Fundamental Symmetries in Atoms and Nuclei

- Nuclear tests: parity violation and the hadronic weak interaction
- Atomic tests: electric dipole moments (and anapole moments)
- Flavor physics: lepton number and separate lepton number
Why use atoms and nuclei in low-energy tests of symmetries?

*It does not necessarily take a handsome man to kill a deer, or to marry a good-looking woman.*  
— Charles Gibson, Native American philosopher  
The Indian Journal, 1903
Why use atoms and nuclei in low-energy tests of symmetries?

It does not necessarily take a handsome man to kill a deer, or to marry a good-looking woman. — Charles Gibson, Native American philosopher

find new physics.

a big accelerator to test the standard model
Atoms and nuclei are laboratories that allow us to

- isolate very weak interactions of interest, by using the spin and parity of atomic/nuclear levels and the kinematics of transitions as filters
  - *double beta decay*: *kinematically isolating a second-order weak interaction*
  - *nuclear level parities*: *to isolate the hadronic weak interaction*

- to enhance the effects of symmetry breaking, by exploiting chance
  - *level degeneracies* or other properties of many-body systems
  - *parity doublets and electromagnetic selection rules*: *PNC effects*
  - *collective nuclear modes*: *atomic electric dipole moments*

- to make use of incredible experimental sensitivities
  - *measurements of atomic level shifts of* $10^{-26}$ eV!
1. Hadronic weak interaction: observable through flavor-changing decays, but

- SM: flavor-changing neutral currents GIM suppressed, unobservable

\[ Z_0 = 0 \quad \text{(tree-level is diagonal)} \]

\[ Z_0 \sim 0 \]

(why one knew there was a third quark)

- therefore to see the hadronic neutral current, must study $\Delta S=0$ interactions
But the only accessible laboratories for studying flavor-conserving weak interactions are the NN system and nuclei

- but such systems have much larger strong and electromagnetic interactions: must use PNC (parity nonconservation) to filter out these nonweak but parity-conserving interactions

- interaction often modeled as a series of one-boson exchanges

\[
\begin{align*}
\pi^\pm, \rho, \omega & \quad \leftrightarrow \quad \text{strong vertex} \\
\text{contains } W, Z & \quad \leftrightarrow \\
\end{align*}
\]
- So what is in that weak vertex $\oplus$? The standard model gives us

$$L^\text{eff} = \frac{G}{\sqrt{2}} \left[ J_W^\dagger J_W + J_Z^\dagger J_Z \right] + h.c.$$  

$$J_W = \cos \theta_C \; J_W^{\Delta S=0} + \sin \theta_C \; J_W^{\Delta S=-1}$$

$$\uparrow \quad \Delta \ell = 1 \quad \uparrow \quad \Delta \ell = 1/2$$

$$L^\text{eff}_{\Delta S=0} = \frac{G}{\sqrt{2}} \left[ \cos^2 \theta_C J_W^0 J_W^0 + \sin^2 \theta_C J_W^1 J_W^1 + J_Z^\dagger J_Z \right]$$

$$\uparrow \quad \uparrow \quad \text{symmetric } \Rightarrow \Delta \ell = 0,2 \quad \Delta \ell = 1 \text{ but Cabibbo suppressed}$$

leads to the expectation that the weak neutral current will dominate nuclear experiments sensitive to isovector PNC
One can describe the resulting NN force in several ways

<table>
<thead>
<tr>
<th>Transition</th>
<th></th>
<th>Δl</th>
<th>n-n</th>
<th>n-p</th>
<th>p-p</th>
<th>NN system exchanges</th>
</tr>
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<tbody>
<tr>
<td>3S1 ↔ 1P1</td>
<td>0</td>
<td>0</td>
<td>x</td>
<td></td>
<td></td>
<td>ρ, ω</td>
</tr>
<tr>
<td>1S0 ↔ 3P0</td>
<td>1</td>
<td>0</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>ρ, ω</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td>ρ</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3S1 ↔ 3P1</td>
<td>0</td>
<td>1</td>
<td>x</td>
<td></td>
<td></td>
<td>π±, ρ, ω</td>
</tr>
</tbody>
</table>

◇ At low energies, in terms of 5 independent SP amplitudes

◇ In analogy with the strong force, as a meon-exchange potential

- one weak, one strong vertex

- exchanges constrained by symmetries, e.g., Barton’s theorem excludes on-shell couplings to neutral scalar mesons (π^0, η, η’…)
  e.g., the work of Donoghue, Desplanques, Holstein
Such a description can make contact with QCD and the SM

Thus these meson-nucleon couplings are a meeting place between particle/nuclear physics:

- work upward from the SM and QCD to calculate these couplings
- work downward from experiment and an effective weak NN to extract Nature’s values for these couplings
\[ 2M_N V^{PNC}(\vec{r}) = i \frac{g_{\pi NN} f_\pi}{\sqrt{2}} \tau_\pi^\times \sigma_+ \cdot \vec{u}_\pi \]

\[-g_\rho \left[ h_\rho^0 \tau_1 \cdot \vec{r}_2 + h_\rho^1 \tau_+ + h_\rho^2 \tau_\pi \right] \left[ (1 + \mu_v) \sigma_+ \cdot \vec{u}_\rho + \sigma_- \cdot \vec{v}_\rho \right] \]
\[-g_\omega \left[ h_\omega^0 + h_\omega^1 \tau_\pi \right] \left[ (1 + \mu_s) \sigma_+ \cdot \vec{u}_\omega + \sigma_- \cdot \vec{v}_\omega \right] \]
\[-\tau_\pi^\times \sigma_+ \cdot \left[ g_\omega h_\omega^1 \vec{v}_\omega - g_\rho h_\rho^1 \vec{v}_\rho \right] \]
\[-\tau_\pi \cdot g_\rho h_\rho^1 \sigma_+ \cdot \vec{u}_\rho \]

where:
\[ \sigma_x \equiv \vec{r}(1) \times \vec{r}(2) \quad \sigma_+ \equiv \frac{1}{2} [\sigma(1) + \sigma(2)] \]
\[ \sigma_- \equiv \sigma(1) - \sigma(2) \quad \tau_\pi \cdot \vec{u}_\rho \frac{1}{2\sqrt{6}} \left[ 3\tau_\pi(1) \tau_\pi(2) - \vec{r}(1) \cdot \vec{r}(2) \right] \]
\[ \vec{u}(\vec{r}) \equiv \left[ \vec{p}, e^{-mr}/4\pi r \right] \quad \vec{v}(\vec{r}) \equiv \left\{ \vec{p}, e^{-mr}/4\pi r \right\} \]

Five principal short-range weak couplings, connected with vector meson exchange
One long-range interaction arising from isovector pion exchange
So the estimation of these couplings in the SM is an important challenge, with broad uncertainties due to the difficulty of the strongly-interacting environment.
These sounds like an effective theory -- long-range pion exchange and short-range contact interactions. Indeed:

- long-range $\pi$ exchange
- intermediate range
- analogous to heavy meson exchange in $m_\rho \to \infty$ limit

Total of 10 low-energy constants arise, with iterated pion exchange determining the mid-range potential.
Experiments: three types, carried out in NN, few, and many-body systems

- rates that would vanish if P conserved -- e.g., PNC $\alpha$ decay -- where observable $\propto |<V^{PNC}>|^2$

- pseudoscalar observables arising PC/PNC interference -- e.g., circular polarization in $\gamma$ decay from an unpolarized nucleus

$$\langle \vec{\sigma}_\gamma \cdot \vec{p} \rangle \leftrightarrow 2 \text{Re} \left[ \frac{\langle f \mid E1 \mid i \rangle}{\langle f \mid M1 \mid i \rangle} \right]$$

- observables involving PC/PNC interference, but not a pseudoscalar: second-order in $<V^{PNC}>$; e.g., odd power of $\cos\theta$ in the angular distribution of $\gamma$s from a decaying nuclear state

- Clearly ideal strategy would be 5+ observables in the NN system

  - but natural PNC/strong scale is $\frac{4\pi G_F m_\pi^2}{g^2_{\pi NN}} \sim 10^{-7}$

  - only one tight constraint so far obtained
- Nuclear measurements

- **advantage:** many opportunities to exploit level degeneracies, selection rules to enhance PNC effects far beyond the natural scale
- **disadvantage:** the complexity of the nuclear physics, which limits the accuracy of extracted PNC couplings

<table>
<thead>
<tr>
<th>Energy</th>
<th>Transition</th>
<th>$^3\text{He}$</th>
<th>$^3\text{H}$</th>
<th>$^3\text{He}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1081</td>
<td>$^1\text{S}_{0}^0$</td>
<td>3134 $^1\text{P}_{1}$</td>
<td>5337 $^1\text{P}_{1}$</td>
<td>3662 $^3\text{P}_{2}$</td>
</tr>
<tr>
<td>1042</td>
<td>$^1\text{S}_{0}^0$</td>
<td>0 $^1\text{S}_{0}^0$</td>
<td>2795 $^1\text{P}_{1}$</td>
<td>2789 $^3\text{P}_{2}$</td>
</tr>
<tr>
<td>3134</td>
<td>$^1\text{S}_{0}^0$</td>
<td>1 $^1\text{S}_{0}^0$</td>
<td>39 keV $^1\text{P}_{1}$</td>
<td>5.7 keV $^3\text{P}_{2}$</td>
</tr>
<tr>
<td>18F</td>
<td>$^1\text{S}_{0}^0$</td>
<td>$^1\text{S}_{0}^0$</td>
<td>110 $^1\text{P}_{1}$</td>
<td>101 keV $^3\text{P}_{2}$</td>
</tr>
<tr>
<td>19F</td>
<td>$^1\text{S}_{0}^0$</td>
<td>$^1\text{S}_{0}^0$</td>
<td>$^1\text{S}_{0}^0$</td>
<td>3/2 $^1\text{P}_{1}$</td>
</tr>
<tr>
<td>21Ne</td>
<td>$^1\text{S}_{0}^0$</td>
<td>$^1\text{S}_{0}^0$</td>
<td>$^1\text{S}_{0}^0$</td>
<td>3/2 $^1\text{P}_{1}$</td>
</tr>
</tbody>
</table>
- Enhancements: example of $^{18}$F

\[ P_\gamma(1081 \text{ keV}) = 2 \text{Re} \left[ \frac{\langle + | V_{PNC} | - \rangle}{39 \text{ keV}} \frac{\langle gs | M1 | + \rangle}{\langle gs | E1 | - \rangle} \right] \]

**typical PNC nuclear matrix**
- element 1.0 eV
- energy scale 100 times typical nuclear scale ~ few MeV

⇒ \( \sim 10^{-5} \) vs natural scale \( 10^{-7} \)

**PC E1:** isoscalar E1 in a self-conjugate nucleus means leading-order forbidden

**PNC M1:** unusually strong (10 W.u.) enhancement \( \sim 110 \)

◊ so expected effect \( \sim 10^{-3} \)

◊ following early work of Barnes et al., heroic efforts by Queens and Florence groups, found a circular polarization smaller than expected

\( (8 \pm 39) \times 10^{-5} \) vs \( (208 \pm 49) \times 10^{-5} \) DDH best value
What about the nuclear physics uncertainties inherent in the nuclear matrix element of the weak NN potential? In some cases there are ways to eliminate much of the uncertainty.

Axial charge operator that controls such a beta decay is essentially identical to \( V_{PNC} \).

Can show that the matrix element we need can be taken from the beta decay measurement.
In the end, one has a rather limited set of accurate, low-energy PNC measurements from which one can reliably extract the underlying PNC couplings -- the couplings we would then like to compare to the results of QCD-based SM calculations.

\[
\begin{align*}
A^+_L (45 \text{ MeV}) &= (-1.57 \pm 0.23) \times 10^{-7} \\
A^+_L (46 \text{ MeV}) &= (-3.34 \pm 0.93) \times 10^{-7} \\
P^{18}_y (1081 \text{ keV}) &= (12 \pm 38) \times 10^{-5} \\
A^{19}_y (110 \text{ keV}) &= (-7.4 \pm 1.9) \times 10^{-5}
\end{align*}
\]

◊ not the whole story, as we have some additional constraints from
- \( \vec{p} + p \) scattering at intermediate energies
- an atomic quantity called the anapole moment (later)
The observables are the circular polarization $P_\gamma$ of the $\gamma$-ray emitted in the decay of the $1081$ keV state in $^{18}$F and angular asymmetry $A_\gamma$ for the decay of the $110$ keV state in polarized $^{19}$F.

Figure 8: Constraints on the PNC meson couplings ($\times 10^7$) that follow from the results in Table 4. The error bands are one standard deviation. The illustrated region contains all of the DDH “reasonable ranges” for the indicated parameters.

Although the PNC parameter space is six-dimensional, two coupling constant combinations, so a simple pattern has not yet emerged.
Some weak conclusions can be drawn

- one key goal -- to measure the weak neutral current by isolating the isovector weak hadronic weak interaction -- not achieved: the size of this contribution is smaller than expected

- but the isoscalar contribution is at least comparable to DDH best values

Superficially similar to the $\Delta I=1/2$ rule

But stronger conclusions will have to await the SNS cold-neutron and other future high-precision measurements
II. Electric Dipole Moments

- Permanent electric dipole moments of an elementary particle or composite system requires time-reversal and parity violation:

\[ H_{edm} = d \cdot \vec{E}' \cdot \vec{s}' \]

By the CPT theorem, a nonzero T-violating edm implies CP violation

- Two important motivations for edm searches
  - CP-odd phases show up generically in the standard model and its extensions: the SM contains two, the QCD $\theta$ parameter and the CKM phase in the quark mixing matrix
  - the need for sufficient CP violation to account for baryogenesis -- which appears to require beyond-the-SM sources
• Experimental sensitivity: the dipole moment of a classical charge distribution is

\[ \mathbf{d} = \int d^3x \, \mathbf{x} \rho(\mathbf{x}) \]

The most stringent limit on an edm is that for the neutral atom \(^{199}\text{Hg}\), \(d(^{199}\text{Hg}) < 2.1 \times 10^{-28}\) e cm.

If this atom were expanded to the size of the earth, such an edm would correspond to a shell of excess charge (difference between + and - charge) at the poles of thickness \(\sim 0.001\) angstroms.

The limit on the precession in the applied field (\(\sim 10^5\) v/m) corresponds to a bound on shifts in atomic levels of \(\sim 10^{-25}\) eV.
• General classification of electromagnetic moments:

<table>
<thead>
<tr>
<th>Multipole</th>
<th>P-even, T-even</th>
<th>P-odd, T-odd</th>
<th>P-odd, T-even</th>
<th>P-even, T-odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_j^M$</td>
<td>even $J \geq 0$</td>
<td>odd $J \geq 1$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>$M_j^M$</td>
<td>odd $J \geq 1$</td>
<td>even $J \geq 2$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>$E_j^M$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>odd $J \geq 1$</td>
<td>even $J \geq 2$</td>
</tr>
</tbody>
</table>

The edm is the C1 moment: additional P-odd, T-odd moments include the C3, C5.... and M2, M4..., if the object has the necessary spin $\geq 1$

• General current for a spin-1/2 fermion:  

$$\langle p' | j_{\mu}^{em} | p \rangle =$$

$$\bar{N}(p') \left( F_1 \gamma_\mu + F_2 \sigma_{\mu\nu} q^\nu + \frac{a(q^2)}{M^2} (q \gamma_\mu - q^2 \gamma_\mu) \gamma_5 + d(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 \right) N(p)$$

Charge    Magnetic    Anapole    Electric Dipole
Aside: the P-odd T-even anapole moment is a quantity important to the previous section: parity violation in the nucleus generates a PNC atomic interaction which depends on the nuclear spin.

One of a set of weak radiative corrections -- generates a V(e)-A(N) interaction that can generate a hyperfine dependence in atomic parity violation experiments.
Some very curious properties: grows as $A^{2/3}$

Consequently, for heavy atoms, is the largest contribution to nuclear-spin-dependent PNC, dominating the tree-level $Z_0$ $V(e)$-$A(N)$ interaction, which is suppressed by $1 - 4 \sin^2 \theta_W$

And is a contact interaction -- just like the ordinary weak interaction

Measured only once - in the JILA Cs experiment

The anapole moment is a toroidal current winding

$$H_{anapole} = \frac{G_F}{\sqrt{2}} \kappa \vec{\alpha} \cdot \vec{I} \rho(\vec{r})$$

with $\kappa$ a function of the PNC weak meson-nucleon couplings
Electric Dipole Moment Experiments

- e/p/n edm experiments break into three general categories
  - neutron edm experiments
  - paramagnetic (unpaired electrons) atoms or molecules with sensitivity to the electron edm
  - diamagnetic atoms (electrons paired, nonzero nuclear spin) with sensitivity to the p and n edm and to CPNC nuclear interactions

- Key limits, done in neutral systems, in units e cm

<table>
<thead>
<tr>
<th>Particle</th>
<th>edm limit</th>
<th>system</th>
<th>SM prediction*</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>$1.9 \times 10^{-27}$</td>
<td>atomic $^{205}$Tl</td>
<td>$10^{-38}$</td>
</tr>
<tr>
<td>p</td>
<td>$6.5 \times 10^{-23}$</td>
<td>molecular TIF</td>
<td>$10^{-31}$</td>
</tr>
<tr>
<td>n</td>
<td>$2.9 \times 10^{-26}$</td>
<td>ultracold n</td>
<td>$10^{-31}$</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>$2.1 \times 10^{-28}$</td>
<td>atom vapor cell</td>
<td>$10^{-33}$</td>
</tr>
</tbody>
</table>

*CKM phase
One of the main motivations for theory work is the variety of 2nd-generation experiments planned; in the case of **hadronic edms**

<table>
<thead>
<tr>
<th>System</th>
<th>Group(s)</th>
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<tbody>
<tr>
<td>ultracold n</td>
<td>ILL, PSI, Munich, SNS</td>
</tr>
<tr>
<td>$^{199}$Hg vapor cell</td>
<td>Seattle*</td>
</tr>
<tr>
<td>$^{129}$Xe (liquid)</td>
<td>Princeton</td>
</tr>
<tr>
<td>$^{225}$Ra (trapped)</td>
<td>Argonne</td>
</tr>
<tr>
<td>$^{213,225}$Ra (trapped)</td>
<td>KVI</td>
</tr>
<tr>
<td>$^{223}$Rn (trapped)</td>
<td>TRIUMF</td>
</tr>
<tr>
<td>deuteron (ring)</td>
<td>BNL (proposed)</td>
</tr>
</tbody>
</table>

*currently running
Simple example: generation of a nuclear edm

- Example of the QCD $\bar{\theta}$ parameter -- one of two sources of SM CPNC
  \[ \bar{\theta} \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \Rightarrow \bar{\theta} \text{ the parameter to be constrained} \]

- Induces a scalar CPNC $\pi NN$ coupling, the leading $\ln (M/m_\pi)$ contribution determined by current algebra
  \[ L_{\pi NN} \rightarrow L_{\pi NN}^{CPNC} = \vec{\pi} \cdot \vec{N} \vec{\tau} (i\gamma_5 g_{\pi NN} + \bar{g}_{\pi NN}) N \]
  \[ |\bar{g}_{\pi NN}| \sim 0.027|\theta| \]

- Nuclear edm = 1-body + polarization + exchange current
• Expect **one-body edm** to be dominated by pion loops, as the lightest meson allows the greatest charge separation

\[ \Rightarrow d_n \sim \frac{e g_{\pi NN} \bar{g}_{\pi NN}}{4\pi^2M} \ln \left( \frac{M}{m_{\pi}} \right) \]

\[ \sim 3.6 \times 10^{-16} \bar{\theta} \text{ e cm} \]

\[ \Rightarrow \bar{\theta} < 10^{-10} \]

Unnatural size of \( \theta \) \( \Rightarrow \) the strong CP problem

• Apparent that the scale

\[ d_n \sim \frac{1}{M} \ln \left( \frac{M}{m_{\pi}} \right) \sim r_N \sim 0.4f \]

so interaction energy depends on the voltage drop across nucleus (with fields of \( 10^5 \) v/m typical)
• The polarization terms depends on the NN CPNC potential

\[ V_{1,2}(r) = -0.9 \, d_n \, m^2_\pi \, \vec{\tau}(1) \cdot \vec{\tau}(2) \left( \vec{\sigma}(1) - \vec{\sigma}(2) \right) \cdot \hat{r} \, \frac{e^{-m_\pi r}}{m_\pi r} \left[ 1 + \frac{1}{m_\pi r} \right] \]

• So we find the overall scale of the polarization term

So the overall scale of polarization term

\[ d_n \, (m_\pi r_N)^2 \, \frac{\Delta E^{\Delta \pi=1}}{\Delta E} \approx 10d_n \, \frac{E^{\Delta \pi=1}}{\Delta E} \]

Possible enhancements in cases of a parity doublet: in some special cases enhancements of \( \sim 1000 \)

• so one expects
  ◇ the polarizability to generally dominate the edm of a heavy nucleus
  ◇ potentially large enhancements in cases where a ground-state parity doublet exists, coupled by a dipole transition of reasonable strength
• One kind of T-odd enhancement comes from collective nuclear motion

Familiar quadrupole case: deformed intrinsic state breaks spherical symmetry, which is restored by the “Goldstone” mode of rotations

Octupole deformation: deformed intrinsic state and its parity reflection can be combined

\[ |\text{even}\rangle = |+\rangle + |-\rangle \]
\[ |\text{odd}\rangle = |+\rangle - |-\rangle \]

Deformation violates T and P: symmetry restored by collective motion, yielding parity doublets

Motivation for \(^{225}\text{Ra}\) measurement: mixing of nearly degenerate states

FIG. 1: (color online). Shape of the microscopically calculated [13] mass distribution in \(^{225}\text{Ra}\), represented here by the surface of a uniform body that has the same multipole moments \(Q_{\lambda 0}\) for \(\lambda=0\ldots4\) as our calculated density.

From Dobaczewski and Engel
• In the previous discussion we have simplified the general case where CP-violating couplings are not restricted to the nucleus.
Also must account for Schiff screening in the case of atomic edms

- Measurable in a diamagnetic atom is the response of a neutral atom to an applied field: what if the edm resides on the nucleus?

- As Schiff and many others have discussed, classical result for a point-like nucleus is that the change in interaction energy linear in $E$ and $d_{nuclear}$

\[ E_{ext} \]

Atom neutral: no net displacement in applied field

Nucleus charged but not accelerated -- sum of applied and induced fields must cancel at the nucleus: no edm interaction energy!
• But there are loopholes due to several corrections (Schiff 63; Sandars 68)
  ◊ nuclear finite-size effects
  ◊ relativistic effects
  ◊ magnetic interactions between the electrons and nucleus

Following Schiff, we can evaluate the finite-size corrections

**The problem:** the naive interaction energy due to a nuclear edm is

$$\Delta E \sim \alpha \, \vec{E}_{\text{ext}} \cdot \vec{J} \, d_N$$

What is the **residual response** after the screening is evaluated? Can attack this as an effective theory in \((R_N / R_A)\)
Expand photon propagator

\[
\frac{1}{|\vec{r}_N - \vec{r}_e|} \sim \frac{1}{|\vec{r}_e|} + \theta(r_N - r_e)[...]
\]

penetration

Exactly cancel (Schiff)

Surviving Contribution
• We find that the surviving contribution is dimensionally of order

\[ \Delta E \sim \alpha \vec{E}_{ext} \cdot \vec{J} \, d_N \left( \frac{R_N}{R_A} \right)^2 \]

• This makes the problem very interesting: an effective theory where the LO and NLO corrections vanish. So must find and evaluate all terms up to NNLO -- what is, up to relative order \( (R_N/R_A)^2 \)

• This includes interactions at the nucleus requiring gradients of the field
  - M2 (T-odd current): \( R_N/R_A \)
  - C3: \( (R_N/R_A)^2 \)

• Opens up some very interesting experimental possibilities, especially for experiments with trapped atoms having nonzero atomic spins -- C3 is the collective octupole operator

\[ \text{C1} \quad \text{e} \quad \text{N} \quad \text{M2} \times \text{M2} \quad \text{or} \quad \text{C3} \times \text{C3} \]
• For a spin-1/2 nucleus the simple, uncancelled finite size contribution to the atomic interaction energy depends on a contact-like atomic potential whose strength is proportional to the “Schiff moment” $S$

$$-4\pi\alpha\langle\psi_p| \hat{S} \cdot [R_A^4 \nabla^3 \delta^3(x)] |\psi_s\rangle$$

where $S$ is given by the nuclear ground-state matrix element

$$\langle gs| \hat{S} |gs\rangle = \frac{1}{10} \left[ \langle gs| \left( \frac{y}{R_A} \right)^2 \bar{y} |gs\rangle - \frac{5}{3Z} \langle gs| \vec{d}_N |gs\rangle \langle gs| \left( \frac{y}{R_A} \right)^2 |gs\rangle \right]$$

explicitly suppressed by $(R_N/R_A)^2$, the penetration finite-size effect

• Some additional and interesting terms arising from second- and other higher-order diagrams, involving nuclear excited states
\(^{199}\text{Hg}\) vapor cells

- Number of \(^{199}\text{Hg}\) atoms: \(10^{14}\)
- Leakage currents at 10 kV: 0.5 – 1 pA
- \(\text{N}_2 + \text{CO}\) buffer gas (500 Torr)
- Paraffin wall coating
- Spin relaxation time: 100 – 200 sec

\(d^{(199}\text{Hg} < 2.1 \times 10^{-28} \text{e cm (95% c.l.) (2001)}}\)

2008 goal is the control of statistical and systematic effects at 1.5 \(\times 10^{-29}\) e cm

Univ Washington/JILA/Princeton
n edm limit $2.9 \times 10^{-26}$ e cm (2006)
goals $5 \times 10^{-27}$ e cm (2010)
$5 \times 10^{-28}$ e cm (2015)
edm flow chart

Fundamental CP-violating Phases

- $d_q, \tilde{d}_q, \theta$

  - NN interaction

  - Schiff moment

  - edms of Hg, Ra (diamagnetic)

- $d_e$

  - edms of Tl,... (paramagnetic)

  - n edm

(based on a slide by Flambaum)
electron edms: experiment and theory

slide from Paul Hamilton
III. Family and Total Lepton Number Conservation

One of the major discoveries of the past decade has been that of neutrino mass and oscillations -- a demonstration that “family number” is not preserved among the leptons.

So we have determined that family-number-violating leptonic processes occur
e.g., $\nu_e \leftrightarrow \nu_\mu$
and have determined the parameters governing this process
$\delta m^2 = m_2^2 - m_1^2$ and $\theta_{12}$

![Graph showing mass squared values with atmospheric and solar oscillations](image-url)
Neutrino mixing status: $\theta_{12}, \theta_{23}$

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
e^{i\phi_1} \nu_2 \\
e^{i\phi_2} \nu_3
\end{pmatrix}
\]

= 
\begin{pmatrix}
1 \\
c_{23} & s_{23} \\
-s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & s_{13}e^{-i\delta} \\
-c_{13}s_{13}e^{i\delta} & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} \\
-s_{12} & c_{12}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
e^{i\phi_1} \nu_2 \\
e^{i\phi_2} \nu_3
\end{pmatrix}

atmospheric $\nu_e$ disappearance solar $\nu_e$ disappearance

results: $\theta_{23} \sim 45^\circ$ $\sin \theta_{13} \leq 0.17$ $\theta_{12} \sim 30^\circ$

We can study oscillations in the three channels defined by $\nu_1, \nu_2, \nu_3$

We also note that the mixing matrix contains three CP-violating phases, further enriching the discussion just finished (with implications for baryogenesis)
Knowing the family number is violated in reactions

\[ \sum_{in} l_e \neq \sum_{out} l_e \]

motivates other tests of family number, including

\[ \mu \rightarrow e + \gamma \]

\[ \mu^- + (N, Z) \rightarrow e^- + (N, Z) \]

with the latter, so far unobserved process being one of the physics goals of FermiLab’s “intensity frontier” program

But today’s last discussion will be of a symmetry so far not found to be broken, total lepton number, and its relevance to neutrino mass

\[ \sum_{in} l_e + l_\mu + l_\tau = \sum_{out} l_e + l_\mu + l_\tau \]
Particle-antiparticle conjugation: We know other SM particles, like the electron have a distinct antiparticle, as $e^- \rightarrow e^+$ under charge conjugation.

But the neutrino has no charge or other distinguishing additive quantum numbers, raising the question -- are the neutrinos produced in $\beta^-$ and $\beta^+$ decay the same?

So we do an experiment:

\[\begin{array}{c}
\text{\(e^+\)} \\
\text{\(\beta^+\) source} \\
\hline
\text{\(\nu_e\)} \\
\hline
\text{\(e^-\)} \\
\end{array}\]

This defines the $\nu_e$ which is then found to produce: $e^-$
and a second one:

- with these definitions of the $\nu_e$ and $\nu_e$, they appear operationally distinct, producing different final states.
- introduce a “charge” to distinguish the neutrino states and to define the allowed reactions, $l_e$, which we require to be additively conserved.

$$\sum_{in} l_e = \sum_{out} l_e$$
- historically connected with the development of the Cl solar neutrino detector: after Pontecorvo’s suggestion, Alvarez did a carefully background study for this detector for a potential reactor experiment, but did not pursue a measurement

- Davis’s BNL program included a Savannah River experiment in which reactor anti-neutrinos

\[ ^{37}\text{Cl} + \bar{\nu}_e \rightarrow ^{37}\text{Ar} + e^- \]

failed to produce Ar, indicating that the $\nu_e$ and $\bar{\nu}_e$ are distinct at $\sim 5\%$, a prejudice embedded in the standard model
These experiments are done virtually in **neutrinoless $\beta\beta$ decay**

parent nucleus (A,Z) \rightarrow (A,Z+1) \rightarrow daughter (A,Z+2)

- only SM fermion where this question of identity under particle-antiparticle conjugation arises: other fermions carry charges
- the arguments make a **tacit assumption** about neutrino helicity
Circa 1957

• parity was used early in the 1920s to classify atomic wave functions and atomic transitions: in 1927 Wigner showed that “Laporte’s rule” was a consequence of the mirror symmetry of the electromagnetic force

• in 1956 Lee and Yang considered the tau-theta puzzle, the apparent existence of a pair of equal-mass mesons, one of which has negative parity and decays into three pions, the other with positive parity and decaying into two pions: observed that the experimental support for parity conservation was limited to the strong and E&M interactions

• parity violation demonstrated by
  ◊ Wu, Ambler, Hayward, Hoppes, and Hudson: observed the angular asymmetry of the $\beta$s from the decay of polarized $^{60}$Co
  ◊ Garwin, Lederman, and Weinrich: established large $\mu$ polarization in $\pi$ $\beta$-decay from the angular distribution of $\mu$-decay electrons

GGS experiment: $v_s$ are left-handed, $\bar{v}_s$ are right-handed,
If the weak interaction produces left-handed $\nu$s and right-handed $\bar{\nu}$s, let’s re-examine
Remove the restriction of an additively conserved lepton number

\[ e^- \xrightarrow{\nu_e} \nu_e \xrightarrow{\nu_e} e^- \]

\[ \text{allowed, with a rate proportional to } G_F^4 \]
and account for suppressed rates by the nearly exact handedness

\[ e^- \rightarrow \nu e_{RH} \rightarrow \nu e_{LH} \rightarrow e^- \]

allowed, but suppressed with a rate proportional to \( G_F^4 \left( \frac{m_\nu}{E_\nu} \right)^2 \)

the \( \gamma_5 \)-invariance is not exact if the \( \nu \) has a mass as the “RH-ed” \( \nu \) state with then contain a small piece of LH-ed helicity proportional to \( m_\nu/E_\nu \)

where \( E_\nu \sim 1/R_{\text{nuclear}} \)

Because of PNC, there is no need for an additively conserved quantum number to distinguish \( \nu \) and \( \overline{\nu} \), unlike the case for other SM fermions
Massive neutrino descriptions

Majorana:
\[ \nu_{\text{LH}} \quad \text{boost} \quad \nu_{\text{RH}} \]

CPT

Dirac:
\[ \nu_{\text{LH}} \quad \nu_{\text{LH}} \quad \nu_{\text{RH}} \quad \nu_{\text{RH}} \]

Lorentz invariance

Boost

or some linear combinations of the two
Let’s see the mass consequences: start with the Dirac eq., project out

\[ \psi_{R/L} = \frac{1}{2} (1 \pm \gamma_5) \psi \]  \quad \quad \quad C \, \psi_{R/L} \, C^{-1} = \psi^c_{R/L} \]

Allow for flavor mixing

\[ L_m(x) \sim m_D \psi(x) \psi(x) \Rightarrow M_D \bar{\Psi}(x) \Psi(x) \]  \quad \quad \quad \Psi_L = \begin{pmatrix} \Psi^e_L \\ \Psi^\mu_L \\ \Psi^\tau_L \end{pmatrix} \]

To give the mass 4n by 4n matrix

\[
(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & M^T_D \\ 0 & 0 & M_D \\ M^*_D & 0 & 0 \\ M^*_D & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}
\]
Observe that the handedness allows an additional generalization

\[ L_m(x) \Rightarrow M_D \bar{\Psi}(x) \Psi(x) + (\bar{\Psi}_L^c(x) M_L \Psi_L(x) + \bar{\Psi}_R^c(x) M_R \Psi_R(x) + h.c.) \]

to give the more general matrix

\[
\begin{pmatrix}
0 & 0 & M_L & M_D^T \\
0 & 0 & M_D & M_R^\dagger \\
M_L^\dagger & M_D^\dagger & 0 & 0 \\
M_D^* & M_R & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Psi_L^c \\
\Psi_R \\
\Psi_L \\
\Psi_R^c
\end{pmatrix}
\]

which has a number of interesting properties

- the eigenvectors are two-component Majorana spinors: 2n of these
- the introduction of \( M_L, M_R \) breaks the global invariance \( \Psi \rightarrow e^{i\alpha} \Psi \) associated with a conserved lepton number
Why is all of this important?

- neutrino masses are physics beyond the standard model

- we have learned that neutrinos are massive but much lighter than other SM fermions? how are we to build a new unified model that accounts for such a disparity?

- neutrinos are special: because they lack charges, we can give them two kinds of masses, resolving our problem

\[
\begin{pmatrix}
0 & M_D \\
M_D & M_R
\end{pmatrix}
\rightarrow
m_{\nu}^{\text{light}} \sim M_D \left(\frac{M_D}{M_R}\right)
\]

(the needed small parameter)

- what we have learned about neutrinos suggests \(M_R \sim 0.3 \times 10^{15}\)GeV

- Majorana masses break total lepton number, and we can search for this symmetry breaking in nuclear experiments
2 Neutrinos meet the Higgs boson

Murayama’s $\nu$ mass cartoon

standard model masses

light Dirac neutrino

LHed Majorana neutrino

$\leftarrow$ the anomalous $\nu$ mass scale
• Majorana masses break total lepton number, and we can search for this symmetry breaking in nuclear $\beta\beta$ decay experiments because of the nuclear pairing force, have a “filter” for isolating a very rare second-order weak process -- one of two possibilities in nature for seeing such an exotic process

Ambitious new experiments like CUORE, Majorana/Gerda, EXO, .... to probe $\nu$ masses to the 10 milli-eV level
The precision frontier is likely to be one of our best windows on physics of and beyond the standard model

◊ clever tricks to isolate interesting interactions: exploit the spins, parities, and energies of atoms and nuclei

◊ exquisite sensitivities: atomic shifts of $10^{-26}$ eV, decay lifetimes of $10^{26}$ years

◊ enhancements of interesting interactions

◊ sensitivities that in some cases are unique, such as sensitivity to GUT scales in the seesaw mechanism

and lots left to do