Hadronic Light-Front Wavefunctions from AdS/QCD

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PANICo8 Eilat, Israel

9-14 November, 2008
• Quarks and Gluons: Fundamental constituents of hadrons and nuclei

• **Quantum Chromodynamics (QCD)**

• New Insights from higher space-time dimensions: *AdS/QCD*

• **Light-Front Holography:** First Approximation to QCD

• **Hadronization at the Amplitude Level**

• **Light Front Wavefunctions:** Analogous to Schrödinger wavefunctions of atomic physics

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ P^+, \vec{P}_\perp \]

Process Independent Direct Link to QCD Lagrangian!

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \]

\[ \sum_{i=1}^{n} x_i = 1 \]

\[ \sum_{i=1}^{n} \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Invariant under boosts! Independent of \( P^\mu \)
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x_i, \vec{k}_\perp i, \lambda_i)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi >= M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space.
Prediction from AdS/QCD: Meson LFWF

\[ \psi_M(x, k_{\perp}^2) \]

\[ \propto \sqrt{x(1-x)} \]

\[ \mu_R = Q/2 < \mu_R < 2Q \]

“Soft Wall” model

de Teramond, sjb

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A Unified Description of Hadron Structure

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

- Elastic form factors
- B-Decays
- Real Compton scattering at high \( t \)
- GPDs
- Distribution Amplitudes
- Deeply Virtual Compton Scattering
- Hadronization at the amplitude level
- Parton momentum distributions TMDs

\[ \sum_{n} \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

\[ \sum_{n} x_i \vec{R}_\perp + \vec{b}_\perp i = \vec{R}_\perp \]

\[ \sum_{n} x_i = 1 \]
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

Off-shell $T$-matrix

$\tau = x^+$
Each element of flash photograph illuminated at same LF time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Eigenstate -- independent of \( \tau \)
Deep Inelastic Electron-Proton Scattering

Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular momentum ....

Gluonic Bremsstrahlung
DGLAP Evolution

Light-Front Quantization:
Rigorous realization of IMF
Deep Inelastic Electron-Proton Scattering

Dynamic Structure Functions

Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular
momentum ....

Final-State Rescattering
Leading-Twist
Sivers Effect, DDIS,
Shadowing, Antishadowing

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<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Square of Target LFWFs</td>
<td>Modified by Rescattering: ISI &amp; FSI</td>
</tr>
<tr>
<td>• No Wilson Line</td>
<td>Contains Wilson Line, Phases</td>
</tr>
<tr>
<td>• Probability Distributions</td>
<td>No Probabilistic Interpretation</td>
</tr>
<tr>
<td>• Process-Independent (in isolation)</td>
<td>Process-Dependent - From Collision</td>
</tr>
<tr>
<td>• T-even Observables</td>
<td>T-Odd (Sivers, Boer-Mulders, etc.)</td>
</tr>
<tr>
<td>• No Shadowing, Anti-Shadowing</td>
<td>Shadowing, Anti-Shadowing, Saturation</td>
</tr>
<tr>
<td>• Sum Rules: Momentum and $J_z$</td>
<td>Sum Rules Not Proven</td>
</tr>
<tr>
<td>• DGLAP Evolution; mod. at large $x$</td>
<td>DGLAP Evolution</td>
</tr>
<tr>
<td>• No Diffractive DIS</td>
<td>Hard Pomeron and Odderon Diffractive DIS</td>
</tr>
</tbody>
</table>

General remarks about orbital angular momentum

\[
\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) = \sum_{n_i} (x_i \vec{R}_{\perp} + \vec{b}_{\perp i}) = \vec{R}_{\perp}
\]

\[
x_i \vec{R}_{\perp} + \vec{b}_{\perp i} = \vec{0}_{\perp}
\]

\[
\sum_{n_i} x_i = 1
\]
Angular Momentum on the Light-Front

\[ A^+ = 0 \] gauge: No unphysical degrees of freedom

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z. \]

Conserved

LF Fock state by Fock State

\[ l_j^z = -i \left( k_1^j \frac{\partial}{\partial k_2^j} - k_2^j \frac{\partial}{\partial k_1^j} \right) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment requires
Nonzero orbital angular momentum.

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Calculation of Form Factors in Equal-Time Theory

*Instant Form*:

\[ \sum \]

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

*Front Form*:

\[ \sum \]

Absent for \( q^+ = 0 \) zero !!
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \\
\left[ -\frac{1}{q_L} \psi_a^\dagger(x_i, k'_\perp i, \lambda_i) \psi_a(x_i, k_\perp i, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_\perp i, \lambda_i) \psi_a(x_i, k_\perp i, \lambda_i) \right] \\
k'_\perp i = k_\perp i - x_i q_\perp \\
k'_\perp j = k_\perp j + (1 - x_j) q_\perp \\
q^2 = -q_\perp^2 \\
q^+ = 0
\]

\[q_{R,L} = q^x \pm iq^y\]

Must have \(\Delta \ell_z = \pm 1\) to have nonzero \(F_2(q^2)\)

**Checked to \(O\alpha^3\) in QED**

Roskies, Suaya, sjb

---

Drell, sjb

\[\Psi_{R,\text{Drell}}\]
\[ \phi(z) \]

- **Light-Front Holography**

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

- **Light Front Wavefunctions:**
  Schrödinger Wavefunctions of Hadron Physics

\[ k_\perp (\text{GeV}) \]

\[ x \]

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Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

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Light-Front Wavefunctions from AdS/QCD

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Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

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String Theory

Bottom-Up

Top-Down
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances.

- Analogous to the Schrodinger Theory for Atomic Physics.

- \textit{AdS/QCD Light-Front Holography}.

- \textit{Hadronic Spectra and Light-Front Wavefunctions}.
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M_{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of \( SO(4,2) \)

\( SO(4,2) \) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of \( SO(4, 2) \) of conformal QCD with the group of isometries of AdS space

\[
 ds^2 = \frac{R^2}{z^2} \left( dx^\mu dx_\mu - dz^2 \right)
\]

\( x^\mu \rightarrow \lambda x^\mu, \ z \rightarrow \lambda z \), maps scale transformations into the holographic coordinate \( z \).

- AdS mode in \( z \) is the extension of the hadron wf into the fifth dimension.

- Different values of \( z \) correspond to different scales at which the hadron is examined.

\[
 x^2 \rightarrow \lambda^2 x^2, \ z \rightarrow \lambda z
\]

\( x^2 = x^\mu dx_\mu \): invariant separation between quarks

- The AdS boundary at \( z \rightarrow 0 \) correspond to the \( Q \rightarrow \infty \), UV zero separation limit.
We will consider both holographic models

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

**We will consider both holographic models**
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $\text{AdS}_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal;** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** \(\alpha_s(Q^2) \simeq \text{const at small } Q^2\)

- **Use mathematical mapping of the conformal group** $\text{SO}(4,2)$ to AdS$_5$ space
Lesson from QED and Lamb Shift:
maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]

\[ \lambda < \Lambda_{QCD} \]

B-Meson

Shrock, sjb

gluon and quark propagators cutoff in IR because of color confinement
Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling
  \[ \alpha_s(Q^2) \text{ finite, } \beta \to 0 \text{ at small } Q^2 \]
- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons

Shrock, sjb
**AdS/CFT**

- Use mapping of conformal group SO(4,2) to AdS$_5$

- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  
  $$x^2_\mu \rightarrow \lambda^2 x^2_\mu \quad z \rightarrow \lambda z$$

- Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates “bag” boundary conditions
  
  $$0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$


**Bosonic Solutions: Hard Wall Model**

- Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m} \).

- Action for massive scalar modes on \( \text{AdS}_{d+1} \):

\[
S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.
\]

- Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
\]

- Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = M^2 \):

\[
[z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 M^2 - (\mu R)^2] \Phi(z) = 0. \quad d = 4
\]

- Solution: \( \Phi(z) \rightarrow z^\Delta \) as \( z \rightarrow 0 \),

\[
\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z M) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4 \mu^2 R^2} \right).
\]

\( (\mu R)^2 = L^2 - 4 \)
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

$$\left[ - \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: orbital angular momentum

**Derived from variation of Action in $AdS_5$**

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
Match fall-off at small $z$ to conformal twist-dimension $\Delta$ at short distances

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi} \gamma_5 D_{\ell_1} \ldots D_{\ell_m} \psi$ ($\Phi_\mu = 0$ gauge). \quad $\Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ \Delta = 2 + L \]

\[ S' = 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
\textbf{AdS Schrödinger Equation for bound state of two scalar constituents:}

\[
\left[ - \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

\textbf{Derived from variation of Action Dilaton-Modified AdS}_5
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Soft Wall Model

Pion mass automatically zero!

$m_q = 0$
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \phi_S(z) = M^2 \phi_S(z)
\]

with eigenvalues \( M^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube): \( M_n^2(L) = 2\pi \sigma (n + L + 1/2) \).

Same slope in \( n \) and \( L \)

- Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \) GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

\[ \alpha(t) \approx \frac{1}{2} + 0.9t \]

Plot of spins of families of particles against their squared mass data tables, but there is some uncertainty about whether they exist.

The function \( \alpha(t) \) is called a Regge trajectory.
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

High \( Q^2 \)
from
small \( z \sim 1/Q \)

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} \]

where \( \tau = \Delta_n - \sigma_n, \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
\[ Q^4 F_1^p(Q^2) \quad [\text{GeV}^4] \]

\[ F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3 \]

- Phenomenological success of dimensional scaling laws for exclusive processes

\[ d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D, \]

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev et al. (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
\left[ z^2 \partial_z^2 - z \left( 1 + 2\kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right),
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \to z Q K_1(\kappa Q) = J(Q, z),
\]

the external current decouples from the dilaton field.
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

\[ q^2(GeV^2) \]

Data Compilation
Baldini, Kloe and Volmer

de Teramond, sjb
See also: Radyushkin

One parameter - set by pion decay constant

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

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Light-Front Wavefunctions from AdS/QCD

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• Analytical continuation to time-like region $q^2 \rightarrow -q^2$. 
  \[ M_\rho = 2\kappa = 750 \text{ MeV} \]

• Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

![Graph showing space and time-like pion form factor]

Space and time-like pion form factor for $\kappa = 0.375 \text{ GeV}$ in the SW model.

• Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi^*_P(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( b = |\vec{b}_\perp| \):

\[
F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\
= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bq_x) \, |\tilde{\psi}(x, b)|^2,
\]

\( \vec{q}_\perp = Q^2 = -q^2 \)
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \left( \frac{1-x}{x} \right) \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|. \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
• Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQK_1(zQ). \)

• Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) \]

• Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

• Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[
\left| \tilde{\psi}_{\bar{q}q/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4}
\]

with \( \zeta = z, \ 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)

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$\psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z)$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)\vec{b}_\perp^2} \phi(\zeta)$$

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b_\perp^2.
\]

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)
\]

G. de Teramond, sjb

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Soft wall confining potential:
Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

\[ M^2 = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \frac{k_\perp^2}{x(1-x)} |\psi(x, \vec{k}_\perp)|^2 + \text{interactions} \]

\[ = \int_0^1 dx \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left( -\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions}. \]

**Change variables**

\((\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2} \)

\[ M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \]

\[ + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \]
Holographic Connection of AdS$_5$ to Light-Front

\[ ds^2 = \frac{R^2}{z^2} (dx^\mu dx_\mu - dz^2) = \frac{R^2}{z^2} (dx^+ dx^- - dx^2_\perp - dz^2) \]

\[ ds^2 = \frac{R^2}{z^2} [-x^2_\perp - dz^2] \text{ at fixed } x^+ = x^0 + x^3 \]

\( ds^2 \) is invariant

if \( (dx_\perp)^2 \rightarrow \lambda^2 (dx_\perp)^2 \) and \( (dz)^2 \rightarrow \lambda^2 (dz)^2 \)

at fixed light-front time

Casimir for \( SO(N) \sim S^{N-1} \) is \( L(L + N - 2) \)

Casimir for \( SO(3) \) is \( L(L + 1) \)

Casimir for \( SO(2) \) is \( L^2 \)
Prediction from AdS/CFT: Meson LFWF

ψ_M(x, k^2_⊥)

\[ \psi_M(x, k^2_⊥) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2_⊥}{2\kappa^2 x(1-x)}} \]

φ_M(x, Q_0) \propto \sqrt{x(1-x)}

“Soft Wall” model

κ = 0.375 GeV

massless quarks

Note coupling \( k^2_⊥, x \)

Connection of Confinement to TMDs
**Hadron Distribution Amplitudes**

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[
\phi_M(x, Q) = \int_Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)
\]

**Lepage, sjb**

Efremov, Radyushkin

Sachrajda, Frishman

Lepage, sjb

Braun, Gardi

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PANIC08
November 10, 2008

Light-Front Wavefunctions from AdS/QCD

Stan Brodsky
SLAC
Second Moment of Pion Distribution Amplitude

\[ < \xi^2 > = \int_{-1}^{1} d\xi \xi^2 \phi(\xi) \]

\[ \xi = 1 - 2x \]

\[ < \xi^2 >_\pi = 1/5 = 0.20 \]

\[ < \xi^2 >_\pi = 1/4 = 0.25 \]

\[ \phi_{\text{asympt}} \propto x(1-x) \]

\[ \phi_{\text{AdS/QCD}} \propto \sqrt{x(1-x)} \]

Lattice (I) \[ < \xi^2 >_\pi = 0.28 \pm 0.03 \]

Lattice (II) \[ < \xi^2 >_\pi = 0.269 \pm 0.039 \]

Donnellan et al.

Braun et al.
\[ F_\pi(Q^2) = \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) \frac{16\pi C_F\alpha_V(Q_V)}{(1-x)(1-y)Q^2} \]

\[ \phi(x, Q_0) \propto \sqrt{x(1-x)} \]

\[ \phi_{\text{asymptotic}} \propto x(1-x) \]

Normalized to \( f_\pi \)

**AdS/CFT:** Increases PQCD leading twist prediction for \( F_\pi(Q^2) \) by factor 16/9
Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!

\[ M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_\pi(x, k_{\perp}) \]
Two-gluon exchange measures the second derivative of the pion light-front wavefunction

\[ M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp}) \]
Key Ingredients in E791 Experiment

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency

Target left intact

Diffraction, Rapidity gap

$M_A = A M_N$

$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2 \frac{d\sigma}{dt}(\pi N \to q\bar{q}N') F_A^2(t)$
Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

• Fundamental test of gauge theory in hadron physics
• Small color dipole moments interact weakly in nuclei
• Complete coherence at high energies
• Clear Demonstration of CT from Diffractive Di-Jets
• Fully coherent interactions between pion and nucleons.

• Emerging Di-Jets do not interact with nucleus.

\[ \mathcal{M}(A) = A \cdot \mathcal{M}(N) \]

\[ \frac{d\sigma}{dq_T^2} \propto A^2 \left( q_T^2 \right) \sim 0 \]

\[ \sigma \propto A^{4/3} \]

Nuclear coherence

\[ F_A^2(q_T^2) \sim e^{-\frac{1}{3} R_A^2 q_T^2} \]
Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

**A-Dependence results:** \( \sigma \propto A^\alpha \)

<table>
<thead>
<tr>
<th>( k_t ) range (GeV/c)</th>
<th>( \alpha )</th>
<th>( \alpha ) (CT)</th>
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<tbody>
<tr>
<td>1.25 &lt; ( k_t ) &lt; 1.5</td>
<td>1.64 ±0.06 -0.12</td>
<td>1.25</td>
</tr>
<tr>
<td>1.5 &lt; ( k_t ) &lt; 2.0</td>
<td>1.52 ± 0.12</td>
<td>1.45</td>
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<tr>
<td>2.0 &lt; ( k_t ) &lt; 2.5</td>
<td>1.55 ± 0.16</td>
<td>1.60</td>
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</table>

\( \alpha \) (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out!

Factor of 7

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman
**E791 Diffractive Di-Jet transverse momentum distribution**

![Graph showing the dependence of the di-jet yield on transverse momentum](image)

**Two Components**

- High Transverse momentum dependence $k_T^{-6.5}$ consistent with PQCD, ERBL Evolution
- Gaussian component similar to AdS/CFT HO LFWF

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**Light-Front Wavefunctions from AdS/QCD**

Stan Brodsky
SLAC
Narrowing of $x$ distribution at higher jet transverse momentum

$x$: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

**Possibly two components:**

Nonperturbative (AdS/CFT) and Perturbative (ERBL)

**Evolution to asymptotic distribution**

$\phi(x) \propto \sqrt{x(1-x)}$

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**Light-Front Wavefunctions from AdS/QCD**

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**Stan Brodsky**

SLAC
possibly two components:

\[ \phi(x) = A_{\text{pert}}(k_{\perp}^2)x(1-x) + B_{\text{nonpert}}(k_{\perp}^2)\sqrt{x(1-x)} \]

narrowing of \( x \) distribution at high jet transverse momentum

\( 1.25 \leq k_t \leq 1.5 \text{ GeV/c} \)

\( 1.5 \leq k_t \leq 2.5 \text{ GeV/c} \)
Construct generator: \[ \Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} - \kappa^2 \zeta \right) \]

and its adjoint \[ \Pi_L^\dagger(\zeta) = -i \left( \frac{d}{d\zeta} + \frac{L + \frac{1}{2}}{\zeta} + \kappa^2 \zeta \right) \]

with commutation relations \[ \left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} - 2\kappa^2 \]

- The LF Hamiltonian

\[ H_{LF} = \Pi_L^\dagger \Pi_L + C \]

is positive definite \[ \langle \phi | H_{LF} | \phi \rangle \geq 0 \] for \( L^2 \geq 0 \), and \( C \geq -4\kappa^2 \)

- Identify the zero mode \( (C = -4\kappa^2) \) with the pion
• Orbital and radial excited states are constructed from the ladder operators from the \( L = 0 \) state.

• Light-front Hamiltonian equation

\[
H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle,
\]

leads to effective LF wave equation

\[
\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)
\]

with effective potential

\[
U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L-1)
\]

eigenvalues

\[
\mathcal{M}^2 = 4\kappa^2(n + L)
\]

and eigenfunctions

\[
\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n + L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L}(\kappa^2\zeta^2).
\]

• Transverse oscillator in the LF plane with \( SO(2) \) rotation subgroup has Casimir \( L^2 \) representing rotations in the transverse LF plane.
Light-Front Holography: Unique mapping derived from
equality of LF and AdS formula for current matrix elements
Light-Front Holography

• LF Hamiltonian Equation in QCD leads to an effective single-variable radial equation

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta)
\]

• Relativistic, frame-independent, tractable

• Radial variable \( \zeta \) separates dynamics from kinematics of spin and orbital angular momentum

• \( \zeta \) measures invariant separation between constituents of hadron

\[ \zeta^2 = b_{\perp}^2 x (1 - x) \]

• L is orbital angular momentum
Example: Two-parton Pion LFWF

- Hard-Wall Model (P-S)

\[
\tilde{\psi}^{HW}_{qq/\pi}(x, b_\perp) = \frac{\Lambda_{QCD} \sqrt{x(1-x)}}{\sqrt{\pi} J_{1+L}(\beta_{L,k})} J_L \left( \sqrt{x(1-x)} |b_\perp| \beta_{L,k} \Lambda_{QCD} \right) \theta \left( \frac{b_\perp^2}{x(1-x)} \right)
\]

- Soft-Wall Model (K-K-S-S)

\[
\tilde{\psi}^{SW}_{qq/\pi}(x, b_\perp) = \kappa^{L+1} \sqrt{\frac{2n!}{(n+L)!}} \left[ x(1-x) \right]^{L+1} |b_\perp|^L e^{-\frac{1}{2} \kappa^2 x(1-x)} b_\perp^2 L_n \left( \kappa^2 x(1-x) b_\perp^2 \right)
\]

Fig: Ground state pion LFWF in impact space: (a) HW model \( \Lambda_{QCD} = 0.32 \) GeV, (b) SW model \( \kappa = 0.375 \) GeV
\[
\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}\]

\[\frac{d}{d\zeta^2} \equiv \frac{k_{\perp}^2}{x(1-x)}\]

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

\[-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_{\perp}^2 + m_1^2}{x} + \frac{k_{\perp}^2 + m_2^2}{1-x}\]
Result: Soft-Wall LFWF for massive constituents

\[ \psi(x, k_\perp) = \frac{4\pi c}{\kappa \sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2} \left( \frac{k_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right)} \]

**LFWF in impact space: soft-wall model with massive quarks**

\[ \psi(x, b_\perp) = \frac{c \kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2} \kappa^2 x(1-x)b_\perp^2 - \frac{1}{2\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]} \]

\[ z \to \zeta \to \chi \]

\[ \chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right] \]
\[ J/\psi \]

LFWF peaks at

\[ x_i = \frac{m_{\perp i}}{\sum_j m_{\perp j}} \]

where

\[ m_{\perp i} = \sqrt{m^2 + k_{\perp}^2} \]

minimum of LF energy denominator

\[ \kappa = 0.375 \text{ GeV} \]

\[ m_a = m_b = 1.25 \text{ GeV} \]
\[ |\pi^+ \rangle = |ud\rangle \]
\[ m_u = 2 \text{ MeV} \]
\[ m_d = 5 \text{ MeV} \]

\[ |D^+ \rangle = |c\bar{d}\rangle \]
\[ m_c = 1.25 \text{ GeV} \]

\[ |B^+ \rangle = |u\bar{b}\rangle \]
\[ m_b = 4.2 \text{ GeV} \]

\[ |K^+ \rangle = |u\bar{s}\rangle \]
\[ m_s = 95 \text{ MeV} \]

\[ |\eta_c \rangle = |c\bar{c}\rangle \]

\[ |\eta_b \rangle = |\bar{b}b\rangle \]

\[ \kappa = 375 \text{ MeV} \]
Light-Front QCD

$$L_{QCD} \to H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_{i} \left[ \frac{m^2 + k_{\perp}^2}{x} \right]_{i} + H_{LF}^{int}$$

$$H_{LF}^{int}: \text{Matrix in Fock Space}$$

$$H_{LF}^{QCD} |\Psi_{h}\rangle = M_{h}^2 |\Psi_{h}\rangle$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in $x^{-}$. Discrete $k^{+}$; frame-independent truncation

Physical gauge: $A^{+} = 0$
Heisenberg Matrix Formulation

\[ H_{LF}^{QCD} | \Psi_h >= M_h^2 | \Psi_h > \]

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Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Pauli, Hornbostel, Hiller, McCartor, sjb

Vary, Harinandrath, Maris, sjb
Use AdS/QCD basis functions!

Stan Brodsky
SLAC
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances
Hadron Dynamics at the Amplitude Level

• LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.

• Relation of spin, momentum, and other distributions to physics of the hadron itself.

• Connections between observables, orbital angular momentum

• Role of FSI and ISIs--Sivers effect

• Higher Fock States give GMOR Relations, Chiral Symmetry Breaking
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Conformal behavior at short distances + Confinement at large distance

Holography

Boost Invariant 3+1 Light-Front Wave Equations

Integrable!

J = 0, 1, 1/2, 3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

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