LATTICE GAUGE THEORY IN TECHNICOLOR

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1. Beyond the Standard Model — more gauge groups, more reps
   • $\beta$ function scenarios

2. Method: Schrödinger Functional (background field method) and the lattice phase diagram

3. Results for:
   The $\beta$ function of the SU(3) gauge theory with $N_f = 2$ fermions in the 6 rep

4. and:
   $m, T \neq 0$: Phase diagram on a finite lattice
BEYOND THE STANDARD MODEL on a lattice:

Strong coupling gauge theories — specifically

- Technicolor; walking
- vs. Unparticles?
- Supersymmetry
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Obstacles to lattice studies:

- Sign problem — odd $N_f, \theta \neq 0, \mu \neq 0$
- Chiral fermions
- Supersymmetry
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Current studies: (Lattice 2008)

- SU(3) with fund rep quarks: $N_f = 8, 12$
- SU(2) with adjoint rep quarks
- SU(3) with sextet quarks
- SU($N$) with 2-index symm rep quarks (quenched)
- SU(2) SUSY
Why this model?

- Banks–Zaks fixed point (Caswell 1974; Banks & Zaks 1981) — Is it really there?
- Scale separation: $C_2(R) = \frac{10}{3}$ vs. $\frac{4}{3}$ for fund rep
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\[
\beta(g^2) = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6 + \ldots
\]

Here \( b_1 > 0, \ b_2 < 0 \) [as in QCD with \( 8.05 < N_f < 16\frac{1}{2} \)]

\[\Rightarrow\] IR-attractive fixed point at \( g_*^2 \approx 10.4 \) — a strong coupling
What can happen NONPERTURBATIVELY?

\[ g^2_{\ast} \text{ weak} \]
IRFP \Rightarrow \text{conformal dynamics at large distances}
\Rightarrow \text{no confinement, no } \chi_{\text{SB}},
\text{no particles!}

[unparticles?]

\[ g^2_{\ast} \text{ strong} \]
\chi_{\text{SB}} \Rightarrow \text{fermions decouple, back to } \beta \text{ fn of pure gauge theory}

[Technicolor \ldots maybe walking]
CALCULATING THE $\beta$ FUNCTION: the Schrödinger Functional

- Wilson fermions — because
  1. boundary values (background field) can be set on a single time slice
  2. control over $N_f$
- SF: fix spatial links $U_i$ on time boundaries $t = 0, L$

Calculate the free energy $\Gamma \equiv -\log Z$ since $\Gamma \equiv \frac{1}{g^2(L)} S_{YM}^{cl}$ gives the running coupling $g^2(L)$.

But we can’t calculate $\Gamma$ directly, so:

**Choose boundary values $U_i$ to depend on a parameter $\eta$. Then**

$$\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S_{YM}}{\partial \eta} - \text{tr} \left( \frac{1}{D_F^\dagger} \frac{\partial (D_F^\dagger D_F)}{\partial \eta} \frac{1}{D_F} \right) \right\rangle = \frac{K}{g^2(L)}, \quad K \equiv \frac{\partial S_{YM}^{cl}}{\partial \eta} = 37.7\ldots$$
EXTRACTING PHYSICS

1. Fix lattice size $L$, couplings $\beta \equiv 6/g_0^2$, $\kappa = \kappa_c(\beta)$

2. Calculate $K/g^2(L)$ and $K/g^2(2L)$. Use common lattice spacing ($= \text{UV cutoff}$) $a = L/4$.

3. Result: Discrete Beta Function

   \[ B(u, 2) = \frac{K}{g^2(2L)} - \frac{K}{g^2(L)} , \]

   a function of $u \equiv K/g^2(L)$. 

The DISCRETE BETA FUNCTION

\[ \frac{u}{g^2} = K \]

\[ 4^4 \rightarrow 8^4 \]

\[ B(u, 2) \text{ crosses zero at } g^2 \approx 2.0 \]

not at \( g^2 \approx 10! \)

\[ \Rightarrow \text{IR theory is CONFORMAL} \]
The DISCRETE BETA FUNCTION

Cf. $6^4 \rightarrow 8^4$
Caveat cursor

- Is there only one, unique running coupling?
  - Perturbatively, yes.
  - If the $q\bar{q}$ potential is almost Coulombic: $V(r) \simeq g^2(r)/r$

- Is it really an IRFP?

- Can we extend the picture off the $\kappa_c(\beta)$ curve?
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"PHASE DIAGRAM" in finite volume

$N_t = 8, 12$: "finite temperature," confinement and chiral phase transition!

⇒ No evidence of scale separation

Note weak coupling at IRFP
**Caveat cursor**

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**ANSWERS** will come from:

- More knowledge of phase diagram
- Checking beta function with more volumes
- Eventually: scaling towards the continuum limit

**MORE QUESTIONS**

- Properties of (near-) conformal theory