On Anomalies and Anomalous Interactions of Vector Particles

Kirill Melnikov and Arkady Vainshtein
work in progress
Introduction

New, very interesting, anomaly mediated interactions were recently introduced by Jeff Harvey, Cris Hill and Richard Hill, arXiv: 0708.1281, 0712.1281.

They argue that anomaly considerations lead to a new interaction which may explain the event excess in the neutrino Mini Boone experiment.

While we have no objections to the general theory we think that its applications to neutrino processes involving anomalous $\omega$ interactions is ambiguous.
Perturbative calculations

Triangle amplitude

\[ T_{\mu\nu} = \langle 0 | \hat{T}_{\mu\nu} | \gamma(q) \rangle \]

\[ \hat{T}_{\mu\nu} = \sum_f 2I^f_3 \hat{T}^f_{\mu\nu} = i \int d^4x e^{ikx} T \{ J_\mu(x) J^5_\nu(0) \} \]

Fermion propagator in the electromagnetic field

\[ S(p) = \frac{1}{\hat{p} - m_f} - \frac{1}{(p^2 - m_f^2)^2} eQ_f \tilde{F}_{\rho\delta} \left( p^\rho \gamma^\delta - \frac{i}{2} m_f \sigma^{\rho\delta} \right) \gamma_5. \]

\[ \tilde{F}_{\rho\delta} = (1/2) \epsilon_{\rho\delta\alpha\beta} F^{\alpha\beta} \]

\[ \hat{T}^f_{\mu\nu} = -\frac{eQ_f^2 N_f}{4\pi^2} \int_0^1 d\xi \frac{\xi(1 - \xi)(k_\mu k^\rho \tilde{F}_{\rho\nu} + k_\nu k^\rho \tilde{F}_{\rho\mu}) - m_f^2 \tilde{F}_{\mu\nu}}{m_f^2 - \xi(1 - \xi)k^2} \]
\[
\hat{T}^{\text{phys}}_{\mu\nu} = \hat{T}_{\mu\nu}(m_f) - \lim_{m_f \to \infty} \hat{T}_{\mu\nu}(m_f) = \hat{T}_{\mu\nu}(m_f) - \frac{eQ_f^2}{4\pi^2} \tilde{F}_{\mu\nu}
\]

\[
= \frac{eQ_f^2 N_f}{4\pi^2 k^2} \left( - k^2 \tilde{F}_{\mu\nu} + k_\mu k_\rho \tilde{F}_{\rho\nu} + k_\nu k_\rho \tilde{F}_{\rho\mu} \right) \left( 1 + \frac{2m_f^2}{\beta k^2} \ln \frac{\beta + 1}{\beta - 1} \right)
\]

Generically

\[
T_{\mu\nu} = \frac{-i|e|}{4\pi^2} \left[ w_T(k^2) \left\{ - k^2 f_{\mu\nu} + k_\mu k_\alpha f_{\alpha\nu} - k_\nu k_\alpha f_{\alpha\mu} \right\} + w_L(k^2) k_\nu k_\alpha f_{\alpha\mu} \right]
\]

Perturbatively

\[
w_f^L = 2w_f^T = \frac{2N_f Q_f^2}{K^2} \left( 1 - \frac{2m_f^2}{\beta K^2} \ln \frac{\beta + 1}{\beta - 1} \right)
\]

No higher order corrections in strong interactions in the chiral limit \( m_f = 0 \). Generalizes the Adler-Bardeen theorem to \( w_T \).

\[
\text{Im} T_{\mu\nu} = \text{Im} T_{\nu\mu} \quad k^2 \text{Im}[w_T(k^2)] = 0, \quad 2 \text{Im}[w_T(k^2)] = \text{Im}[w_L(k^2)]
\]

‘2003 Czarnecki, Marciano and A.V.

‘2003 A.V.

‘2004 Knecht, Peris, Perrottet and E. de Rafael

‘2006 Jegerlehner and Tarasov
Non-perturbative effects and OPE

\[ \hat{T}_{\mu\nu} = \sum_i c^{(i)\alpha_1...\alpha_i}_\mu\nu O^{(i)}_{\alpha_1...\alpha_i}. \]

The leading operator

\[ O_F = |e|/(4\pi^2) \tilde{F}_{\alpha\beta}. \]

The leading non-perturbative effect comes from four-fermion operator

\[ \frac{8\pi\alpha_s Q_q}{k^6} \bar{q} t^\alpha (\gamma_\alpha \hat{k} \gamma_\mu - \gamma_\mu \hat{k} \gamma_\alpha) q \otimes \bar{q} t^\beta (\gamma_\nu \hat{k} \gamma_\alpha - \gamma_\alpha \hat{k} \gamma_\nu) \gamma_5 q. \]

\[ \Delta^{(6)} w_T[u, d] = -\alpha_s(K^2) \frac{(0.7 \text{ GeV})^4}{K^6}. \]

The model

\[ w_T[u, d] = \frac{1}{m_{a_1}^2 - m_\rho^2} \left( \frac{m_{a_1}^2 - m_\pi^2}{K^2 + m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{K^2 + m_{a_1}^2} \right). \]
What is different in case of baryon current considered by HHH?

\[ T_{\mu \nu}^{H,f} = \frac{-1}{4\pi^2} \left[ w_L^f(q^2) q_\nu q^\sigma \tilde{f}_{\sigma \mu} + w_T^f(q^2) \left( -q^2 \tilde{f}_{\mu \nu} + q_\mu q^\sigma \tilde{f}_{\sigma \nu} - q_\nu q^\sigma \tilde{f}_{\sigma \mu} \right) \right] \]

where perturbatively (notations a bit different from before)

\[ w_L^f = 2 w_T^f = \frac{2}{Q^2} \]

It’s easy to check then

\[ q_\mu T_{\mu \nu}^{H} = \frac{1}{4\pi^2} q^\mu \tilde{f}_{\mu \nu}, \]
\[ q_\nu T_{\mu \nu}^{H} = \frac{1}{4\pi^2} q^\nu \tilde{f}_{\mu \nu}, \]

what is consistent with the HHH relations for divergences of the axial current and vector baryon current.

The last extra term is just a local counterterm \(-2 \tilde{f}_{\mu \nu}\). It is the contact \(Z\omega\gamma\) interaction discussed by HHH. The first logitudinal term contains the massless pion pole. Only the second, transversal term receives nonperturbative corrections.
We see that the $\omega$ exchange with momentum is proportional to

$$\frac{q^2}{q^2 - m_\omega^2}$$

for the conserved vector current and contains a local nonpole term in addition when the vector current is not conserved. It does not matter which of the currents we use to define the residue of the pole, i.e. $Z\omega\gamma$ coupling.

Then in application to neutrino reactions at small $q^2$ we cannot fixed the answer without fixing nonpole terms. It is our understanding that introduction of nonconserved baryonic current remains just a formal exercise not related to observables.