SYMMETRY, CONFINEMENT AND
THE PHASE DIAGRAM OF QCD.

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BASED ON WORK IN COLLABORATION WITH

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1. WHY SYMMETRY?

- EXPERIMENT

\[
\frac{n_q}{n_p} < 10^{-27} \quad \text{or} \quad n_q \approx 10^{-12} \quad \text{S.C.H.}
\]

\[
\sigma_q = \sigma(p+p+q(q)+X) < 10^{-26} \text{cm}^2 \quad \sigma_q \approx \sigma_{\text{tor}} = 10^{-25} \text{cm}^2
\]

A FACTOR \approx 10^{-15}!

- NATURAL EXPLANATION:

\[
n_q = 0 \quad \sigma_q = 0
\]

DUE TO A SYMMETRY,

\[
\downarrow
\]

THE DECONFINING TRANSITION IS ORDER-DISORDER, CANNOT BE A Crossover!

2. SUPERCONDUCTIVITY

\[
\frac{\bar{s}_{\text{sc}}}{s_{\text{norm.}}} < 10^{-16} \quad \Rightarrow \quad \bar{s}_{\text{sc}} = 0
\]

Higgs BREAKING OF E.H. U(1)
3-flavor phase diagram

$T_{\chi}^{nr=3} \sim 155 \text{ MeV}$

$T_{\chi}^{nr=2} \sim 175 \text{ MeV}$

$T_d \sim 270 \text{ MeV}$

$\mathbf{m}_{PS}^{\text{crit}} \simeq 2.5 \text{ GeV}$

$\mathbf{m}_{PS}^{\text{crit}} \simeq 300 \text{ MeV}$

BIELEFELD GROUP
II. WHAT SYMMETRY?

- COLOR AN EXACT SYMMETRY

- QUENCHED (NO QUARKS)

  \[ Z_n : \langle L \rangle \] \quad \text{"THE ORDER PARAMETER"}

  QUARKS BREAK \( Z_n \).

- CHIRAL SYMMETRY \( \exists \) AT \( m_q = 0 \), AND IS

  RESTORED AT \( T_c \).

  IN \( N_f = 2 \) ADJOINT QCD THE TWO TRANSITIONS ARE DISTINCT:

  CHIRAL RESTORED AT \( T_c' > T_c \) : DECONFINEMENT IS ORDER,

  CHIRAL A CROSSOVER. [Karsch et al., Cossu et al.]

- ONLY KNOWN WAY TO GET EXTRA

  SYMMETRY : DUALITY [Kramers Wannier 41, Seiberg Witten]

  EXCITATIONS WITH TOPOLOGICALLY NON TRIVIAL BOUNDARY CONDITIONS.

  2+1 DIM. VORTICES (NON TRIVIAL \( \Pi_1 \))

  3+1 DIM. MONOPOLES (NON TRIVIAL \( \Pi_2 \))
MONOPOLES ["tHooft '74, Polyakov '74]

SO(3) HIGGS MODEL: HEDGHEGOG GAUGE

\[ \phi \xrightarrow{r \to \infty} \frac{k^a}{r} \]

A non trivial mapping \( S_2 \to SO(3) \).

tHooft Tensor

\[ F_{\mu \nu}^a = \bar{\phi} \varepsilon_{\mu \nu}^a - \frac{1}{3} \bar{\phi} \left( D_\mu \phi \wedge D_\nu \phi \right) \]

\[ \bar{J}_\nu = \frac{1}{3} \frac{2}{r^3} + \text{Dirac String} \]

In compact formulation (lattice) string is invisible

\[ \nabla^2 \bar{J}_\nu = \frac{1}{3} \delta^2 \langle \nu \rangle \]

Violation of Bianchi identities

\[ \partial_\mu F^\mu_{\nu \lambda} = J^\nu_{\lambda} \neq 0 \]

Dual symmetry

Geometry independent of the gauge group \( (\text{AdS}_5 \times \text{S}^5) \)

\[ F^a_{\mu \nu} \quad (a = 1, 2, \ldots, r) \]

\( r \) = rank of the group

\[ j^a_\nu = \partial_\mu F^\mu_{a \nu} \neq 0 \quad \partial_\nu j^a_\nu = 0 \]

\( q^a \) magnetic charges \( U(1)^a \)

\( U(1)^a \) Higgs broken (dual superconductor)

\( U(1)^a \) restored (normal)

Order parameters \( \langle \mu^a \rangle \) \( (\text{AdS}_5; \text{AdS}_2 \times \text{S}^2) \)

\( \mu^a \) carries \( q^a \neq 0 \)

Checked on the lattice.
"Transition": a rapid change at some value $T_c$ of a parameter $T$. At $T = T_c$, a peak of susceptibilities. [Fig. 2]

- Change in the heat content:
  - Peak off the specific heat $C_v$.

- **Crossover**: no discontinuity at $T_c$ as $V \to \infty$.

- **First Order**: discontinuity in heat content: $C_v(T_c) \to \infty$ as $V \to \infty$.

- **Second Order**: free energy continuous, $C_v$ discontinuous, $\frac{dC_v}{dT}$ divergent.

Stating that a transition is a crossover means no discontinuity at any order: impossible with finite resolution. Sometimes possible with support of history.
\( N_f = 2 \). If chiral degrees of freedom dominate ren. group + \((4 - \xi) \Rightarrow [\text{ Pisovnik et al. 84}] \)

(i) \((m = 0)\): 2nd order O(4), crossover \( m \neq 0 \)
- Tricritical point

(ii) \( m = 0 \): 1st order, 1st order \( m \neq 0 \)

- Finite size scaling, \( \tau = 1 - T/T_c \)

\[
c_V - c_0 \approx \frac{\alpha}{V^{1/2}} \phi_c \left( \frac{\tau}{V^{1/2}}, m L_5 \right)
\]

\( \Lambda = \int (c_V - c_0) d\tau \) [Latent heat]

\( \Lambda \approx V^{-0.35} \) \( \sim O(4) \) \( \Lambda \approx \text{cutoff 1st order} \)

\( \Lambda = \left\{ \begin{array}{l}
\alpha = -0.24 \\
y_\rho = 1.34 \\
y_h = 1.48
\end{array} \right. \)

\( \Lambda = \left\{ \begin{array}{l}
\alpha = 1 \\
y_\rho = 3 \\
y_h = 3
\end{array} \right. \)

(i) Keep second scaling variable \{ \text{fixed} \}
- Check scaling in the first \{ \text{O(4)} \} \text{ no Bielefeld, MILC, Tsukuba} \text{ 1st OK.}

(ii) Keep first variable fixed, \( V \to \infty \)

\[
c_V - c_0 \approx m^{13} f_c \left( \tau V^{1/45} \right) \quad \text{[O(4) no]}
\]

\[
c_V - c_0 = V \phi_c \left( \tau V, m \right) \quad \text{1st order}
\]

\text{OK but first term negligible.}
\[ l_e = 4 \quad l_s = 6, 20, 24, 32, 48 \text{ on the way} \]

First Term Small: Very Weak First Order
Work in Progress at Large Volumes \( l_s \rightarrow \infty \)

- \( O(4) \) Excluded; First Order Consistent with Data But Needs Check at Large \( V \).

- Phase Diagram Still Controversial.

Observation of Tri-Critical Point in Heavy Ion Experiments Very Important.

IV Summary

- Nature Suggests a Symmetry Behind Confinement: No Crossovers
- Geometry \( \rightarrow \) Dual Superconductivity
- Phase Diagram: Under Debate