathbf{A^-} = -\,\partial^i \mathbf{A^a_i} + \frac{\mathbf{g}}{\partial^+} \mathbf{J^+_a}$$

Light Cone Wave Function

$$\mathbf{H}^{\rm LC}_{\rm QCD}\,|\boldsymbol{\Psi}\rangle \;=\; \mathbf{E}\,|\boldsymbol{\Psi}\rangle$$

Born-Oppenheimer adiabatic approximation: We split the modes into hard and soft. The hard (valence) modes with $k^+ \; > \; \Lambda$ scatter of the target. They act as an external shock wave current $j^+_a \; = \; \delta(x^-)\,\rho^a$ for the soft modes.



The boost opens a window above Λ with the width $\sim\;\delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho\;\sim\;1$; gluon emission $\sim\alpha_s\,\rho$, LO = one gluon, NLO = 2 gluons/quarks

Denote soft glue (quark) creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

Treat ρ as non-abelian background field: SU(N) algebra: $[\rho^a, \rho^b] = i f^{abc} \rho^c$

$$\mathbf{H}_{\rm LC\,QCD} = \mathbf{H}[\rho, \mathbf{a}, \mathbf{a}^\dagger] = \mathbf{H}_V[\rho] + \mathbf{H}_{\rm free}[\mathbf{a}, \mathbf{a}^\dagger] + \mathbf{H}_{\rm int}[\rho, \mathbf{a}, \mathbf{a}^\dagger]$$

LCWF with no soft gluons

$$\mathbf{H}_V\,|v, 0_a\rangle = \mathbf{E}_0\,|v, 0_a\rangle\,; \qquad \mathbf{a}\,|v, 0_a\rangle = 0\,; \qquad \mathbf{E}_0 = 0$$

LCWF with soft gluon/quark dressing

$$|\Psi\rangle = \Omega(\rho, \mathbf{a}, \mathbf{a}^\dagger)\,|v, 0_a\rangle\,; \qquad \Omega^\dagger\,\mathbf{H}_{\rm LC\,QCD}\,\Omega = \mathbf{H}_{\rm diagonal}$$

Find Ω in perturbation theory

- Dilute limit $(\rho \to 0)$ – usual perturbation theory in $\mathbf{H}_{\rm int}$
- Dense limit $(\rho \sim 1/\alpha_s)$ – all order resummation in a strong background field

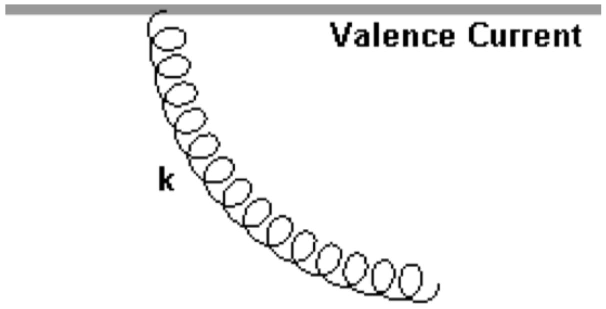
LCWF at LO (dilute case)

First order (g) perturbation theory

Eikonal coupling between valence and soft gluons due to separation of scales

$$\mathbf{H_{int}} = -\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\frac{g\,k_i}{\sqrt{2}\,|k^+|^{3/2}}\left[a_i^{\dagger a}(k^+,\,k_\perp)\;\rho^a(-k_\perp) + a_i^a(k^+,\,-k_\perp)\;\rho^a(k_\perp)\right]$$

$$|\Psi_{LO}\rangle = \Omega\,|0_a\rangle = \mathcal{N}\,|0_a\rangle - \sum_i |i\rangle\,\frac{\langle i|\,H_{int}\,|0_a\rangle}{E_i} \qquad\qquad \langle\Psi_{LO}|\Psi_{LO}\rangle = 1 \rightarrow \mathcal{N}$$



A cloud of WW gluons dressing the valence ones

$$\Omega_Y(\rho \to 0) \equiv C_Y = \mathrm{Exp}\left\{ i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^Y \Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z)) \right] \right\}$$

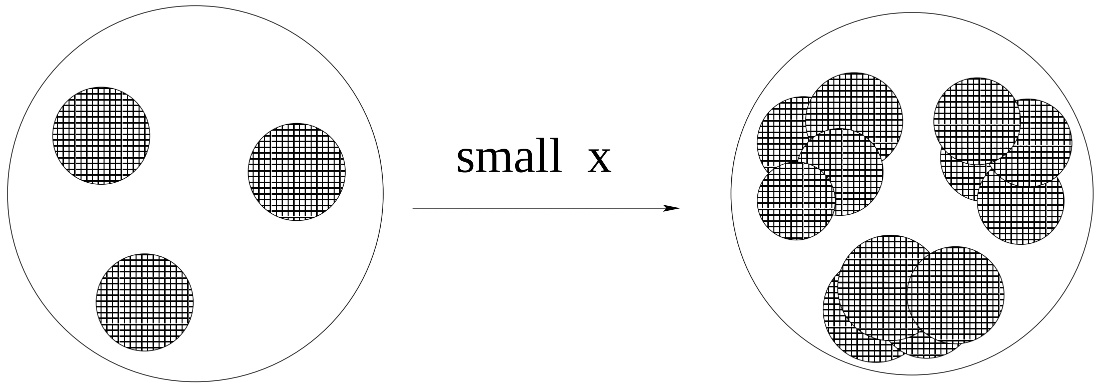
The classical Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(z-x)_i}{(z-x)^2} \rho^a(x)$$

The operator C dresses the valence charges by a cloud of the WW gluons

The LCWF is a starting point for numerous semi-inclusive calculations, such as single/double inclusive particle production.

Dilute regime: $\delta\rho \sim \rho \quad \rightarrow \quad \rho \simeq \mathbf{e}^{\mathbf{c}\,\mathbf{Y}}$ **BFKL** $\qquad s = \exp[Y] = 1/x$



Evolution is generated by boost. Accelerated (color) charged particles radiate

Fast particles emit softer ones

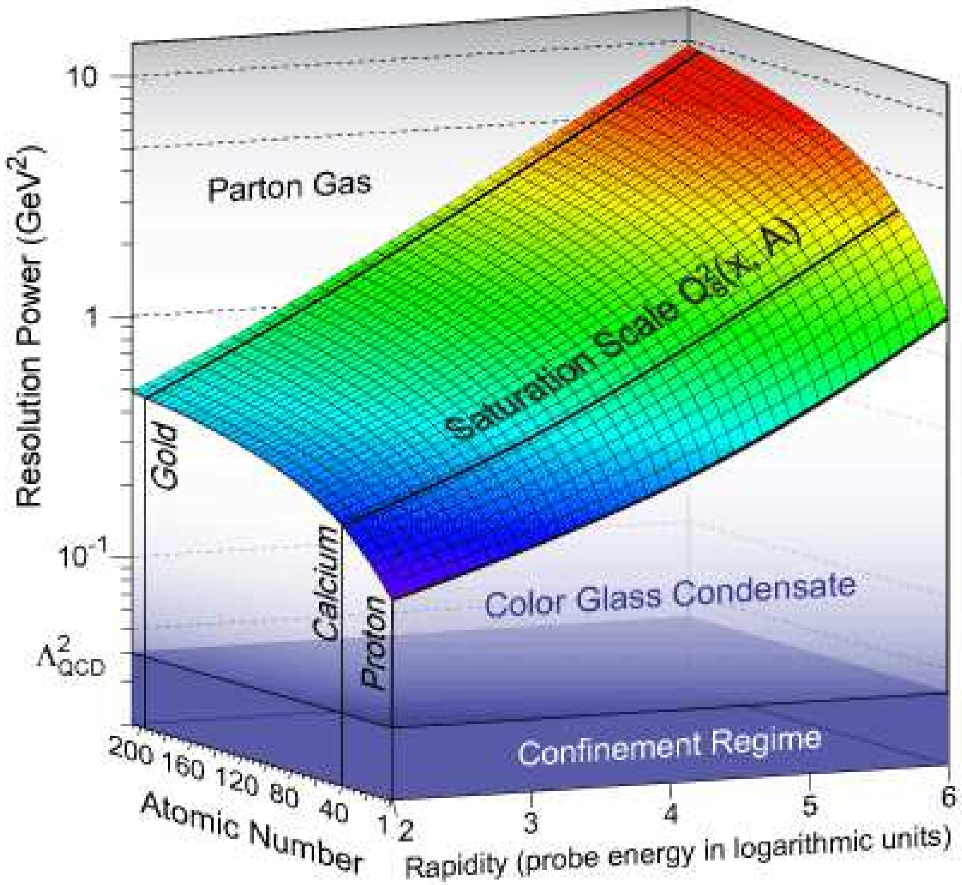
High energy limit = soft gluon emission approximation

Exponential growth of gluon densities leads to unitarity violation.

At high densities the growth should be slowed down due to non-linear effects.

Transition to a non-linear regime is characterized by emergence of a new scale $\mathbf{Q_s}$, known as saturation scale.

$\mathbf{Q_s} \gg \mathbf{\Lambda_{QCD}}$ **and perturbative methods are applicable.**



Physics is more perturbative.

Classical background fields are strong

Atomic number enhancement

$$\mathbf{Q_s^2 \sim A^{1/3}}$$

motivation for EIC

Saturation occures when the parton density becomes of the order $1/\alpha_s$.
This high density of (color) charges produces strong (non-abelian) classical fields, the Color Glass Condensate.

The situation is somewhat similar to non-linear QED in high intensity lasers.

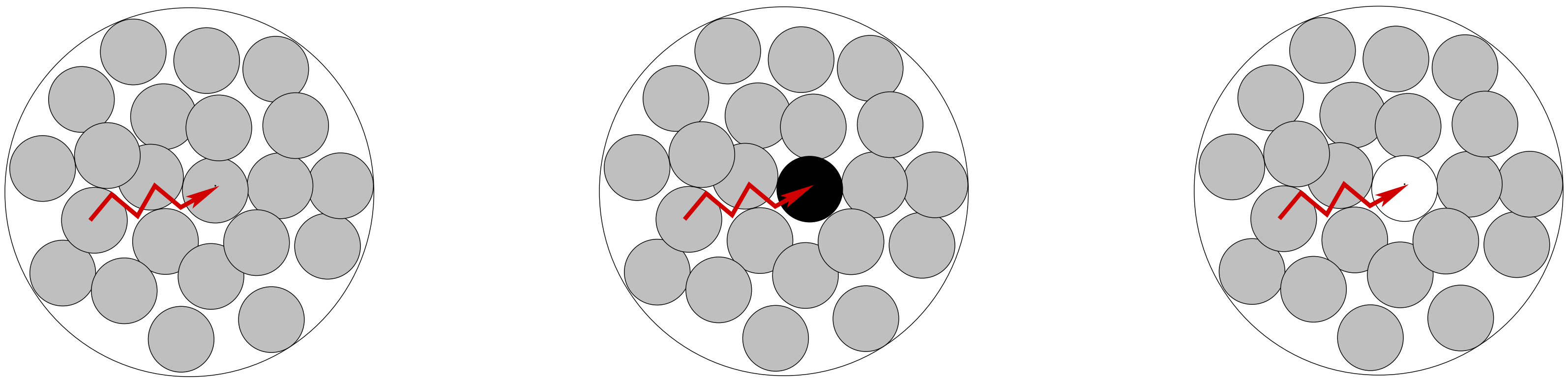
Inside Color Glass Condensate (CGC)

Dense regime:

(1) Hadron is almost black

(2) Emission probability is independent of density

(3) "Bleaching of color"



Random walk $\rho \sim \sqrt{\mathbf{Y}}$ $\quad \mathbf{\Omega = B\,C,}$ **where B - is a Bogoliubov operator**

Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

LCWF at NLO

ML and Yair Mulian, arXiv:1610.03453

- g^3 + normalisation at g^4

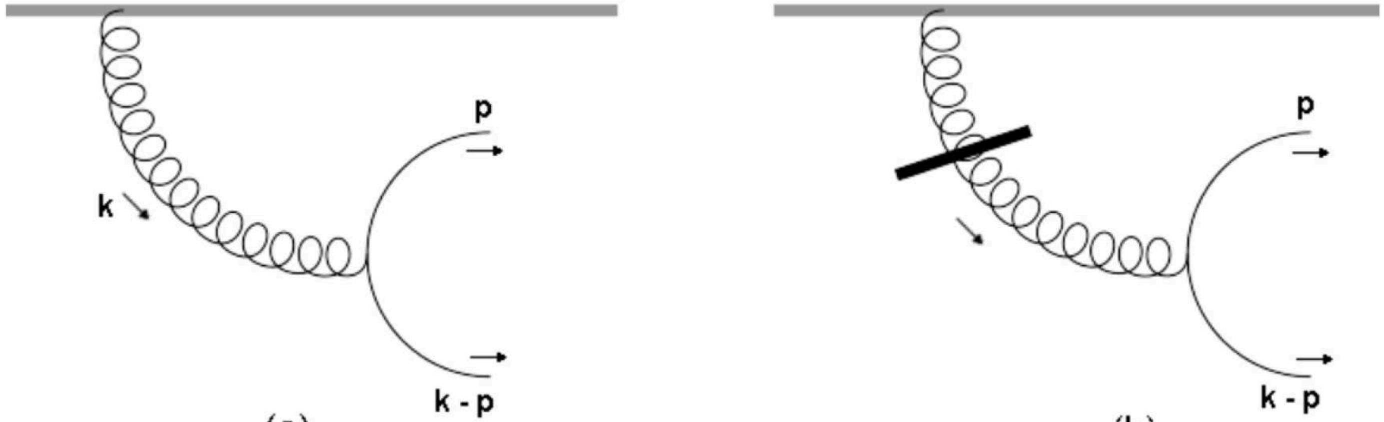
$$|\Psi_{NLO}\rangle = \mathcal{N}\,|0\rangle + \sum_i |i\rangle \left[-\frac{\langle i|\,H_{int}\,|0\rangle}{E_i} + \frac{\langle i|\,H_{int}\,|j\rangle\,\langle j|\,H_{int}\,|0\rangle}{E_i\,E_j} + \right.$$
$$\left. + \frac{\langle i|\,H_{int}\,|0\rangle\,\langle j|\,H_{int}\,|0\rangle^2\,(2\,E_j - E_i)}{2\,E_i^2\,E_j^2} - \frac{\langle i|\,H_{int}\,|j\rangle\,\langle j|\,H_{int}\,|k\rangle\,\langle k|\,H_{int}\,|0\rangle}{E_i\,E_j\,E_k} \right]$$

i runs over one gluon, two gluons, and two quarks

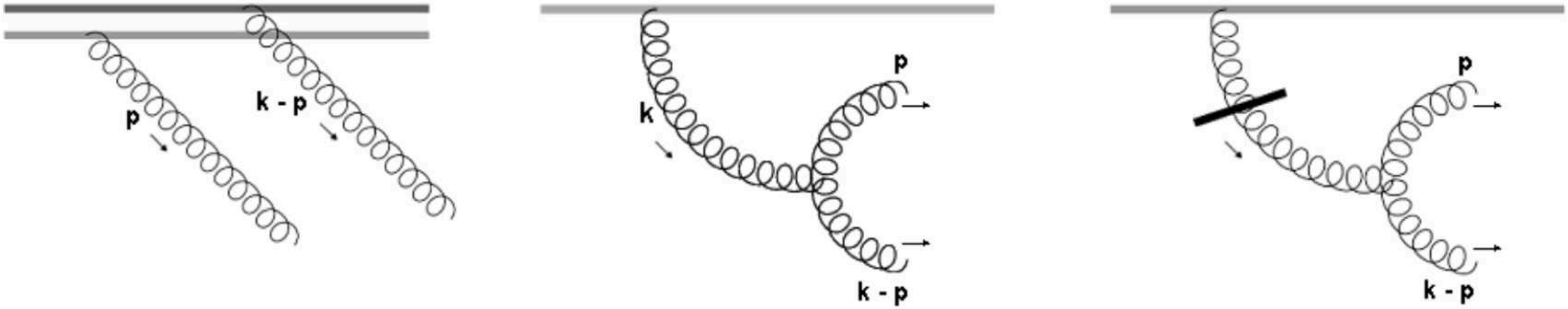
Operator valued matrix elements

Accounts for first non-linear/saturation effects in the projectile

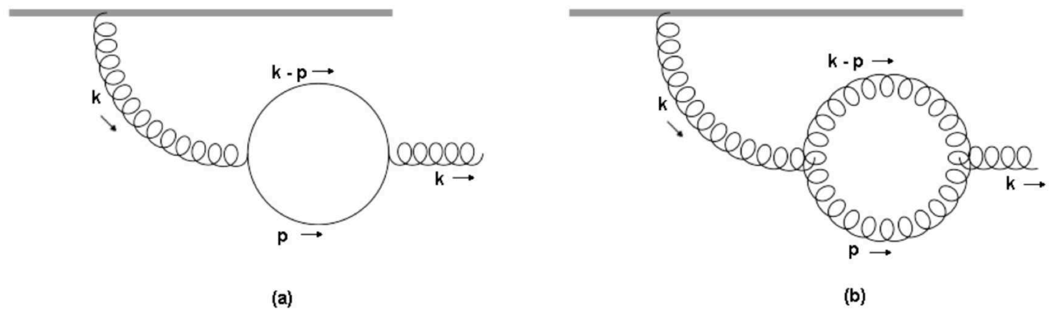
$$\langle \Psi_{NLO}|\Psi_{NLO}\rangle = 1 \to |\mathcal{N}|\,; \qquad \mathcal{N} = |\mathcal{N}|\,e^{i\phi}$$

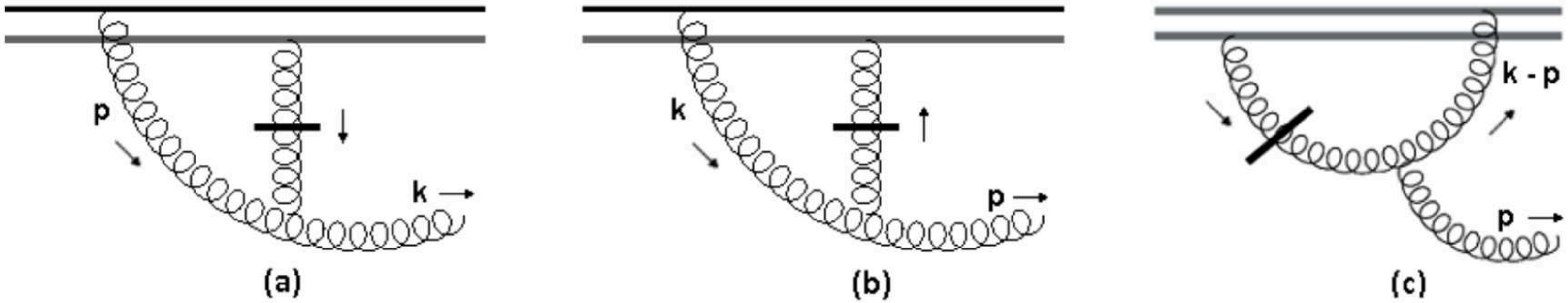


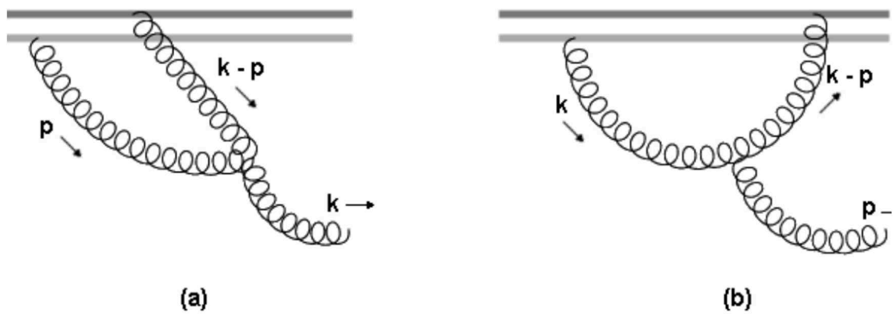
(a) (b)

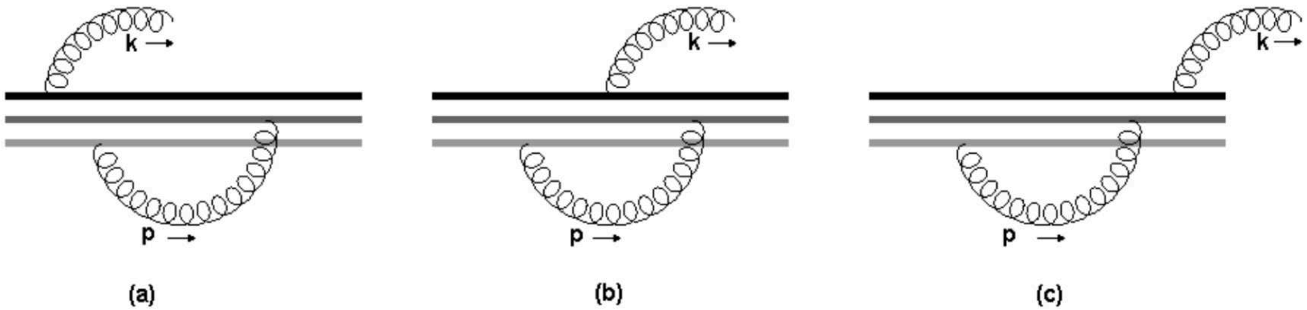


(a) (b) (c)


k - p
k
k
p
(a)
k - p
k
k
p
(b)


p
k
(a)
k
p
(b)
k - p
p
(c)


k - p
p
k
(a)
k
k - p
p
(b)


k
p
(a)
k
p
(b)
k
p
(c)

Phase of the LCWF @ NLO

$$\mathcal{N} \;=\; |\mathcal{N}|\,\mathbf{e}^{\mathbf{i}\phi}$$

Beyond perturbation theory: Born-Oppenheimer adiabatic approximation

$$\langle \mathbf{v}| \otimes \langle\psi|\,\mathbf{H_V}\,|\psi\rangle \otimes |\mathbf{v}\rangle \;\simeq\; \langle \mathbf{v}|\,\mathbf{H_V}\,|\mathbf{v}\rangle \qquad \textbf{or} \qquad \langle\psi|\;\mathbf{H_V}\;|\psi\rangle_{\mathbf{soft}} \;\simeq\; \mathbf{0}$$

the dynamics of the soft modes does not significantly affect that of the valence.

Berry connection

$$\langle\psi|\;\frac{\delta}{\delta\rho^{\mathrm{d}}(\mathbf{w})}\;|\psi\rangle \;=\; \mathbf{0} \qquad \rightarrow\; \phi$$

High Energy Scattering

Projectile averaged S-matrix:

$$\mathbf{\Sigma(Y) \equiv \langle 0_a | \hat{S} | 0_a \rangle \, ;} \qquad \mathbf{\Sigma(Y + \delta Y) = \langle 0_a | \Omega^\dagger \, \hat{S} \, \Omega \, | 0_a \rangle}$$

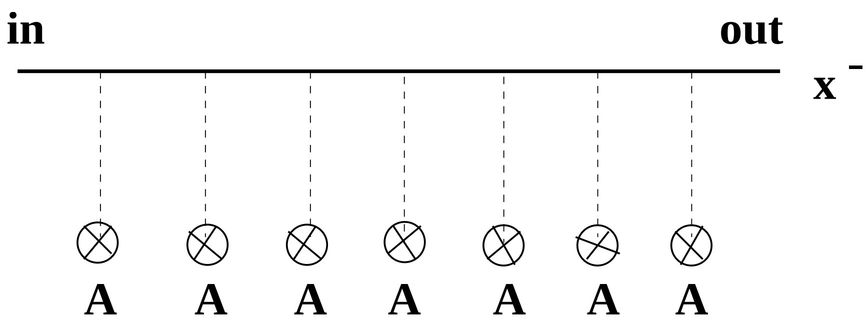
$$\mathbf{\Sigma(Y + \delta Y) = e^{-\delta Y \, H} \, \Sigma(Y)}$$

$$\mathbf{e^{-\delta Y \, H} \simeq 1 - \delta Y \, H + \frac{1}{2} \delta Y^2 \, H^2 \, \ldots}$$

$$\mathbf{H = H^{LO}(\alpha_s) + H^{NLO}(\alpha_s^2) + \ldots} \qquad \mathbf{H = H[\rho^t, \, \delta/\delta\rho^t]}$$

So far we know Ω. What is left is to specify $\hat{S}$

Eikonal scattering approximation



Eikonal scattering is a color rotation
Eikonal factor does not depend on rapidity

In the light cone gauge ($A^+ = 0$) the large target field component is $A^- = \alpha^t$.

$$S(x) = \mathcal{P} \exp\left\{ i \int dx^+ \, T^a \, \alpha_t^a(x, x^+) \right\} . \qquad \text{"}\Delta\text{"} \alpha^t = \rho^t \quad (YM)$$

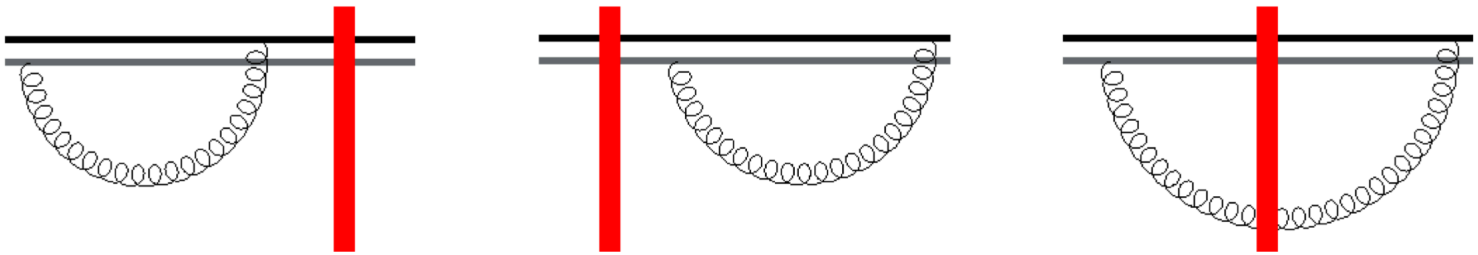
$$|in\rangle = |z, b\rangle \; ; \qquad |out\rangle = |z, a\rangle \; ; \qquad |out\rangle = S \, |in\rangle$$

LO JIMWLK Hamiltonian

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

The JIMWLK Hamiltonian is a limit of H for dilute partonic system ($\rho^{\mathrm{p}} \to 0$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$$H_{LO}^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2} \left\{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) S_A^{ab}(z) J_R^b(y) \right\}$$



Here $\rho^{\mathrm{p}} \to \mathrm{J_L}$ and $\hat{\mathrm{S}}\rho^{\mathrm{p}} \to \mathrm{J_R}$ are left and right SU(N) generators:

$$\mathbf{J^a_L(x) S^{ij}_A(z) = (T^a S_A(z))^{ij}\, \delta^2(x - z)} \qquad \mathbf{J^a_R(x) S^{ij}_A(z) = (S_A(z) T^a)^{ij}\, \delta^2(x - z)}$$

- $\mathbf{H^{JIMWLK}}$ contains all the LO BFKL / BKP / TPV physics

JIMWLK Hamiltonian @ NLO

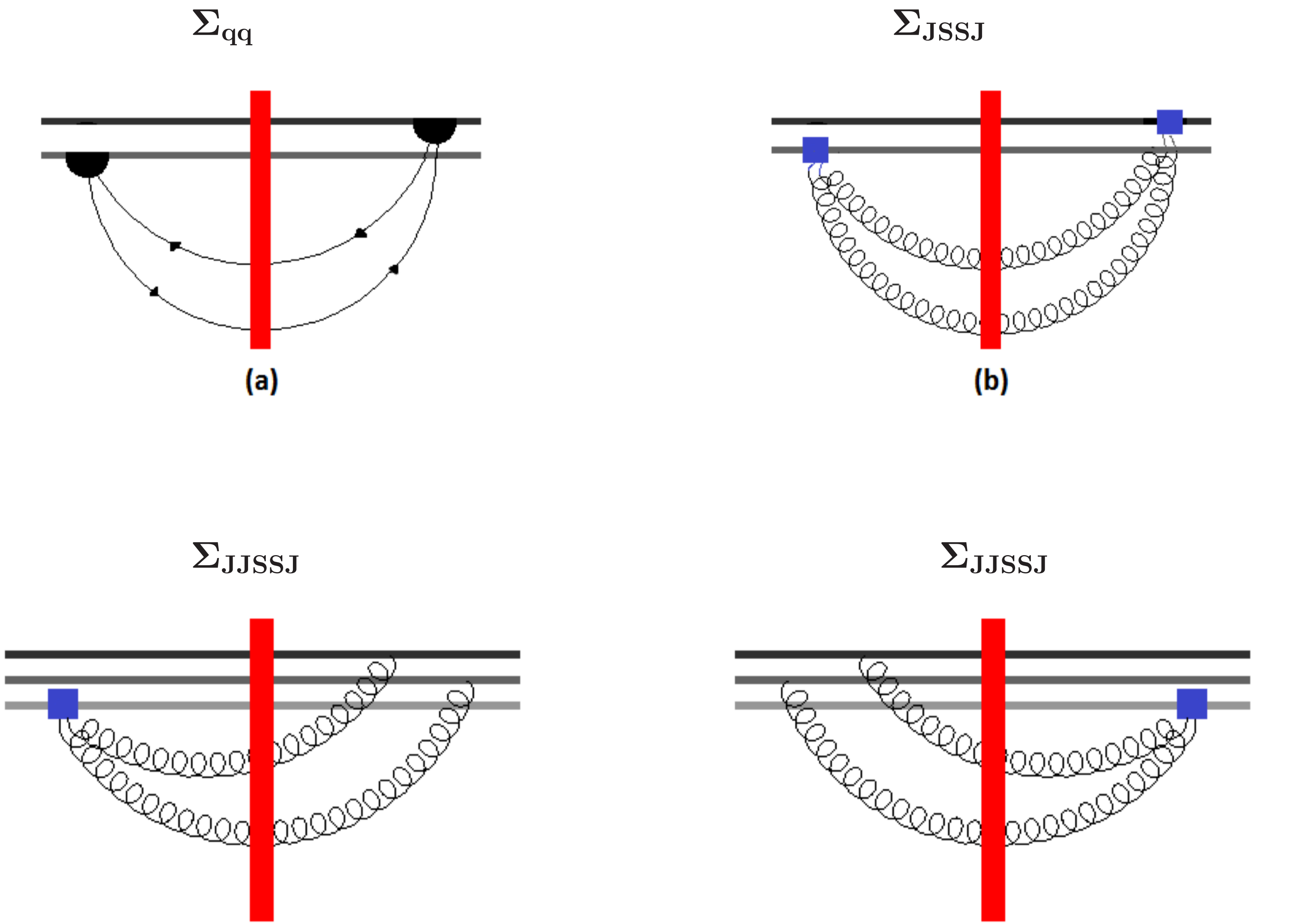
$$\mathbf{\Sigma} \;=\; \langle \psi_{\mathbf{NLO}} |\; \hat{\mathbf{S}} \;| \psi_{\mathbf{NLO}} \rangle \quad = \quad \langle \rho,\; \mathbf{0_a} |\; \mathbf{\Omega}^{\dagger}\, \hat{\mathbf{S}}\; \mathbf{\Omega}\, | \rho, \mathbf{0_a} \rangle$$

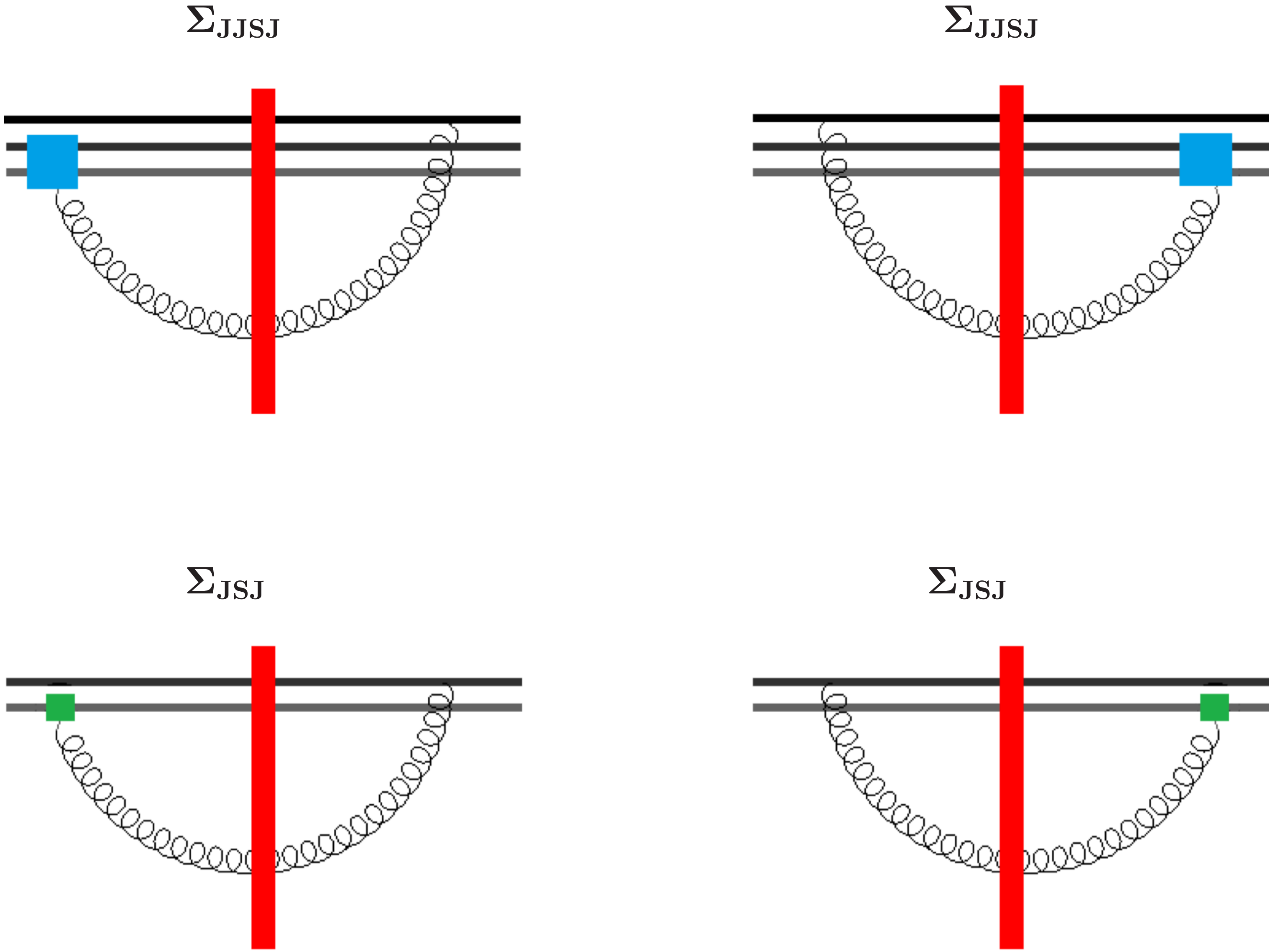
$$= \Sigma^{LO} + \Sigma_{q\bar{q}} + \Sigma_{JJSSJ} + \Sigma_{JSSJ} + \Sigma_{JJSJ} + \Sigma_{JSJ} + \Sigma_{JJSSJJ} + \Sigma_{JJJSJ} + \Sigma_{virtual}.$$

$$\mathbf{\Sigma}_{\dots} \;=\; \mathbf{\Sigma}^{\mathbf{NLO}}_{\dots}(\delta \mathsf{Y}) \;+\; \mathbf{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}}_{\dots}$$

$$\mathbf{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}} \;=\; \frac{\mathbf{1}}{\mathbf{2}} \left(\delta \mathbf{Y}\, \mathbf{H}^{\mathbf{LO}}_{\mathbf{JIMWLK}}\right)^{\mathbf{2}}$$

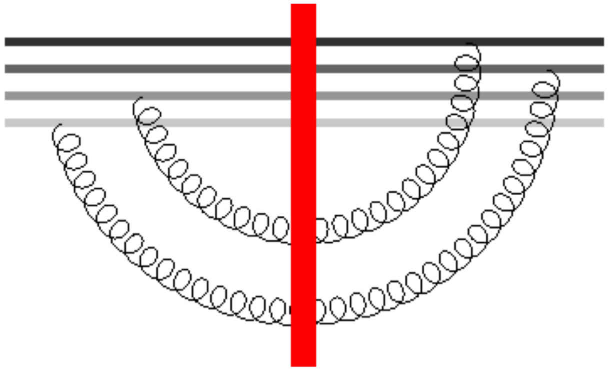
$$\mathbf{\Sigma}^{\mathbf{NLO}}(\delta \mathsf{Y}) \;\to\; \mathbf{H}^{\mathbf{NLO\,JIMWLK}}$$


$\Sigma_{\mathbf{qq}}$
$\Sigma_{\mathbf{JSSJ}}$
(a)
(b)
$\Sigma_{\mathbf{JJSSJ}}$
$\Sigma_{\mathbf{JJSSJ}}$

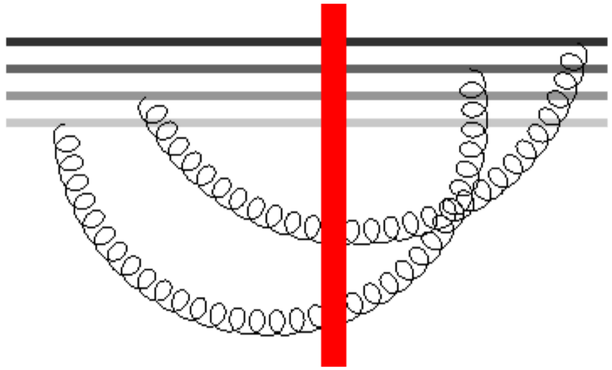


$\Sigma_{\mathbf{JJSJ}}$
$\Sigma_{\mathbf{JJSJ}}$
$\Sigma_{\mathbf{JSJ}}$
$\Sigma_{\mathbf{JSJ}}$

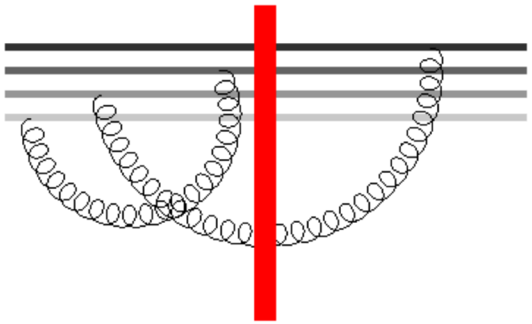
$\Sigma_{\mathbf{JJSSJJ}}$

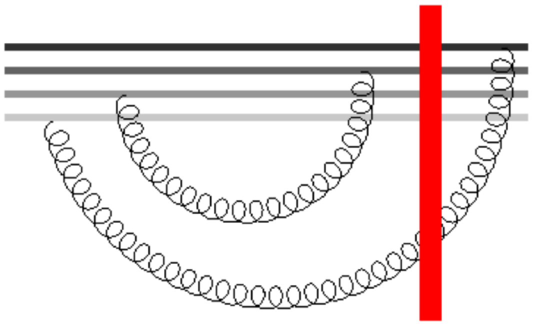


$\Sigma_{\mathbf{JJSSJJ}}$

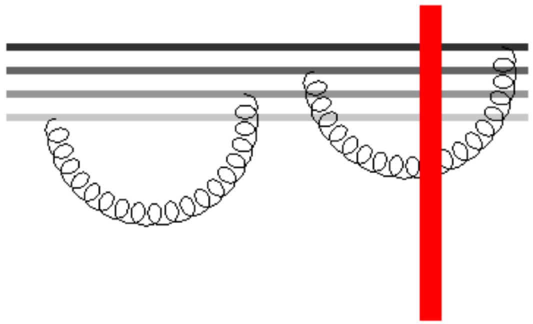


$\Sigma_{\mathbf{JJJSJ}}$





$\Sigma_{\mathbf{JJJSJ}}$



And also $\Sigma_{\mathbf{virtual}}$; $\quad \phi \rightarrow JJJ$

JIMWLK Hamiltonian @ NLO

Alex Kovner, ML and Yair Mulian (2013)

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S_F^\dagger(z)\,t^a\,S_F(z')t^b]\,J_R^b(y)\;-\;J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\Big]$$

$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$

$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w)\;-\;J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$

Symmetries: $\mathbf{SU_L(N)\times SU_R(N)}$ **CPT, Unitarity**

NLO Kernels (for gauge invariant operators)

$$K_{JJSSJ}(w;x,y;z,z') = -i\frac{\alpha_s^2}{2\,\pi^4}\left(\frac{X_i Y'_j}{X^2 Y'^2}\right)$$
$$\times\left(\frac{\delta_{ij}}{2(z-z')^2} + \frac{(z'-z)_i W'_j}{(z'-z)^2 W'^2} + \frac{(z-z')_j W_i}{(z-z')^2 W^2} - \frac{W_i W'_j}{W^2 W'^2}\right)\ln\frac{W^2}{W'^2}$$

$$K_{JJSJ}(w;x,y;z) \;=\; -\,i\,\frac{\alpha_s^2}{4\,\pi^3}\left[\frac{X\cdot W}{X^2\,W^2} - \frac{Y\cdot W}{Y^2\,W^2}\right]\ln\frac{Y^2}{(x-y)^2}\;\ln\frac{X^2}{(x-y)^2},$$

$$K_{q\bar{q}}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\left\{\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2} - \frac{2}{(z-z')^4}\right\}$$

$$X = x - z,\quad X' = x - z',\quad Y = y - z,\quad Y' = y - z',\quad W = w - z$$

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Bigg[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]}$$

$$+\frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2} + \frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2} - \frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Bigg] + \tilde{K}(x,y,z,z').$$

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \Big(\frac{67}{9} - \frac{\pi^2}{3}\Big)N_c - \frac{10}{9}n_f\Big]$$

$$-\frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z').$$

Here μ is the normalization point, $b = \frac{11}{3}N_c - \frac{2}{3}n_f$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\Big[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z')$$

$$+K_{JJSSJ}(y;y,x;z,z')\Big]$$

Is the JIMWLK Hamiltonian Conformally invariant?

Alex Kovner, ML and Yair Mulian (2014)

Scale invariance is trivial. Lets focus on inversion. Introduce $\mathbf{x_\pm = x_1 \pm i\, x_2}$

$$\text{Inversion transformation}: \quad x_+ \to 1/x_- \, ; \qquad\qquad x_- \to 1/x_+$$

A "naive" representation $\mathcal{I}_0$ of the inversion transformation is

$$\mathcal{I}_\mathbf{0}: \quad \mathbf{S(x_+, x_-) \to S(1/x_-, 1/x_+)} \ , \qquad \mathbf{J_{L,R}(x_+, x_-) \to \frac{1}{x_+ x_-} J_{L,R}(1/x_-, 1/x_+)} \, .$$

Conformal invariance (in the gauge invariant sector) @LO:

$$\mathcal{I}_\mathbf{0} \, \mathbf{H^{LO\,JIMWLK}} \, \mathcal{I}_\mathbf{0} \; = \; \mathbf{H^{LO\,JIMWLK}}$$

No (naive) Conformal invariance @NLO:

$$\mathcal{I}_\mathbf{0} \, \mathbf{H^{NLO\,JIMWLK}} \, \mathcal{I}_\mathbf{0} \; = \; \mathbf{H^{NLO\,JIMWLK}} \, + \, \mathcal{A}$$

QCD is not conformally invariant beyond tree level, but $\mathcal{N} = 4$ SUSY is.

JIMWLK Hamiltonian IS conformally invariant! (in $\mathcal{N}=4$)

S **forms a non-trivial representation of the conformal group:**

$$\mathcal{I}: S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathcal{I}: H^{LO} \to H^{LO} - \mathcal{A}$$

Here δS is of order α_s. The condition is that the net anomaly cancels:

$$\mathcal{I}\,(\mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}})\,\mathcal{I} \;=\; \mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}}$$

We have constructed $\mathcal{I}$ perturbatively: $\qquad \mathcal{I} \;=\; (\mathbf{1} + \mathcal{C})\,\mathcal{I}_0\,.$

$$\mathcal{C} \;=\; -\,\frac{\mathbf{1}}{\mathbf{2}}\,\frac{\alpha_{\mathrm{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{x},\mathbf{y},\mathbf{z}} \ln\left[\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}\mathbf{a}^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\right]\times$$

$$\times\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\left\{\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{x})\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{y}) + \mathbf{J}^{\mathbf{a}}_{\mathbf{R}}(\mathbf{x})\mathbf{J}^{\mathbf{a}}_{\mathbf{R}}(\mathbf{y}) - \mathbf{2}\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{x})\mathbf{S}^{\mathbf{ab}}_{\mathbf{A}}(\mathbf{z})\mathbf{J}^{\mathbf{b}}_{\mathbf{R}}(\mathbf{y})\right\}$$

For an arbitrary operator $\mathcal{O}$ (s, B, H^{JIMWLK},...) we define its conformal extension:

$$\mathcal{O}^{conf} \;=\; \mathcal{O} + \frac{1}{2}\,[\mathcal{C},\,\mathcal{O}]\,; \qquad [s^{conf} \text{ by Balitsky and Chirilli (arXiv : 0903.5326)}]$$

Summary and Outlook

- LCWF-based formalism is a very efficient approach to high energy/high density QCD. With the LCWF known both @LO and @NLO, we are able to reproduce and generalise all known results in the field.

- We have first constructed and then independently derived the JIMWLK Hamiltonian @NLO, which is a perturbative version of QCD RFT.

- The field of High Energy QCD has finally reached the stage of precision NLO-based phenomenology and a lot of activity is being directed towards this goal.

- Yet, there are known problems @NLO – the corrections to cross sections are negative and large. The expansion is unstable. Additional resummations/RG improvements are required and we are working on that.