$B(\tau^- \rightarrow \eta \pi^- \nu_\tau)$, the $a_0(980)$, and New Weak Interactions

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Outline

• Motivation

• Estimate of $B(\tau^- \to \eta \pi^- \nu_\tau)$:
  – Vector contribution
  – Scalar contribution

• Sensitivity to new interactions

• Implications of various $B(\tau^- \to \eta \pi^- \nu_\tau)$ values
Motivation

• $\tau^- \rightarrow \eta \pi^- \nu_\tau$ is sensitive to new-physics
• CLEO limit on $B(\tau^- \rightarrow \eta \pi^- \nu_\tau) < 1.4 \times 10^{-4}$ (90% CL)
• Chiral perturbation theory: $B(\tau^- \rightarrow \eta \pi^- \nu_\tau) \approx 1.3 \times 10^{-5}$
  – Assumes $a_0(980) \rightarrow \eta \pi^-$ contribution dominates, and that $a_0(980)$ is a $q\bar{q}$ state
  – However, $a_0(980)$ is probably a 4-quark, $u\bar{d}s\bar{s}$ state
• At this level, $\rho^- (770) \rightarrow \eta \pi^-$ could also lead to a vector contribution

• Therefore, we...
  – calculate the $\rho$ contribution to $\tau^- \rightarrow \eta \pi^- \nu_\tau$
  – perform an alternative calculation of the $a_0$ contribution
  – comment on sensitivity of $B(\tau^- \rightarrow \eta \pi^- \nu_\tau)$ to new weak interactions
Vector contribution

• Note: $B(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)$ is large (25.5%) and completely dominated by $\rho^-$ contribution
• So expect $\rho^-$ to also dominate the vector contribution to $\tau^- \rightarrow \eta \pi^- \nu_\tau$, with branching fraction

$$B_{L=1}(\tau^- \rightarrow \eta \pi^- \nu) = \left(\frac{g_{\rho \eta \pi}}{g_{\rho \pi \pi}}\right)^2 \left(\frac{p_{\rho \rightarrow \eta \pi}}{p_{\rho \rightarrow \pi \pi}}\right)^3 B(\tau^- \rightarrow \rho^- \nu)$$
Obtaining $g_{\rho \eta \pi}$ Coupling

• $\rho^{-}\rightarrow\eta\pi^{-}$ has not been observed, but we can obtain $g_{\rho \eta \pi}$ from the Dalitz-plot distribution of $\eta\rightarrow\pi^{+}\pi^{-}\pi^{0}$
  – Method used by Ametller & Bramon, PRD 24, 1325 (1981)
  – Now more precise data, access to more terms in Dalitz-plot distribution

• Assume the decay has 3 contributions: scalar, $\rho^{+}$ and $\rho^{-}$:
  – ($\rho^{0}$ is forbidden due to C conservation)

\[
M_{\eta\rightarrow\pi^{+}\pi^{-}\pi^{0}} \equiv M_{+0} = M_{S} + M_{\rho^{+}} + M_{\rho^{-}}
\]

• Scalar part has flat distribution in the $\pi^{+}\pi^{-}\pi^{0}$ Dalitz plot, and is also the only contribution to $\eta\rightarrow3\pi^{0}$
\[ |\mathcal{M}_S|^2 = 8(2\pi)^3 m_\eta \Gamma_\eta \mathcal{B}(\eta \to \pi^0 \pi^0 \pi^0) \frac{6\sqrt{3}}{Q^2 S_1} \frac{3!}{9} \]

= 0.065,

\[ Q \equiv m_\eta - 3m_\pi \]

\[ S_1 \equiv \int dX \, dY = 2.75 = \text{area of Dalitz plot} \]

Dalitz plot variables

\[ X = \frac{\sqrt{3}(T_{\pi^+} - T_{\pi^-})}{Q}, \quad Y = \frac{3(T_{\pi^0} - 1)}{Q} \]

Write vector part as:

\[ M_{\rho^\mp} = -g_{\rho\eta\pi} g_{\rho\pi\pi} \frac{(P_{\eta} + P_{\pi^\pm}) \cdot (P_{\pi^\mp} - P_{\pi^0})}{(P_{\pi^\mp} + P_{\pi^0})^2 - m_\rho^2 + i\Gamma_\rho m_\rho} \]

The coupling we are after
• Expand squared amplitude to 3rd order in

\[ r = \frac{m_\eta Q}{m_\rho^2 - m_\eta^2 / 3 - m_\pi^2 - i\Gamma_\rho m_\rho} = 0.14 - 0.03i \]

• Obtain total rate relative to scalar part:

\[ |M_{+-0}|^2 \propto 1 + \alpha Y + \beta Y^2 + \gamma X^2 + \delta \left( Y^3 + YX^2 \right) \]

• Where

\[ \alpha = -4g_\eta \rho \pi g_\rho \pi \pi \Re \{ \mathcal{M}_S^* r \} \frac{1}{|\mathcal{M}_S|^2} \]

\[ \beta = \left[ -\frac{4}{3} g_\eta \rho \pi g_\rho \pi \pi \Re \{ \mathcal{M}_S^* r^2 \} + 4(g_\eta \rho \pi g_\rho \pi \pi)^2 |r|^2 \right] \frac{1}{|\mathcal{M}_S|^2} \]

\[ \gamma = \frac{4}{3} g_\eta \rho \pi g_\rho \pi \pi \Re \{ \mathcal{M}_S^* r^2 \} \frac{1}{|\mathcal{M}_S|^2} \]

\[ \delta = \left[ \frac{4}{9} g_\eta \rho \pi g_\rho \pi \pi \Re \{ \mathcal{M}_S^* r^3 \} + \frac{8}{3} (g_\eta \rho \pi g_\rho \pi \pi)^2 \Re \{ r(r^2)^* \} \right] \frac{1}{|\mathcal{M}_S|^2} \]
Dalitz-plot parameters measured by KLOE (arXiv:0707.2355):

\[
|M_{+-0}|^2 \propto 1 - 1.09Y + 0.124Y^2 + 0.057X^2 + 0.14Y^3 + ?YX^2
\]

Fig. 2. Dalitz-plot distribution for the whole data sample. The plot contains 1.34 millions of events in 256 bins.
Comparing the Y coefficients, get:

\[ g_{\eta \rho \pi} g_{\rho \pi \pi} = \frac{1.09}{4} \frac{\mathcal{M}_s}{\mathcal{R}(r)} = 0.51 \]

Extract \( g_{\rho \pi \pi} \) from \( \rho \to \pi \pi \) width:

\[ g_{\rho \pi \pi} = \sqrt{\frac{6\pi m_\rho^2 \Gamma_\rho}{p_\rho^{3 \to \pi \pi}}} = 6.0 \]

So the vector contribution to \( B(\tau \to \eta \pi \nu) \) is

\[
B_{L=1} = \left( \frac{g_{\rho \eta \pi}}{g_{\rho \pi \pi}} \right)^2 \left( \frac{p_{\rho \to \eta \pi}}{p_{\rho \to \pi \pi}} \right)^3 B(\tau^- \to \rho^- \nu) \approx 3.6 \times 10^{-6}
\]

Taken to be real

Consistent w. Ametller & Bramon
Cross-checks

• The other coefficients are a test of the model:

\[ |M_{+-0}|^2 \propto 1 - 1.09Y + 0.27Y^2 + 0.05X^2 + 0.03(Y^3 - YX^2) \]

• Compare with KLOE measurement:

\[ |M_{+-0}|^2 \propto 1 - 1.09Y + 0.124Y^2 + 0.057X^2 + 0.14Y^3 \]

• Floating \( \arg(M_S) = 15^\circ \) improves agreement only slightly

• Also check ratio of BR’s:

\[
\frac{B(\eta \rightarrow \pi^+\pi^-\pi^0)}{B(\eta \rightarrow 3\pi^0)} = \begin{cases} 
0.7, \text{ measured} \\
0.71, \text{ model + KLEO parameters} \\
0.76, \text{ model + our parameters}
\end{cases}
\]
Scalar Contribution to $B(\tau^{-} \rightarrow \eta \pi^- \nu_{\tau})$

- Chiral perturbation theory calculates assumed $a_0(980)$ is a $q\bar{q}$ state and are complicated
- We conduct a simpler estimate and arrive at a similar result:
  - Vector current is conserved up to $m_d - m_u$:
    $\nabla^\mu \bar{u}(x) \gamma_\mu d(x) = (m_d - m_u)\bar{u}(x)d(x) + \text{EM term}$

- We estimate the scalar matrix element by relating the P-wave states $a_0(980)$ & $a_1(1260)$:
  $$\frac{B_{L=0}(\tau^{-} \rightarrow a_0^{-}(980)\nu)}{B(\tau^{-} \rightarrow a_1^{-}(1260)\nu)} \sim 1.3 \left| \frac{\langle 0|S|a_0(980)\rangle}{\langle 0|A|a_1(1260)\rangle} \right|^2 \left( \frac{m_d - m_u}{m_{a_1(1260)}} \right)^2$$

  Phase space

  $\sim 1$, since fixed by quark-model wave functions

  $B_{L=0} \sim 10^{-5}$
Limits on New Physics

- $B(\tau^-\rightarrow\eta\pi^-\nu)$ can be used to put bounds on new scalar interactions up to the SM expectation $B(\tau^-\rightarrow\eta\pi^-\nu) \sim 10^{-5}$
- A limit $B(\tau^-\rightarrow\eta\pi^-\nu) < 3 \times 10^{-5}$ implies
  \[
  \frac{M_{\text{Scalar}}}{M_W} > \left(3 \times 10^{-5}\right)^{\frac{1}{4}} \sim 13
  \]
  for the same couplings as in the SM.
- Competitive with limits from angular distributions in nuclear $\beta$ decay, $\sim 4$ (expected to improve to $\sim 7$ and then to $\sim 15$)
- The two limits are complementary:
  - $\beta$-decay: 1st-generation couplings
  - $\tau^-\rightarrow\eta\pi^-\nu$: 3rd-generation couplings
Conclusions

• Our estimates
  - \( B_{L=0}(\tau \rightarrow \eta \pi^- \nu) \sim 10^{-5} \)
  - \( B_{L=1}(\tau \rightarrow \eta \pi^- \nu) \approx 3 \times 10^{-6} \)

imply the following for the measured value of \( B_{L=0}(\tau \rightarrow \eta \pi^- \nu) \):

• \( \approx 3 \times 10^{-6} \), especially with a \( \rho^-(770) \) peak ⇒
  - No surprises

• \( \approx 10 \times 10^{-6} \), especially with a \( a_0^-\langle980\rangle \) peak ⇒
  - \( a_0^-\langle980\rangle \) is a q\( \bar{q} \) state after all

• \( > 30 \times 10^{-6} \), especially with scalar dominance ⇒
  - Possibly new scalar interactions, \( M_S \sim 13 M_W \) for weak coupling

• Note that BaBar has limit \( B(\tau \rightarrow \eta'\pi^- \nu) < 7.2 \times 10^{-6} \)
  - Contributions from additional intermediate resonances
Backup Slides
Arguments for 4-quark $a_0(980)$

- $\Gamma(a_0(980)) \sim 50$ MeV, compare with $\Gamma(\rho(770)) \sim 150$ MeV
- $B(a_0(980) \rightarrow K\bar{K}) / B(a_0(980) \rightarrow \eta\pi) = 18\%$
  despite highly limited phase space in $KK$ final state
- $a_0(980)$ production much suppressed wrt. $q\bar{q}$ states.
  - This suppression might be smaller in heavy ion collisions due to high quark multiplicity, especially $s$ quarks
• 2\textsuperscript{nd}-class currents: Weinberg, Phys. Rev. 112, 1375 (1958):
  – 1\textsuperscript{st}-class currents: $G J_1 G^{-1} = -\xi J_1$, JPG = $0^{++}$, $0^{--}$, $1^{+-}$, $1^{-+}$
  – 2\textsuperscript{nd}-class currents: $G J_2 G^{-1} = +\xi J_2$, JPG = $0^{+-}$, $0^{-+}$, $1^{++}$, $1^{--}$
  – where $\xi = 1(-1)$ for S, A, P (V, T) and $G = C e^{i\pi I_2}$:

• Full expression for $\eta \rightarrow 3\pi^0$:

\[
|M_S|^2 = 8(2\pi)^3 m_\eta \Gamma_\eta B(\eta \rightarrow \pi^0 \pi^0 \pi^0) \frac{6\sqrt{3}}{Q^2 S_1} \frac{3!}{9}
\]

\[= 0.065,
\]

• Rho decay amplitude:

\[
\Gamma(\rho \rightarrow \pi\pi) = \frac{p_{\rho \rightarrow \pi\pi}}{32\pi^2 m_\rho^2} \int |M_\rho|^2 d\Omega
\]

\[M_\rho = g_{\eta\rho\pi} \varepsilon_\mu \left(P_{\pi^+} - P_{\pi^-}\right)\]