A review of neutrino physics tailored to PANIC ‘08.
The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!
These discoveries come from the observation of

*neutrino flavor change* (neutrino oscillation).
The Physics of Neutrino Oscillation
The Neutrino Flavors

We define the three known flavors of neutrinos, $\nu_e$, $\nu_\mu$, $\nu_\tau$, by W boson decays:

- $W \nu_e \rightarrow e$
- $W \nu_\mu \rightarrow \mu$
- $W \nu_\tau \rightarrow \tau$

As far as we know, neither $\nu \rightarrow \ell$ interaction ever occurs. With $\alpha = e, \mu, \tau$, $\nu_\alpha$ makes only $\ell_\alpha$ ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).
If neutrinos have masses, and leptons mix, we can have —

Give $\nu$ time to change character

$\nu_\mu \rightarrow \nu_\tau$

The last decade has brought us compelling evidence that such flavor changes actually occur.
Flavor Change Requires *Neutrino Masses*

There must be some spectrum of at least 3 neutrino mass eigenstates $\nu_i$:

\[(\text{Mass})^2\]

\[\nu_1 \quad \nu_2 \quad \nu_3\]

Mass ($\nu_i$) $\equiv m_i$
Flavor Change Requires *Leptonic Mixing*

The neutrinos $\nu_{e, \mu, \tau}$ of definite flavor 

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_{\alpha} > = \sum_i U^*_{\alpha i} |\nu_i> .$$

**Neutrino of flavor** $\alpha = e, \mu, \text{ or } \tau$

**PMNS Leptonic Mixing Matrix**

**Neutrino of definite mass** $m_i$
This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

\[
L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\ell}_{L\alpha} \gamma^\lambda \nu_{L\alpha} W_{\lambda}^- + \bar{\nu}_{L\alpha} \gamma^\lambda \ell_{L\alpha} W_{\lambda}^+ \right)
\]

\[
= -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{L i} W_{\lambda}^- + \bar{\nu}_{L i} \gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W_{\lambda}^+ \right)
\]

If neutrino *masses* are described by an extension of the SM, $U$ is unitary. Then —

\[
\text{Amp}\left(W \rightarrow \ell_\alpha + \nu_\alpha; \nu_\alpha \rightarrow \ell_\beta + W\right) \propto \sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i} = \delta_{\beta\alpha}, \text{ as observed.}
\]
Neutrino Flavor Change (Oscillation) in Vacuum

\[ \mathcal{L}^+ (\text{e.g. } \mu) \rightarrow (\nu_\alpha) \]

\[ (\nu_\alpha) \rightarrow (\nu_\beta) \]

\[ \mathcal{L}^- (\text{e.g. } \tau) \]

\[ \text{Source} \quad W \quad \text{Target} \]

Approach of B.K. & Stodolsky

\[ \text{Amp} = \sum \text{Amp} \]

\[ \mathcal{L}^+ \quad \text{Source} \]

\[ U_{\alpha_i}^* \quad W \quad U_{\beta_i} \]

\[ \nu_i \quad e^{-im_i^2 \frac{L}{2E}} \quad \text{Target} \]
\[ \text{Amp} \left[ \nu_\alpha \rightarrow \nu_\beta \right] \]

\[ = \sum_i \text{Amp} \left[ W U_{\alpha i}^* e^{-i m_i^2 \frac{L}{2E}} \right] \]

\[ = \sum_i U_{\alpha i}^* e^{-i m_i^2 \frac{L}{2E}} U_{\beta i} \]
Probability for Neutrino Oscillation in Vacuum

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 = \]

\[ = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \]

\[ + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E}) \]

where \( \Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \)
For Antineutrinos –

We assume the world is CPT invariant.
Our formalism assumes this.
\[ P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta)^{\text{CPT}} = P(\nu_\beta \rightarrow \nu_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*) \]

Thus,

\[ P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) = \]

\[ = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \]

\[ \pm 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E}) \]

A complex U would lead to the CP violation

\[ P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta) \]
1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} \]

Flavor change $\Rightarrow$ $\nu$ Mass

2. If there is no mixing,

\[ \Rightarrow U_{\alpha i}U_{\beta \neq \alpha, i} = 0, \text{ so that } P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta}. \]

Flavor change $\Rightarrow$ Mixing
3. One can detect \((\nu_\alpha \rightarrow \nu_\beta)\) in two ways:

- See \(\nu_\beta \neq \alpha\) in a \(\nu_\alpha\) beam (Appearance)
- See some of known \(\nu_\alpha\) flux disappear (Disappearance)

4. Including \(\hbar\) and \(c\)

\[
\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (eV^2) \frac{L(km)}{E(\text{GeV})}
\]

\[
\sin^2[1.27\Delta m^2 (eV)^2 \frac{L(km)}{E(\text{GeV})}] \text{ becomes appreciable when its argument reaches } \mathcal{O}(1).
\]

An experiment with given \(L/E\) is sensitive to

\[
\Delta m^2 (eV^2) \gtrsim \frac{E(\text{GeV})}{L(km)}.
\]
5. Flavor change in vacuum oscillates with L/E. Hence the name “neutrino oscillation”. {The L/E is from the proper time $\tau$.}

6. $P(\nu_\alpha \rightarrow \nu_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us

![Diagram showing mass squared differences](attachment:image.png)
7. Neutrino flavor change does not change the total flux in a beam. It just redistributes it among the flavors.

\[ \sum_{\text{All } \beta} P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) = 1 \]

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the active flux disappears:

\[ \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}} \]
8. When there are, in effect, only two flavors and two mass eigenstates —

\[
U = \begin{bmatrix}
    U_{\alpha 1} & U_{\alpha 2} \\
    U_{\beta 1} & U_{\beta 2}
\end{bmatrix} = \begin{bmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
    e^{i \xi} & 0 \\
    0 & 1
\end{bmatrix}
\]

For \( \beta \neq \alpha \),

\[
P(\nu_{\alpha} \leftrightarrow \nu_{\beta}) = \sin^2 2\theta \sin^2 (\Delta m^2 \frac{L}{4E})
\]

For no flavor change,

\[
P(\nu_{\alpha} \rightarrow \nu_{\alpha}) = 1 - \sin^2 2\theta \sin^2 (\Delta m^2 \frac{L}{4E})
\]
Neutrino Flavor Change In Matter

\[ V_W = +\sqrt{2}G_F N_e \quad (- \text{for } \bar{\nu}_e) \]

Fermi constant

#e/vol
The fractional importance of matter effects on an oscillation involving a vacuum splitting $\Delta m^2$ is —

$$\frac{[(G_{\text{Fermi}}/\sqrt{2})N_e]}{[\Delta m^2/4E]} \equiv x.$$  

The matter effect —

— Grows with neutrino energy $E$
— Is sensitive to $\text{Sign}(\Delta m^2)$
— Reverses when $\nu$ is replaced by $\bar{\nu}$
Energy Dependence of the Matter Effect In the Sun

Solar neutrinos are all born as $\nu_e$.

The *solar matter effect*, which enhances the conversion of solar $\nu_e$ into neutrinos of other flavors, should grow with neutrino energy $E$.

Until recently, only the high-energy $^8\text{B}$ solar neutrinos, with $E \sim 8$ MeV, had been studied in detail.

Now *Borexino* has studied, in the same detector, both the $^8\text{B}$ neutrinos and the monoenergetic $^7\text{Be}$ neutrinos, which have $E = 0.862$ MeV.
The $\nu_e$ survival probability does appear to decrease with energy, as predicted.
What We Have Learned
The \((\text{Mass})^2\) Spectrum

\[ \Delta m^2_{\text{atm}} \quad \text{or} \quad \Delta m^2_{\text{sol}} \]

\(\nu_3 \quad \nu_2 \quad \nu_1\)

Normal

\(\Delta m^2_{\text{sol}} \approx 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \approx 2.4 \times 10^{-3} \text{ eV}^2\)
Are There More Than 3 Mass Eigenstates?

When only two neutrinos count,

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left[ 1.27\Delta m^2 \left( eV^2 \right) \frac{L(km)}{E(GeV)} \right] \]

\( \Delta m^2_{\text{sol}} = 7.6 \times 10^{-5} eV^2 \)
\( \Delta m^2_{\text{atm}} = 2.4 \times 10^{-3} eV^2 \)

\( \sim 1 \text{eV}^2 \)

\( \text{in contrast to} \)

\( \text{At least 4 mass eigenstates} \) \( \text{At least 1 } \nu_{\text{Sterile}}\)
Is the LSND Signal Genuine Neutrino Oscillation?

MiniBooNE results prior to November 9, 2008 suggest that the answer is —

No.
While awaiting further news —

*We will assume there are only 3 neutrino mass eigenstates.*
Leptonic Mixing

This has the consequence that —

$$|\nu_i> = \sum_\alpha U_{\alpha i} |\nu_\alpha>.$$  

Flavor-\(\alpha\) fraction of \(\nu_i\) = \(|U_{\alpha i}|^2\).

When a \(\nu_i\) interacts and produces a charged lepton, the probability that this charged lepton will be of flavor \(\alpha\) is \(|U_{\alpha i}|^2\).
The spectrum, showing its approximate flavor content, is

\begin{align*}
\nu_1 & \quad \nu_2 \quad \nu_3 \\
\Delta m^2_{\text{atm}} & \quad \sin^2 \theta_{13} \quad \Delta m^2_{\text{sol}} \\
(Mass)^2 & \\
\end{align*}

or

\begin{align*}
\nu_1 & \quad \nu_2 \quad \nu_3 \\
\Delta m^2_{\text{atm}} & \quad \sin^2 \theta_{13} \quad \Delta m^2_{\text{sol}} \\
(Mass)^2 & \\
\end{align*}

Normal

Inverted

\begin{align*}
\nu_e & \left[ |U_{ei}|^2 \right] \\
\nu_\mu & \left[ |U_{\mu i}|^2 \right] \\
\nu_\tau & \left[ |U_{\tau i}|^2 \right] \\
\end{align*}
The Mixing Matrix

\[
U = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix} \times \begin{bmatrix}
c_{13} & 0 & s_{13} e^{-i \delta} \\
0 & 1 & 0 \\
-s_{13} e^{i \delta} & 0 & c_{13}
\end{bmatrix} \times \begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
c_{ij} \equiv \cos \theta_{ij}
\]
\[
s_{ij} \equiv \sin \theta_{ij}
\]

\[
\theta_{12} \approx \theta_{\text{sol}} \approx 34^\circ, \quad \theta_{23} \approx \theta_{\text{atm}} \approx 38-52^\circ, \quad \theta_{13} \lesssim 10^\circ
\]

\[
\delta \text{ would lead to } \mathcal{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq \mathcal{P}(\nu_\alpha \to \nu_\beta). \quad \text{\(\mathcal{CP}\)}
\]

But note the crucial role of \(s_{13} \equiv \sin \theta_{13}\).
What Processes Oscillate?
KamLAND Evidence for Oscillatory Behavior

Survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ of reactor $\bar{\nu}_e$

$L_0 = 180$ km is a flux-weighted average travel distance.

$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ actually oscillates!
Oscillation Comes From the Coherence of Intermediate $\nu$ States
Neutrinos In *Final* States Are *Incoherent*

In Electron-Capture decays —

\[ \Gamma(\text{Ion}_1 \rightarrow \text{Ion}_2 + \nu; t) = \sum_i \Gamma(\text{Ion}_1 \rightarrow \text{Ion}_2 + \nu_i; t) \]

or —

\[ \Gamma(\text{Atom}_1 \rightarrow \text{Atom}_2 + \nu; t) = \sum_i \Gamma(\text{Atom}_1 \rightarrow \text{Atom}_2 + \nu_i; t) \]

The different mass eigenstates \( \nu_i \) contribute *incoherently* to the decay rate.

*The total decay rate should not oscillate.*

*(Unless the rate for decay to each \( \nu_i \) does.)*
Litvinov et al. —

EC decays of H-like $^{140}$Pr and $^{142}$Pm ions in a storage ring at GSI oscillate.

![Graph showing oscillation and exponential decay of $^{142}$Pm](Image)
Vetter et al. —

EC decays of neutral $^{142}$Pm atoms at LBNL do not oscillate.

Faestermann et al.: EC decays of neutral $^{180}$Re atoms in Munich do not oscillate either.
Theoretical Opinion

Neutrino mixing can make decay rates oscillate:
Faber, Ivanov, Kienle, Kleinert, Lipkin, Reda

No it cannot:
Cohen, Gal, Giunti, Glashow, Kienert, Kopp, Ligeti, Lindner, Merle, Peshkin

Lipkin: Putting the mother ion in a magnetic field, which is done only in the GSI experiment, makes a difference.

To be continued ......
The Open Questions
• What is the absolute scale of neutrino mass?

• Are neutrinos their own antiparticles?

• Are there “sterile” neutrinos?

We must be alert to surprises!
• What is the pattern of mixing among the different types of neutrinos?

What is $\theta_{13}$?

• Is the spectrum like $\equiv$ or $\equiv$?

• Do neutrino – matter interactions violate CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?
• What can neutrinos and the universe tell us about one another?

• Is CP violation involving neutrinos the key to understanding the matter–antimatter asymmetry of the universe?

• What physics is behind neutrino mass?
The Importance of Some Questions, and How They May Be Answered
What Is the Absolute Scale of Neutrino Mass?
How far above zero is the whole pattern?

Cosmology may one day give us an answer.

Ultimately, there will be no substitute for a decisive Laboratory determination.
Does $\bar{\nu} = \nu$?
What Is the Question?

For each mass eigenstate $\nu_i$, and given helicity $h$, does —

- $\bar{\nu}_i(h) = \nu_i(h)$ (Majorana neutrinos)

or

- $\bar{\nu}_i(h) \neq \nu_i(h)$ (Dirac neutrinos)?

Equivalently, do neutrinos have Majorana masses? If they do, then the mass eigenstates are Majorana neutrinos.
Majorana Masses

Out of, say, a left-handed neutrino field, \( \nu_L \), and its charge-conjugate, \( \nu^c_L \), we can build a Majorana mass term —

\[
m_L \nu_L \nu^c_L
\]

Quark and charged-lepton Majorana masses are forbidden by electric charge conservation.

Neutrino Majorana masses would make the neutrinos very distinctive.
The objects $\nu_L$ and $\nu_L^c$ in $m_L\nu_L\nu_L^c$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed.

$m_L\nu_L\nu_L^c$ induces $\nu_L \leftrightarrow \nu_L^c$ mixing.

As a result of $K^0 \leftrightarrow \bar{K}^0$ mixing, the neutral K mass eigenstates are —

$$K_{S,L} \equiv (K^0 \pm \bar{K}^0)/\sqrt{2} \quad \bar{K}_{S,L} = K_{S,L}.$$  

As a result of $\nu_L \leftrightarrow \nu_L^c$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu_L + \nu_L^c = "\nu + \bar{\nu}". \quad \bar{\nu}_i = \nu_i.$$
To Determine Whether Majorana Masses Occur in Nature
The Promising Approach — Seek Neutrinoless Double Beta Decay $[0\nu\beta\beta]$
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)

$(\bar{\nu})_R \rightarrow \nu_L : A \ (tiny) \ Majorana \ mass \ term$

$\therefore \ 0\nu\beta\beta \rightarrow \bar{\nu}_i = \nu_i$
We expect the dominant mechanism to be —

$$\sum_{i} U_{ei} \nu_i \overline{\nu_i} \nu_i W^- W^-$$

Nuclear Process

Then —

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum_{i} m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$
45

Takes 1 ton

\( m_{\beta\beta} \)

Takes 100 tons

95% CL

Target sensitivity for coming experiments

\( m_{\beta\beta} \text{ For Each Hierarchy} \)
Mixing, Mass Ordering, and CP
The Central Role of $\theta_{13}$

Both CP violation and our ability to tell whether the spectrum is normal or inverted depend on $\theta_{13}$.

If $\sin^2 2\theta_{13} > 10^{-2.3}$, we can study both of these issues with intense but conventional accelerator $\nu$ and $\bar{\nu}$ beams, produced via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$.

Determining $\theta_{13}$ is an important step.
Reactor Experiments To Determine $\theta_{13}$

Looking for disappearance of reactor $\bar{\nu}_e$, which have $E \sim 3$ MeV, while they travel $L \sim 1.5$ km is the cleanest way to determine $\theta_{13}$.

$$P(\bar{\nu}_e \text{ Disappearance}) =$$
$$= \sin^2 2\theta_{13} \sin^2 [1.27\Delta m^2_{\text{atm}} (\text{eV}^2) L(\text{km})/E(\text{GeV})]$$
Accelerator Experiments

Accelerator neutrino experiments can also probe $\theta_{13}$. Now it is entwined with other parameters.

In addition, accelerator experiments can probe whether the mass spectrum is normal or inverted, and look for CP violation.

All of this is done by studying $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ while the beams travel hundreds of kilometers.
The Mass Spectrum: \( \equiv \) or \( \equiv \)?

Generically, grand unified models (GUTS) favor —

—

—

GUTS relate the Leptons to the Quarks.

However, Majorana masses, with no quark analogues, could turn \( \equiv \) into \( \equiv \).
How To Determine If The Spectrum Is Normal Or Inverted

Exploit the *matter effect* on accelerator neutrinos. This effect depends on the parameter —

\[
x \equiv \left[ \left( \frac{G_{Fermi}}{\sqrt{2}} \right) N_e \right] / \left[ \frac{\Delta m_{atm}^2}{4E} \right]
\]

Electron density \(\downarrow\) \(m^2(\Longleftarrow) - m^2(\Longrightarrow)\)

Matter affects \(\nu\) and \(\bar{\nu}\) oscillation *differently*, leading to:

\[
\begin{align*}
P(\nu_\mu \rightarrow \nu_e) & > 1 ; \quad \Longleftrightarrow \\
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) & < 1 ; \quad \Longleftrightarrow
\end{align*}
\]

*Note fake CP*

*Note dependence on the mass ordering*
Q: Does matter still affect $\nu$ and $\bar{\nu}$ differently when $\bar{\nu} = \nu$?

A: Yes!

The weak interactions violate parity. Neutrino – matter interactions depend on the neutrino polarization.
Do Neutrino Interactions Violate CP?

The observed CP in the weak interactions of quarks cannot explain the Baryon Asymmetry of the universe.

Is leptonic CP, through Leptogenesis, the origin of the Baryon Asymmetry of the universe?

(Fukugita, Yanagida)
Leptogenesis In Brief

The most popular theory of why neutrinos are so light is the —

See-Saw Mechanism

\[(\text{Yanagida; Gell-Mann, Ramond, Slansky;}) \quad \text{Mohapatra, Senjanovic; Minkowski}\]

The **very** heavy neutrinos $N$ would have been made in the hot Big Bang.
The heavy neutrinos $N$, like the light ones $\nu$, are Majorana particles. Thus, an $N$ can decay into $\ell^-$ or $\ell^+$. If neutrino oscillation violates CP, then quite likely so does $N$ decay. In the See-Saw, these two CP violations have a common origin: One Yukawa coupling matrix.

Then, in the early universe, we would have had different rates for the CP-mirror-image decays –

$$N \rightarrow \ell^- + \phi^+ \quad \text{and} \quad N \rightarrow \ell^+ + \phi^-$$

This would have led to unequal numbers of leptons and antileptons (Leptogenesis).

Then, Standard-Model Sphaleron processes would have turned $\sim 1/3$ of this leptonic asymmetry into a Baryon Asymmetry.
How To Search for $\not{\mathbb{C}}P$
In Neutrino Oscillation

Look for $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$
Q: Can CP violation still lead to $P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e) \neq P(\nu_\mu \rightarrow \nu_e)$ when $\overline{\nu} = \nu$?

A: Certainly!

Compare

$$\sum_i \pi^+ \rightarrow \mu^+ \rightarrow \nu_i \rightarrow \nu_e \rightarrow e^-$$

with

$$\sum_i \pi^- \rightarrow \mu^- \rightarrow \overline{\nu}_i \rightarrow \overline{\nu}_e \rightarrow e^+$$

$$U_{\mu i}^* \exp(-im^2_i L/2E) U_{ei}$$

$$U_{\mu i} \exp(-im^2_i L/2E) U_{ei}^*$$
Enjoy PANIC
2008