Testing Nucleonic Forces with Three Nucleon Reactions

H. Witała¹, J. Golak¹, R. Skibiński¹, W. Gloeckle², H. Kamada³, A. Nogga⁴

¹Jagiellonian University, Cracow, Poland
²Ruhr Universitaet, Bochum, Germany
³Kyushu Institute of Technology, Kitakyushu, Japan
⁴Kernforschungzentrum Juelich, Juelich, Germany

- equations
- 3N force effects
- relativistic effects
- summary
3N scattering:

- the elastic scattering transition amplitude \( \langle \phi' | U | \phi \rangle \):

\[
\langle \phi' | U | \phi \rangle = \langle \phi' | P G_0^{-1} + PT | \phi \rangle
\]

- the breakup transition amplitude:

\[
\langle \vec{p} \vec{q} | U_0 | \phi \rangle = \langle \vec{p} \vec{q} | (1 + P)T | \phi \rangle
\]

- Faddeev type equation for T:

\[
T | \phi \rangle = tP | \phi \rangle + (1 + tG_0)V_4^{(1)}(1 + P) | \phi \rangle + tPG_0T | \phi \rangle + (1 + tG_0)V_4^{(1)}(1 + P)T | \phi \rangle
\]

- we solve it in a partial wave momentum space basis
3N scattering:

The dynamical input:

- off-shell t-matrix $t$ calculated from a given NN potential $V$ by the Lippmann-Schwinger equation

\[ t = V + VG_{0}t \]

- $3NF V_{4}$ with $V_{4}^{(i)}$ a part of it which is symmetrical under exchange of nucleons $j$ and $k$ (ijk cyclical = 1,2 or 3)

\[ V_{4} = V_{4}^{(1)} + V_{4}^{(2)} + V_{4}^{(3)} \]
CD Bonn: solid

Nd data: dots

CD Bonn: solid
CD Bonn + TM99: dashed
AV18 + TM99: squares
AV18 + UrIX: circles
light shade band: AV18, CD Bonn, NijmI, NijmII


solid line: AV18+UrIX

circles: RIKEN pd data

solid circles and squares: RCNP nd data

open circles: RCNP pd data

K.Sekiguchi ..., P.R.C65,034003(2001)  
Y.Maeda..., P.R.C76,014004(2007)
light shade band: AV18, CD Bonn, NijmI, NijmII


solid line: AV18+UrIX

circles: IUCF pd data
light shade band: AV18, CD Bonn, NijmI, NijmII
solid line: AV18+UrIX
circles: IUCF pd data
Nonrel. and rel. 3N calculations:

- form of the free propagator $G_0$ differs
- different choice of momenta used to describe the configuration of three nucleons:
  
  **nrel**: the standard Jacobi momenta $(\vec{p}, \vec{q})$
  
  **rel**: relative momentum in the 2N (2-3) c.m. subsystem $\vec{k}$ and momentum $\vec{q}$ of the nucleon (1) in the 3N c.m. system $(\vec{k}, \vec{q}')$

- interacting 2N subsystem (2-3) has **nonzero total momentum** ($\vec{p} = -\vec{q}$ in the 3N c.m. system): it leads in **relativistic** case to the **boosted potential** $V$

  $$V \equiv \sqrt{\left[2\sqrt{\vec{k}^2 + m^2} + \nu_{rel}\right]^2 + \vec{p}^2}$$
  $$\quad - \sqrt{\left[2\sqrt{\vec{k}^2 + m^2}\right]^2 + \vec{p}^2}$$

- The Lorentz transformation from 2N c.m. to 3N c.m. is performed along the **total momentum** of the 2N subsystem, which in general is **not parallel** to momenta of these nucleons. This leads to a **Wigner rotation** of spin states. When defining 3N partial wave states care must be taken about spin states.
$d(n,n)n, \ E_{\text{lab}}^n=250$ MeV

light shade band: AV18, CD Bonn, NijmI, II
dark shaded band: NN+TM99
solid: nonrelativistic CD Bonn
dashed: relativistic CD Bonn
d(n,n)d

CD Bonn
\[ d(n,n_1n_2)p \]

\[ E_{\text{lab}}^n = 200 \text{ MeV} \]

dashed line: nonrelativistic
solid line: relativistic

fixed \( \theta_2 = 37.5^\circ \) and \( \phi_{12} = 180^\circ \)

a) \( \theta_1 = 27.5^\circ \)
b) \( \theta_1 = 32.5^\circ \)
c) \( \theta_1 = 37.5^\circ \)
d) \( \theta_1 = 42.5^\circ \)
e) \( \theta_1 = 47.5^\circ \)
f) \( \theta_1 = 52.5^\circ \)
g) \( \theta_1 = 57.5^\circ \)
h) \( \theta_1 = 62.5^\circ \)
pd data: J. Zejma..., PRC 55, 42 (1997)

pd data: W. Pairsuwan..., PRC 52, 2552 (1995)
- parameter-free $2\pi$ exchange 3NF:

$$V_{\text{TPE}}^{3\text{NF}} = \sum_{i\neq j\neq k} \frac{1}{2} \left( \frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \circ \vec{q}_i)(\vec{\sigma}_j \circ \vec{q}_j)}{(q_i^2 + M^2_\pi)(q_j^2 + M^2_\pi)} F^{\alpha\beta}_{ijk} \tau_i^{\alpha} \tau_j^{\beta}$$

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta} \left[ -\frac{4c_1 M^2_\pi}{F^2_\pi} + \frac{2c_3}{F^2_\pi} \vec{q}_i \circ \vec{q}_j \right] + \sum_\gamma \frac{c_4}{F^2_\pi} \epsilon^{\alpha\beta\gamma} \tau_k^{\gamma} \vec{\sigma}_k \circ \left[ \vec{q}_i \times \vec{q}_j \right]$$

- one-pion exchange 3NF depends on constant $c_D$:

$$V_{\text{OPE}}^{3\text{NF}} = -\sum_{i\neq j\neq k} \frac{g_A}{8F^2_\pi} c_D \frac{\vec{\sigma}_j \circ \vec{q}_j}{q_j^2 + M^2_\pi} (\vec{\tau}_i \circ \vec{\tau}_j)(\vec{\sigma}_i \circ \vec{q}_j)$$

- contact 3NF depends on constant $c_E$:

$$V_{\text{cont}}^{3\text{NF}} = \frac{1}{2} \sum_{j\neq k} c_E (\vec{\tau}_j \circ \vec{\tau}_k)$$

FAC A: $c_D = -1.1128$, $c_E = -0.6590$, $\Lambda = 500$ MeV

FAC B: $c_D = 8.1413$, $c_E = -2.0289$, $\Lambda = 500$ MeV
Summary

- no doubt 3NF is required to understand 3N reactions
- small relativistic effects in elastic Nd scattering
- with increased energy importance of shorter range 3NF components
- application of higher order $\chi$PT 3NF contributions will help to explain discrepancies in elastic scattering cross sections and spin observables seen at energies above $\sim$100 MeV